MODERN ELECTRICAL ENGINEERING

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PREPARED UNDER
THE EDITORSHIP OF

MAGNUS MACLEAN M.A. D.Sc.

PROFESSOR OF ELECTRICAL
ENGINEERING IN THE ROYAL
TECHNICAL COLLEGE GLASGOW

In Six Volumes: Profusely Illustrated with Diagrams and Plates

VOLUME I

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PREFACE

The success which has attended the two separate issues, in recent years, of *Modern Electric Practice*, has been so gratifying that the publishers have asked me to edit a similar work on *Modern Electrical Engineering*. Accordingly, I have, as on the two previous occasions, obtained the co-operation of contributors, each of whom is an expert in his own department of study and practice.

The separate articles are:—

1. Electric and Magnetic Measurements.
2. Alternating-current Measurements.
3. Continuous-current Generators.
4. Alternating-current Generators.
5. Direct-current Motors.
6. Alternating-current Motors—
   - A. Synchronous Motors.
   - B. Alternating-current Commutator Motors.
   - C. Induction Motors.
   - D. Rotary Converters.
7. Static Transformers.
8. Storage Batteries.
9. Switches and Switch-gear.
10. Electric Mains.
11. High-pressure Transmission of Electrical Energy
12. Electric Lighting and Wiring.
13. Arc Lamps.
14. Incandescent Lamps.
15. Electric Tramways.
16. Electric Traction on Railways.
17. Electrical Appliances in use on Railways.
18. Electric Vehicles.
19. Steam Boilers.

20. Steam-engines and other Prime Movers.
21. Condensing and Auxiliary Plant.
22. Electrochemistry and Electrometallurgy.
23. Telegraphy.
24. Telephony.
25. Electricity in Mining.
26. Electric Cranes.

My cordial thanks are due to the contributors for their hearty co-operation and for their readiness to accept suggestions made with a view to the symmetrical arrangement and completeness of all the articles.

I have also to acknowledge gratefully the willingness with which manufacturers supplied prints, tracings, and blocks of the various pieces of apparatus described in the text.

MAGNUS MACLEAN.

The Royal Technical College,
Glasgow, 1918.

LIST OF CONTRIBUTORS

WM. ALLAN, Assistant Superintending Engineer, Post Office Engineering Department, North Wales District. [§ 24. *Telephony.*]

R. D. ARCHIBALD, B.Sc., A.M.I.C.E., Head of Electrical Engineering Department, Technical College, Dundee. [§ 4. *Alternating-current Generators.*]

FRANK AYTON, Corporation Electricity Department, Ipswich. [§ 18. *Electric Vehicles.*]

F. F. P. BISACRE, B.A.(Cantab.), B.Sc.(London), A.M.I.C.E., 32 Victoria Street, London. [§ 16. *Electric Traction on Railways*; § 17. *Electrical Appliances in Use on Railways.*]

G. G. BRAID, A.M.I.C.E., Tramways Department, Glasgow. [§ 15. *Electric Tramways*, I to IV and VII to IX.]

ALFRED CAMPION, Professor of Metallurgy, Royal Technical College, Glasgow. [§ 22. *Electrochemistry and Electrometallurgy.*]

W. C. CLINTON, B.Sc., M.I.E.E., Assistant Professor of Electrical Engineering, University College, London. [§ 13. *Arc Lamps.*]

FRANK J. HAWKINS, A.M.I.E.E., British Thomson-Houston Co., Ltd., Rugby. [§ 14. *Incandescent Lamps.*]

THOMAS HETHERINGTON, Head Post Office, Glasgow. [§ 23. *Telegraphy.*]

ALEXANDER JACK, M.I.Mech.E. [§ 26. *Electric Cranes.*]

J. J. H. JOHNSTON, M.A., A.M.I.E.E., A.Am.I.E.E., Tramways Department, Glasgow. [§ 15. *Electric Tramways*, V and VI.]

HENRY M'EWEN, Electricity Department, Glasgow. [§ 9. *Switches and Switchgear.*]

D. J. MACKELLAR, B.Sc., Royal Technical College, Glasgow. [§ 1. *Electric and Magnetic Measurements*; § 2. *Alternating-current Measurements*; § 6D. *Rotary Converters.*]

WM. MARDEN, Siemens Brothers, Stafford. [§ 7. *Static Transformers.*]

J. H. MARTIN, Chapel Coal Co., Newmains. [§ 25. *Electricity in Mining.*]

A. L. MELLANBY, D.Sc., Professor of Mechanical Engineering, Royal Technical College, Glasgow. [§ 20. *Steam-engines and other Prime Movers.*]

R. B. MITCHELL, Electricity Department, Glasgow. [§ 10. *Electric Mains*; § 12. *Electric Lighting and Wiring.*]

R. RANKIN, B.Sc., Pritchet & Gold, and Electrical Power Storage Co., London. [§ 8. *Storage Batteries.*]

R. ROYDS, M.Sc., A.M.I.Mech.E., Royal Technical College, Glasgow. [§ 21. *Condensing and Auxiliary Plant.*]

S. P. SMITH, D.Sc., City Guilds (Engineering) College, London. [§ 6A. *Synchronous Motors*; § 6B. *Alternating-current Commutator Motors.*]

J. K. STOTHERT, Babcock & Wilcox, Glasgow. [§ 19. *Steam Boilers.*]

H. VICKERS, M.Sc., British Westinghouse Co., Manchester. [§ 3. *Continuous-current Generators*; § 5. *Direct-current Motors*; § 6C. *Induction Motors.*]

B. WELBOURN, British Insulated and Helsby Cables, Prescot. [§ 11. *High-pressure Transmission of Electrical Energy.*]

CONTENTS

VOL. I

1. ELECTRIC AND MAGNETIC MEASUREMENTS

2. ALTERNATING-CURRENT MEASUREMENTS

3. CONTINUOUS-CURRENT GENERATORS

4. ALTERNATING-CURRENT GENERATORS

I Electric and Magnetic Measurements

CHAPTER I

MECHANICAL AND MAGNETIC UNITS

Introduction.—There can be little doubt that the extremely rapid development of electrical engineering has been in a large measure due to the early adoption of a sound system of units.

The absolute system of units introduced in 1832 by Gauss, and developed by Weber, was made a practical reality in this country through the work of the British Association Committee on Electrical Standards (1861–70).

It was mainly owing to the exertions of Lord Kelvin that this committee was formed. Among engineers of that day there seems to have been a considerable section of opinion in favour of arbitrary practical units. The committee saved electrical engineering from the chaos the adoption of such a system would have involved, by establishing without hesitation the *absolute* system of units. The committee carefully pointed out that the word *absolute* is used in this sense as opposed to the word *relative*, and by no means implies that the measurement is accurately made or that the unit is necessarily perfect, but only that the measurement, instead of being a simple comparison with an arbitrary quantity of the same kind, is made with reference to certain other fundamental units of another kind treated as postulates.

In building up a system of physical units it is found necessary to choose arbitrarily *three* units as a basis of the system; all the other units may then be very simply derived from the three whose magnitudes have been arbitrarily fixed, and which are called the three *fundamental* units. If we attempted to choose more than three, say four, units *arbitrarily*, then we should find that such a random choice of the magnitudes of four units would no longer enable us to maintain the simplicity aimed at, and inconvenient constants would be introduced in referring the other units to the four chosen at random. *Any* three units might be selected as the fundamental units; but, for the sake of simplicity and convenience, the three units which have been adopted as *fundamental units* are those of *length*, *mass*, and *time*. Not only are these units known to everybody, but measurements in terms of them are, in common everyday life as well as in scientific investigations, more frequent than any other measurements. All other physical units may now be derived from the three fundamental units, and are for this reason termed *derived* units.

VOL. I

The units of length, mass, and time finally chosen by the committee were the centimetre, the gramme, and the second respectively. For ordinary use the absolute or C.G.S. electrical units are found to be inconveniently small or large as the case may be. This has led to the adoption of a set of *practical units* each of which is equal to the corresponding C.G.S. unit multiplied by an integral power of 10.

An important property of the C.G.S. system of electrical units may be pointed out in passing: The units bear a direct relation to the mechanical unit of work. Thus, unit current flowing in a conductor of unit resistance does a unit of work (or its equivalent) in unit time.

It is scarcely necessary to emphasize the importance, to the student of electrical engineering, of a sound knowledge of electrical units and their relationships. In the following chapters an attempt will be made to give a connected account of the absolute and practical systems of units, to indicate how the more important electric and magnetic measurements may be carried out, and to describe the instruments used in such measurements.

Mechanical Units (C.G.S. System).—In order to lessen the labour of numerical computation it is very convenient to select a system of physical units in which the relationships are simple and direct, not only between the units themselves, but also between each unit and its multiples used as auxiliary practical units.

The metric system possesses these advantages, and for that reason is used by electrical engineers almost exclusively. By the use of such a system not only is the arithmetical part of a calculation simplified, but in many cases the necessity for remembering constants is done away with. In the more cumbrous British system the relations between the units and their multiples are such that awkward multipliers and divisors are constantly introduced into arithmetical work. Owing to the familiarity of the British system commercially it is occasionally more convenient to express experimental results in British units; for example, the output of a motor would usually be expressed in brake horse-power and not in watts.

Unit of Length.—The C.G.S. unit of length is the centimetre (usually written cm.). This unit was originally intended to represent one-thousand-millionth of the distance along a meridan arc, passing through Paris, from the pole to the equator. It is now defined to be one-hundredth of the distance between two marks on a certain bar of platinum-iridium kept at Paris. The units of area and volume are directly derived from the unit of length. The *unit of area* (1 sq. cm.) is the area of a square whose side is 1 cm. long; and the *unit of volume* (1 cu. cm.) is the volume of a cube whose edge is 1 cm. long.

Unit of Mass.—The unit of mass is the *gramme*, and was originally intended to represent the mass of a cubic centimetre of water at its maximum density. It is now defined to be one-thousandth of the mass of a certain block of platinum kept at Paris.

Unit of Time.—The unit of time is the second and is a certain fraction of the *mean* solar day. The units of velocity and accleration are derived directly from the units of length and time; the *unit of velocity* being 1 cm. per sec., and the *unit of acceleration* 1 cm. per sec. per sec.

Unit of Force.—The *dyne* is the unit of force, and is that force which acting alone upon a mass of *1 gramme* produces in it an acceleration of *1 cm. per sec. per sec.*

It also represents the force which acting alone for *1 sec.* upon a mass of *1 gramme*, originally at rest, imparts to it a velocity of *1 cm. per sec.*

If a mass of 1 gramme be allowed to fall freely under the action of its own weight at the earth's surface, it is found to move with an acceleration of 981 cms. per sec. per sec. Hence the gravitational pull exerted by the earth on a mass of 1 gramme, or the *weight* of a gramme-mass, is equal to 981 dynes. The gramme-weight is the *gravitational* unit of force, and is, as we have just seen, much larger than the absolute unit.

Unit of Work.—The *erg* is the unit of work, and is the work done by a force of *1 dyne* acting in its own direction through a distance of *1 cm.*

The *joule* is an auxiliary practical unit of work and is equal to 10^7 ergs.

Unit of Activity or Power.—The unit of activity is that rate of working in which 1 erg of work is done per sec. The *watt* is an auxiliary unit of activity and is equal to a rate of working of *10^7 ergs per sec.*, or 1 joule per sec.

Relationships between Metric and British Units.—As it is sometimes necessary to express a quantity in terms of the units of the one system when it has, perhaps, been determined in terms of the units of the other system, the relationships between the foregoing metric units (both absolute and practical) and the corresponding British units are set out in tabular form below:—

Unit of.	Metric Unit. Absolute C.G.S.	Metric Unit. Practical.	British Unit. Absolute Foot-Lb.-Sec.	British Unit. Practical.	Relationships.
Length	1 cm.	—	1 foot	—	1 foot = 30.48 cms. : 1 inch = 2.54 cms.
Mass	1 gramme	—	1 pound	—	1 pound = 453.59 grammes.
Force	1 dyne	1 kg.-wt.	1 poundal	1 lb.-wt.	1 poundal = 13,825 dynes.
Work	1 erg	1 joule 1 metre-kg.	1 foot-poundal	1 foot-lb.	1 foot-poundal = 421,399 ergs. 1 foot-pound = 1.356×10^7 ergs. 1 foot-pound = 0.1382 metre-kg. 1 foot-pound = 1.356 joules. 1 metre-kg. = 9.81×10^7 ergs. = 7.23 ft.-lbs.
Activity	1 erg per sec.	1 watt	1 foot-poundal per sec.	1 H.P.	1 horse-power = 746×10^7 ergs. per sec. 1 horse-power = 746 watts. 1 horse-power = 550 ft.-lbs. per sec. 1 watt = 0.738 ft.-lb. per sec. 1 K.W. = 1000 watts. 1 K.W. = 1.34 H.P.

Magnetic Units (C.G.S. System).—The mechanical units already developed may now be applied to the derivation of some of the more important magnetic units. In order to arrive at the definitions of the various quantities a short discussion of magnetic phenomena is necessary.

A magnetized piece of iron or steel possesses the property of attracting pieces of iron, steel, and certain other magnetic materials (cobalt, nickel, and a few compounds). It also possesses the property of taking up a definite position when suspended or pivoted horizontally so as to be capable of rotation about a vertical axis. The end of the magnet which points approximately north is termed the north-seeking or north pole of it, the other end being termed the south-seeking or south pole. The property of attraction is mainly confined to the poles or ends of the magnet. Experiment shows that the north ends or south ends of two magnets repel each other, while a north end attracts a south end.

In order to enable us to study *quantitatively* the laws of magnetic attraction and repulsion, it becomes necessary to define a *unit magnetic pole* (also called unit quantity of magnetism). If we attempt to isolate a single magnetic pole by breaking a magnet, we find that this merely

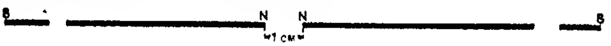


Fig. 1

results in the production of two complete magnets, each with a north and south pole. In order to enable us to study the effects due to a *single* pole, we have to use an extremely long magnet. If then we confine our attention to points in the immediate neighbourhood of one of the poles the effect due to the other may, on account of its distance, be neglected.

Imagine, then, two very long magnets, similar in every respect, placed in line, and with their like poles facing each other, as in fig. 1, and let suitable arrangements be provided for measuring accurately the force exerted between them when the poles are 1 cm. apart.[1] Suppose, further, that by suitable means we are able to vary simultaneously and equally the degree of magnetization of the two magnets, and let this be so adjusted that the repulsion between them amounts exactly to 1 dyne. We then agree to say that each pole is a *unit magnetic pole.* Hence the following definition:—

Unit Magnetic Pole.—*A unit magnetic pole is one which, when placed at a distance of 1 cm. from an equal pole, acts on it with a force of 1 dyne, the intervening medium being supposed to be air.*

If we suppose that a unit pole is placed at a distance of 1 cm. from another pole, and that the force exerted between them amounts to m dynes, we should say that the strength of the second pole is m. From this definition it follows that the force exerted between two poles varies as the product of their pole-strengths.

[1] The medium surrounding the magnets is supposed to be air, and their lengths are assumed to be very great in comparison with 1 cm., so that the action due to the distant poles may be left out of consideration.

As a result of experiment, we find that the force exerted between two poles of given pole-strength varies *inversely as the square of the distance between them.*[1]

Inverse Square Law.—From the above it is seen that magnetic attractions and repulsions follow an inverse square law, which may be expressed algebraically as follows:—

$$f = \frac{m_1 m_2}{r^2}.$$

Where f = mutual force exerted between the two poles of strengths m_1 and m_2 respectively,

r = distance from one pole to the other.

It must be noted that this form of the equation only applies to non-magnetic media; but this is of little importance practically, since all fluid media are non-magnetic.

Magnetic Moment.—The magnetic moment of a magnet is defined to be the product of the pole-strength into the distance between the poles.

Intensity of Magnetization.—The intensity of magnetization of a piece of material is defined to be the magnetic moment per unit of volume. As an example, let us consider the case of a straight bar magnet:—

Let m = pole-strength of magnet,
l = distance between the poles,
V = volume of the magnet,
I = intensity of magnetization,
M = moment of the magnet.

$$\text{Then } M = ml,$$

$$\text{and } I = \frac{M}{V} = \frac{ml}{V}.$$

If the magnet is such that the poles may be assumed to reside at the ends, then the distance "l" between the poles will be equal to the total length of the magnet. If "a" is the cross-section of the magnet, then—

$$V = al,$$

$$\text{and } I = \frac{ml}{al} = \frac{m}{a}.$$

Or the intensity of magnetization is equal to the pole-strength per unit area of cross-section of the magnet.

Magnetic Force.—A unit pole placed at any point in a magnetic field will experience a force of definite magnitude and in a definite direction. This force is defined to be the *magnetic force*[2] at that point.

[1] The law of the inverse square of the distance is sometimes called the *law of nature*, on account of its very general applicability. It holds not only for magnetic, but also for gravitational and electric attractions and repulsions, and in other cases. The researches which led up to the experimental verification of this law are fairly numerous, the most important being those of Coulomb and Gauss, especially the latter. A detailed account of Gauss's proof will be found in J. J. Thomson's *Elements of the Mathematical Theory of Electricity and Magnetism* (Cambridge University Press).

[2] The term "magnetic intensity" is sometimes used instead of magnetic force, but its use is not to be recommended, one obvious reason being the probability of confusion with "intensity of magnetization".

Magnetic Field-strength.—A *magnetic field* is defined to be a region throughout which magnetic force exists. The magnitude of the magnetic force at any point is a measure of the *magnetic field-strength* at that point.

Lines of Magnetic Force.—The conception of lines of magnetic force is of importance, since it enables us to visualize a magnetic field. A line of magnetic force may be defined as being an imaginary line which by its direction at any point indicates the direction of the magnetic force at that point, and which is in a state of longitudinal tension and lateral compression. Clearly any number of lines of magnetic force could be drawn in any magnetic field. For simplicity in calculation it is agreed to limit the number to *one per unit area per unit of field-strength* at any point in the field, the unit area, 1 sq. cm., being taken at right angles to the direction of the lines of force. It follows, therefore, that in a uniform magnetic field of unit strength there is one line of force per unit area, or the number of lines of force per unit area at any point in any magnetic field is numerically equal to the field-strength at that point, it being understood that the unit area is always taken at right angles to the lines of force.

Number of Lines of Force due to a Unit Pole.—Since the mutual force existing between two unit poles placed near one another acts in the direction of the straight line joining the two poles, it follows that the lines of force issuing from or entering into a unit pole are everywhere perpendicular to the surface of a sphere circumscribed about the unit pole. Imagine that a sphere of 1 cm. radius is circumscribed about the unit pole. Every line of force must pass through the surface of this sphere, and, as we have just seen, every line will be perpendicular to the surface. The field strength at all points on the surface of the sphere is unity, since from the definition of unit pole the force exerted on another unit pole at unit distance is unity. Also, since one line of force corresponds to unit field strength, it follows that one line of force passes through each square centimetre of the surface of the sphere.

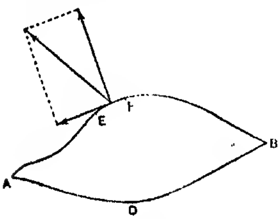


Fig. 2

The area of a sphere of 1 cm. radius is 4π sq. cms. Hence *the total number of lines issuing from*[1] *or entering into the unit of pole is* 4π.

Magnetomotive Force and Difference of Magnetic Potential.—The following definitions are very important, and familiarity with them is essential to a clear understanding of magnetic-circuit problems.

Consider any magnetic field in which two points A and B are connected by *any* closed curve such as A C B D in fig. 2.

Let a unit pole be placed at A, and carried from A round the closed curve until A is reached again. The unit pole will experience a magnetic force at every point on the curve, and consequently in passing from A to B a certain amount of work will be done on or by the unit pole. When

[1] If the lines of force issue from the pole, it is a north pole; if they enter into the pole, it is a south pole.

A has been reached again, the conditions will be the same as they originally were, and no gain or loss of energy will have taken place. Consequently the work done in passing from A to B is equal and opposite to the work done in passing from B to A. Hence the work done in conveying a unit pole from A to B is *independent of the path* taken by the unit pole. The total work done in conveying a unit pole once round a closed curve in a magnetic field is defined to be the *magnetomotive force* round that curve. The magnetomotive force round any closed curve in any magnetic field, other than one produced by an electric current, is zero. A knowledge of the magnetomotive force produced by a given current flowing in a given coil of wire is of the utmost importance in magnetic-circuit calculations, and will be treated in detail in Chapter III.

The work done in carrying the unit pole from A to B is defined to be the *difference of magnetic potential* between A and B. In order to determine the total amount of work done, we may imagine the path ACB divided up into a large number of small parts or elements, such as EF. By resolving the magnetic force into two components, one along and the other at right angles to EF, and taking the product of the length of EF and the component of magnetic force along it, we get the work done in moving over the distance EF. The sum of all such products will give the magnetic potential difference between A and B. It is obvious from this that *the mean value of the magnetic force along any curve is equal to the difference of magnetic potential between its ends, divided by the length of the curve.* If work has to be done by a unit *north* pole in overcoming the magnetic forces between A and B, then B is said to be at a *higher* potential than A. On the other hand, if the unit north pole has work done upon it by the magnetic forces—i.e. if the unit pole tends to move of itself from A to B—B is said to be at a *lower* potential than A. Hence a unit north pole tends to move from places at a higher to places at a lower magnetic potential.

The C.G.S. unit of magnetomotive force (more usually written M.M.F.) is identical with that of difference of magnetic potential, and is equal to 1 erg per unit pole.

CHAPTER II

THE ELECTRIC CURRENT AND ITS EFFECTS

Effects of an Electric Current.—A discussion of modern views on the passage of electricity through metallic conductors would be out of place here, and it must suffice to state that, if a difference of electric potential be maintained between the ends of a metallic conductor, what is called an electric current will flow in the conductor.

The current is found to be capable of producing several important effects.

The Magnetic Effect.—The space surrounding a current-carrying conductor is the seat of magnetic forces. A magnetic needle placed near

and parallel to a conductor will, when a current is passed through the conductor, tend to set itself at right angles to the conductor.

This phenomenon was first observed by the Danish philosopher Oersted in 1820, and is historically interesting as being the first indication of a direct relation between electricity and magnetism. In fig. 3 a simple apparatus is shown by means of which Oersted's experiment may be demonstrated.

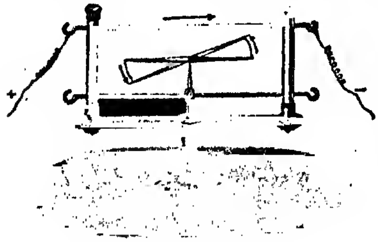


Fig. 3.—Oersted's Experiment

The lines of magnetic force set up by the electric current are, in the case of a long straight conductor, concentric circles, whose centres lie along the axis of the conductor.

The direction of the current and the direction of the lines of magnetic force have a definite relation to one another. Various rules have been given for rapidly determining this relation, but perhaps the most generally useful is the "*corkscrew rule*", which may be stated as follows: *The direction of the current and the direction of the lines of magnetic force are related in the same way as the advance and rotation of a right-handed corkscrew.*

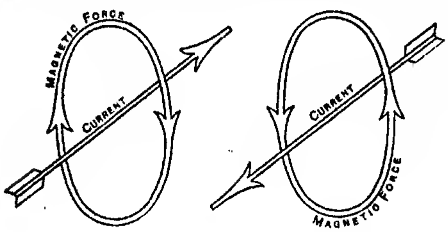


Fig. 4

As an example of the application of this rule we may refer to fig. 3. In driving the screw from left to right (i.e. in the direction of the current as shown in the figure) the rotation will be clockwise, indicating that the lines of magnetic force pass down in front of the wire and into the plane of the paper below the wire. Consequently the north pole of the needle will be urged into the plane of the paper; i.e. looking at the needle from above, it will appear to be deflected in a counter-clockwise direction.

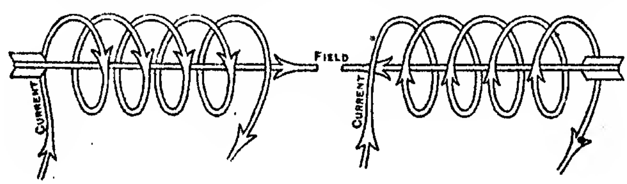


Fig. 5

The practical applications of the magnetic effect of a current are of the highest importance in electrical engineering. The generation of electrical

power on a large scale is only rendered possible by the use of an electric current to produce the magnetic field required in the generators.

Electric motors also depend for their operation on an electrically produced magnetic field.

Many electric control mechanisms consist of a solenoid and soft-iron plunger. When a current is passed through the solenoid the plunger is drawn into the strongest part of the magnetic field produced.

Electromagnets are largely used in ironworks for the handling of scrap-iron, &c.

The magnetic effect provides a means of detecting the presence of an electric current, and suitably contrived instruments, such as mirror galvanometers can be made highly sensitive, so that exceedingly small currents can be easily measured.

Heating Effect.—When an electric current flows in a conductor heat is produced in the conductor.

The amount of heat produced depends on the magnitude of the current and the resistance of the conductor. By suitably adjusting these quantities high temperatures may be obtained.

In practice, advantage is taken of the heating effect of an electric current to obtain the necessary temperature in electric heating and cooking apparatus, in electric lamps, and in electric furnaces, welders, &c.

Chemical Action of a Current.—Certain liquids are capable of conveying an electric current, but their behaviour differs from that of metallic conductors, inasmuch as the liquid undergoes a definite chemical decomposition during the passage of the current.

These liquids are called *electrolytes* and are chemical *compounds* either in the liquid state or in solution in some solvent. The metallic conductors immersed in the liquid, and conveying the current into and out of the electrolyte, are called *electrodes*; the one by which the current enters being called the *anode*, and that by which it leaves, the *cathode*.

The view commonly held at present regarding electrolysis may be briefly stated as follows: When a salt is melted or dissolved the metallic and non-metallic atom groups which in the solid state are bound together by electric forces are to a certain extent dissociated, i.e. the metallic and non-metallic groups are free to move about among one another. The extent of this dissociation in the case of solutions depends on the nature of the solvent and on the relative amounts of the salt and the solvent. Of known solvents water produces the greatest degree of dissociation. These dissociated atom groups are called *ions*, and they each carry a definite electric charge. The ions exhibit none of the ordinary chemical properties, but do so as soon as they give up their electric charge and *cease to be ions.*

The metallic ions (or ions behaving as metallic ions) carry a *positive* charge, while the non-metallic ions carry a *negative* charge. If two electrodes are introduced into an electrolyte, one being maintained at a certain positive potential and the other at a certain negative potential, then the positively charged ions will pass to the negative electrode (cathode), while the negatively charged ions will pass to the positive

electrode (anode).[1] If the ions did not give up their charge on reaching the electrodes the action would soon cease. However, when the ions touch the electrodes they do give up their charge, *cease to be ions*,[2] and appear in their free chemical state. Fresh ions are formed in the electrolyte and the action continues. The double procession of ions to the electrodes constitutes an electric current passing through the solution from the anode to the cathode. It frequently happens that the primary products of the electrolysis of a salt do not appear at the electrodes, since they react with either the material of the electrodes or the electrolyte itself and form other substances.

This secondary action is purely chemical.

Laws of Electrolysis.—The weight of a substance liberated from an electrolyte when a *constant* current is flowing through the electrolyte is proportional to the *time* during which the current flows. Further, if currents of different strengths are allowed to flow during equal intervals of time through any given electrolyte, the weights of either product liberated are proportional to the *strengths of the currents*. The total amount of chemical decomposition in any given electrolyte is therefore proportional to the product of the *current* and the *time* during which it flows, or, in other words, to the *quantity of electricity* which has passed through the electrolyte. This result was first established by Faraday, and is known as Faraday's First Law of Electrolysis. The weight of a substance[3] liberated by a given quantity of electricity depends on the chemical equivalent of the substance. The *electrochemical equivalent* of a substance is defined to be the weight of that substance liberated by the passage of one practical unit of quantity (1 coulomb) through the electrolyte. Faraday's Second Law of Electrolysis states that *the electrochemical equivalent of a substance is proportional to its chemical equivalent.* Thus, if the electrochemical equivalent of *one* element is carefully determined once and for all, the electrochemical equivalents of all the other elements (and atom-groups) can easily be calculated from their valencies and the known atomic weights of the elements. If the electrochemical equivalent of a substance is known, we have a means of measuring a current by determining the amount of that substance liberated in a given time, and dividing this by the product: time × electrochemical equivalent.

The most suitable electrolytes for this purpose are silver nitrate ($AgNO_3$) and copper sulphate ($CuSO_4$). The former may be used for the measurement of small, and the latter of large currents. An arrangement which enables a current to be measured by means of its chemical effect is termed a *voltameter*.

This will be referred to again later, in dealing with the calibration of ammeters.

The determination of the electrochemical equivalent of an element is one of the most important measurements in physics, since it provides a

[1] This only takes place when the difference of potential between the electrodes exceeds a certain amount.

[2] It should carefully be noted that an ion is an atom group *carrying an electric charge.*

[3] The word "substance" must be understood as m[illegible]g "an element *or* atom group".

cheap and comparatively simple method of determining currents in absolute measure.

Hence this determination has formed the subject of numerous researches, one of the most important of which is that carried out by Lord Rayleigh and Mrs. Sidgwick, who, using an absolute current balance, determined the electrochemical equivalent of silver. The value arrived at by them was about .001118 gramme per coulomb (or .01118 gramme per absolute unit of quantity).

The practical applications of the chemical effect of an electric current are very important commercially. Electrolysis is largely employed in the refining of metals, more especially copper. Electrolytically deposited copper is remarkable for its purity and high conductivity. Electroplating, electrogilding, and the production of electrotype plates are also important applications.

Definition of C.G.S. Unit of Current.—The magnitude of a current is taken to be proportional to the magnetic force which it produces. Thus, if a certain current flowing in a given coil produces a definite magnetic force at a given point, and if we increase the current until the magnetic force at the same point is doubled, we agree to say that the current in the second case is twice as great as in the first.

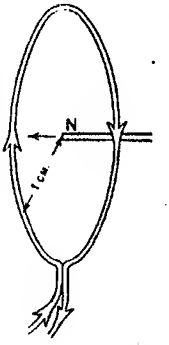


Fig. 6

The C.G.S. or "absolute" unit of current is arrived at as follows: Imagine a circular conductor, of 1 cm. radius, as shown in fig. 6, to be conveying a current whose magnitude may be varied at will, and let a unit north pole be placed at the centre of the circular conductor. The force acting on this pole will, by a simple application of the right-handed-screw rule, be from right to left, as shown in the sketch. Let us now suppose that the magnitude of the current is adjusted until the force acting on the unit pole becomes equal to 2π dynes.[1] The current is then said to be a unit C.G.S. current. We thus have the following definition:—

The absolute unit of current is a current which, flowing round a circular conductor of 1 cm. radius, produces a magnetic force of 2π dynes at the centre of the circle.

At first sight it might appear as if a more convenient constant than 2π—say unity—were preferable.

The reason for choosing 2π is that (since the total length of the conductor is 2π cms.) the magnetic force at the centre, corresponding to 1 cm. of the conductor, is equal to 1 dyne.

The definition of the *C.G.S. unit of quantity* follows at once from the definition of the C.G.S. unit of current. The C.G.S. unit of quantity is defined to be that quantity of electricity which has passed round a circuit when *one C.G.S. unit of current* has been flowing in the circuit for *one second.*

[1] The force is supposed to be that due solely to the current in the circular conductor; the earth's field, or any other magnetic fields, being supposed to be either non-existent or suitably compensated.

Practical Realization of the C.G.S. Unit of Current.—The arrangement described in connection with the definition of the unit of current is not one which could be conveniently employed for the experimental determination of currents in absolute measure. For this purpose several methods may be employed, and these are diagrammatically represented in fig. 7. At (*a*) we have a *standard galvanometer*, consisting of two fixed coils, C_1 and C_2, placed parallel to one another and at a distance apart equal to their mean radius; between them is a suspended or pivoted magnetic needle. If the dimensions of the coils and number of turns in them are known, the value of the current in absolute measure may be calculated from the observed deflection of the needle, provided the value of the horizontal component of the earth's field is known; this latter absolute measurement must thus precede the current measurement. At (*b*) we have a *standard electrodynamometer*; this differs from (*a*) in having a

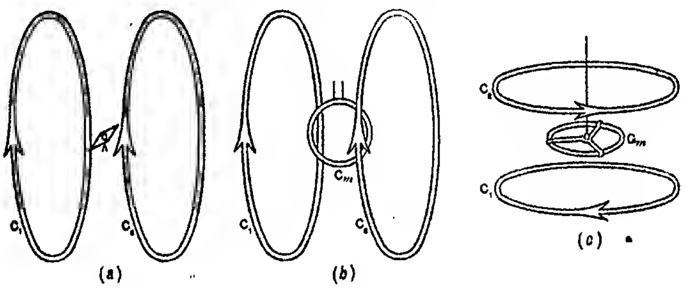


Fig. 7

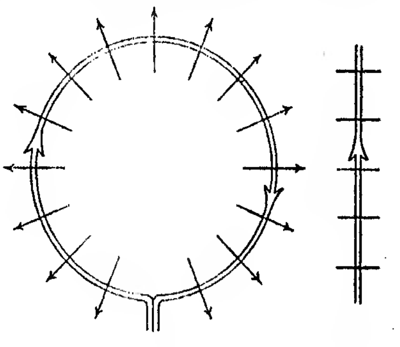
small suspended coil, C_m, instead of a needle. At (*c*) is shown the most satisfactory method for the absolute measurement of current. The instrument, known as a *current balance*, consists of two fixed horizontal coils, C_1 and C_2, and a smaller movable coil, C_m, which is suspended from the beam of a balance. The three coils are connected in series, and the directions of the currents are indicated by arrows. A laborious mathematical calculation enables the pull exerted by the fixed on the movable coil to be determined in terms of the current and constants of the coils (their dimensions and number of turns); and if this pull be experimentally found, by ordinary weighing, we have a means of finding the current.

Instruments such as those briefly described above are expensive to construct and require a considerable amount of care and skill in their use. They form the ultimate or primary standards which have enabled us to realize experimentally the theoretically defined absolute unit of current.

Definition of the Practical Unit of Current.—For practical purposes it is found more convenient to adopt a unit of current smaller than the C.G.S. unit. This unit is called the *ampere*, and has the value 10^{-1} in terms of the C.G.S. unit. It may be represented with sufficient accuracy for practical purposes by that unvarying current which, when passed

through a solution of silver nitrate under certain specified conditions, deposits silver at the rate of 0.001118 gramme per second.

Force exerted on a Current-carrying Conductor lying in a Magnetic Field.—Let us consider the special case taken as a basis for the definition of the C.G.S. unit of current. The force acting on the unit pole and the circular conductor is a mutual one. The current-carrying conductor tends to move in one direction, while the unit pole tends to move in the opposite direction. The only effect of the unit pole is to produce a magnetic field which at any point in the conductor is at right angles to the tangent at that point. Also, since every point in the conductor is at unit distance (1 cm.) from the unit pole, the field-strength (due to the pole) is unity at every point in the conductor. The total force acting on the conductor is 2π dynes, and, since the total length of the conductor is 2π cms., it follows that the force per cm. of the conductor is 1 dyne. The direction of the force is perpendicular both to the conductor and to the magnetic field.



(a) (b)

Fig. 8

Clearly, since this force is solely due to the presence of the unit pole, or rather to the presence of a magnetic field of unit strength and everywhere perpendicular to the conductor, the force will not be in any way altered if the conductor is straightened out (as in fig. 8*b*) and placed in a magnetic field of unit strength which is everywhere perpendicular to the conductor. We thus arrive at the following important result:—

A conductor one centimetre long and carrying a unit current, when placed in a magnetic field of one line per square centimetre, which is everywhere perpendicular to the conductor, experiences a force of one dyne, which is in a direction at right angles both to the conductor and to the magnetic field.

If the current be I C.G.S. units, the length of the conductor l cms., and the strength of field H C.G.S. units, then the force f acting on the conductor is HlI dynes.

The direction of the force may be readily determined from a consideration of the shape of the lines of force surrounding the conductor. Fig. 9*a* represents the lines of force due to the current[1] alone. If the direction of the magnetic field is vertically downwards, then fig. 9*b* represents the form of the resultant field surrounding the conductor.

Remembering the inherent properties of longitudinal tension and lateral

[1] ⊕ represents a current flowing away from the observer. ⊙ represents a current flowing towards the observer.

compression of the lines of force, we see at once that they will tend to straighten themselves.

In order to permit of this the conductor will require to move from right to left. The direction of the force acting on the conductor is thus from right to left.

The lines of force shown in the accompanying diagram are not in

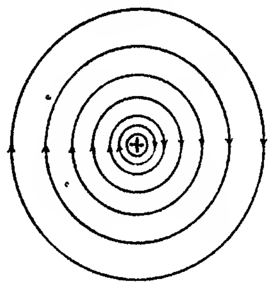


Fig. 9a

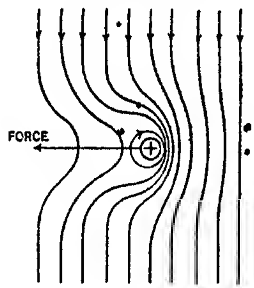


Fig. 9b

any way to be taken quantitatively. The diagram is merely intended to represent the general form of the field.

Left-hand Rule.—A very convenient rule for determining the relation connecting the direction in which the conductor tends to move with the direction of the field and current has been given by Dr. Fleming and is illustrated in fig. 10. If the thuMb, the Forefinger, and the mIddle finger of the *left* hand be held so as to form a system of three mutually perpendicular lines, the forefinger pointing along the direction of the magnetic field, and the middle finger along that of the current, then the thumb will indicate the direction of the force acting on the conductor.

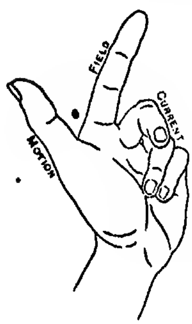


Fig. 10

Work done in increasing the Magnetic Flux linked with an Electric Circuit.—If a straight conductor of length l cms., carrying a current of I C.G.S. units, and lying in a magnetic field of strength H (so that its length is at right angles to the field) is moved parallel to itself through a distance D cms. in a direction opposed to that of the force f acting on it, then a certain amount of work will be expended in moving the conductor.

$$\text{Since } f = H l I \text{ dynes.}$$
$$\text{Work done} = H l I D \text{ ergs.}$$

Now $D l$ is the area of the rectangle swept out by the conductor during the motion, and since H also represents the number of lines of magnetic force per sq. cm. of that area, $H l D$ is equal to the number of lines of force (or the magnetic flux) cut by the conductor.

Hence, *the work done is equal to the current multiplied by the total magnetic flux cut by the conductor.*

This result is quite general, and holds also for any electric *circuit* in which the *linkages* of lines of magnetic force with the circuit is increased, whether by increasing the strength of the field, by mechanical displacement of the circuit, or by enlargement of the circuit.

The effect is also reversible: that is to say, if the flux linkages are decreased instead of increased, there will be a gain instead of an expenditure of energy.

CHAPTER III

FUNDAMENTAL M.M.F. AND E.M.F. THEOREMS

Fundamental M.M.F Theorem.—The following theorem is of great importance, inasmuch as it supplies the ultimate basis for the various magnetic-circuit calculations which constantly occur in the design of electrical apparatus.

Let us take the case of a simple circuit, such as A B C (fig. 11), carrying a current of I C.G.S. units, and consider any closed curve D E F, which is so chosen as to be *linked* with the electric circuit.

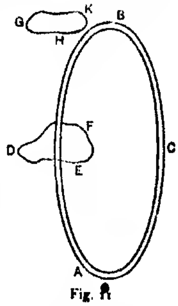


Fig. 11

Let a unit magnetic pole be carried round the closed curve. By the definition of M.M.F. the work done in carrying the unit pole round D E F gives the M.M.F. round that curve. Now, as the pole is carried round the curve, every magnetic line due to the pole will cut the circuit, and the total number of lines cut will be equal to that due to the unit pole, i.e. to 4π. But in order to make the circuit cut 4π lines an expenditure of energy is required of amount 4π I. Hence the M.M.F. round the closed curve is equal to 4π I.[1]

The result obtained, it will be observed, is quite independent of the *shape* of the curve, since the reasoning employed does not involve a consideration of the shape. The essential point is the *linkage* of the closed curve with the circuit; for if we consider a closed curve, such as G H K, which is *not* linked with the circuit, then although, in carrying a unit pole round it, the lines of force cut the circuit in a certain direction during part of the journey, they cut it (or sweep across it) *in the opposite direction* during the remainder of the journey. Thus, if the first part of the journey involves an *expenditure* of energy, the second corresponds to a *gain* of energy, the total gain or loss of energy becoming zero, i.e. the M.M.F. round the closed curve vanishing.

[1] The algebraical sign of the M.M.F. is determined by the direction followed in going round the curve, a positive sign being taken if the M.M.F. corresponds to a *gain* of energy, and a negative sign if it corresponds to an *expenditure* of energy.

If instead of a simple circuit we consider a coil of S turns conveying a current I, and determine the M.M.F. round any closed curve which is linked with *every* turn of the coil, such as the curve K L M in fig. 12, then since the work done in carrying a unit pole round a single turn of the coil is $4\pi I$, and since the coil consists of S turns, the total work done in going round K L M is $4\pi I S$.

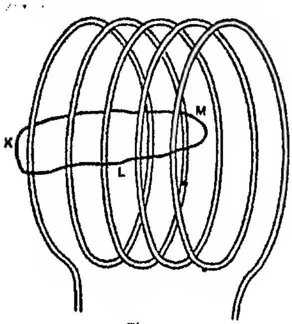


Fig. 12

The expression I S still denotes the total flow of current through the closed curve (or total current linked with it). The relation established is a perfectly general one, and may be briefly put thus:—

The M.M.F. round any closed curve is equal to 4π times the total current linked with it.

In practical calculations, although the M.M.F. is expressed in C.G.S. units, the current is invariably taken in practical units (amperes). Since the ampere is equal to 10^{-1} C.G.S. units,

$$\begin{aligned} \text{M.M.F. (in C.G.S. units)} &= \frac{4\pi}{10} X \\ &= 1.257\ X. \\ \text{Conversely } X &= \frac{1}{1.257}\ \text{M.M.F.} \\ &= 0.8\ \text{M.M.F. (approx.).} \end{aligned}$$

Where X is the product of the number of turns and the current in *amperes*, and is known as the *ampere-turns* of the coil.

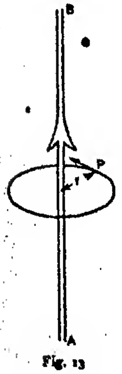


Fig. 13

Applications of M.M.F. Theorem.—The solutions of the following problems are arrived at directly from the preceding theorem and serve to illustrate its wide applicability.

1. To find the magnetic field strength at any point outside a long straight conductor carrying a current of I C.G.S. units.

In fig. 13, let A B be the conductor, and P the point at which the field strength is required. Imagine a circle to be drawn through P with its centre on the axis of the conductor and its plane perpendicular to it. The lines of magnetic force due to the current are concentric circles having their centres on the axis of the conductor, and consequently the field-strength at every point on the circle through P is the same.

Let H equal this field-strength, and let a unit pole be carried once round this circle. The force acting on the unit pole is H.

$$\text{Hence, work done} = H \times 2\pi r,$$

where r is the distance of P from the axis of the conductor.

From the definition of M.M.F.

$$\text{M.M.F.} = \text{work done} = 2\pi r \text{H}.$$

But by the M.M.F. theorem

$$\text{M.M.F.} = 4\pi \text{I}.$$

The two expressions for the M.M.F. must be equal.

$$\therefore 2\pi r \text{H} = 4\pi \text{I},$$

$$\text{and H} = \frac{2\text{I}}{r}.$$

From the above equation we see that the field-strength at any point varies directly as the current, and inversely as the distance of the point from the conductor.

Since a unit pole placed in a unit field experiences a unit force, the magnetic force at P is equal to $\frac{2\text{I}}{r}$.

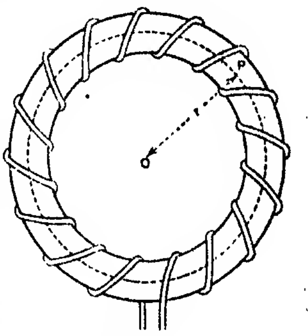


Fig. 14

2. To find the magnetic field-strength at any point inside a coil wound in the form of an anchor-ring.

In fig. 14, let P be the point at which it is required to determine the field-strength. Imagine a circle to be drawn through P with its centre on the axis of the ring and its plane perpendicular to it. As before, the field-strength is the same at every point on this circle. In carrying a unit pole once round this circle $2\pi r$H ergs of work will be done, where r is the radius of the circle.

$$\text{Hence M.M.F.} = 2\pi r \text{H}.$$

$$\text{Also M.M.F.} = 4\pi \text{S I}.$$

$$\therefore \text{H} = \frac{2\text{S I}}{r}.$$

The field-strength is thus not constant over the cross-section of the ring, but diminishes as the distance from the centre of the ring increases.

In the case of a ring of large diameter in comparison with the diameter of its cross-section the field-strength may be taken as uniform over the cross-section without introducing any serious error.

3. To find the magnetic field-strength at any point well inside a straight coil of which the length is great compared with its diameter.

Such a coil or solenoid may be considered as part of an anchor-ring coil of infinite radius. From the result already obtained for the anchor-ring coil

$$\text{H} = \frac{4\pi \text{S I}}{2\pi r}.$$

This may be written in the form

$$H = 4\pi I \frac{S}{2\pi r}.$$

Now $\frac{S}{2\pi r}$ *is* the number of turns S_1 per unit length of the solenoid.

$$\text{Or } H = 4\pi S_1 I.$$

So long, therefore, as S_1 is constant, H is independent of the value of r. We may now suppose r to increase indefinitely; then any finite portion of the ring, of length great in comparison with the cross-section, may be regarded as approximately straight, forming a *solenoid.* If—which is permissible in the case of a very long solenoid—we neglect the effect of the distant portions of the anchor-ring, we may suppose these to be removed without materially affecting the intensity inside the solenoid at points not near the ends. We thus get $H = 4\pi S_1 I$, approximately, for points well inside the solenoid.

If, in the last problem, the current be measured in amperes, we see that $H = 1.257$ times the ampere-turns per cm. length of the solenoid.

Electric Force.—It will be convenient at this point to introduce a conception regarding electricity analogous to the conception of a unit magnetic pole, which we have already made use of in dealing with magnetic problems.

This conception is that of a *unit electric charge.* Imagine a spherical conductor of indefinitely small dimensions. By suitable means allow 1 C.G.S. *electromagnetic*[1] unit quantity of electricity to be conveyed on to the conductor.

The conductor now carries a unit charge which is positive or negative according as the current flowed towards or away from the conductor during charging.

Since the dimensions of the conductor are indefinitely small, the unit charge may be considered to be concentrated at a point.

If such a unit point-charge be brought into the neighbourhood of any other charged conductors, it will experience a definite force acting in a definite direction, and this force (measured in dynes) gives the value of the *electric force* at the point where the unit charge is placed. Any region throughout which electric intensity is exerted is spoken of as an *electric field.*

Definition of E.M.F. and P.D.—The definitions of electromotive force or E.M.F. and of difference of electric potential or P.D., are analogous to the definitions of M.M.F. and difference of magnetic potential, and are arrived at in a similar way.

Consider any two points A and B, as in fig. 2, lying in an electric field, and imagine these points to be connected by any closed curve A C B D. Let a unit charge be placed at A and carried round the curve A O B D until A is again reached. In moving the unit charge from one point to a neigh-

[1] It must be understood that the electromagnetic system of units is being used throughout. We are not here concerned with the electrostatic system, and to prevent confusion all mention of this system has been avoided.

bouring one a certain amount of work will be done on or by the unit charge.

The total work done in transferring the unit charge from A to B is defined to be the difference of electric potential or P.D. between A and B. If the unit charge is a positive one, and work is done *by* it in moving from A to B, B is said to be at a higher potential than A. Conversely, if work is done *on* the unit charge, B is said to be at a lower potential than A.

The electromotive force, or E.M.F., round the curve is defined to be the work done in carrying a unit charge once round the curve. In this case the E.M.F. is clearly zero, since when the unit charge has returned to A the conditions are exactly as they were when the unit charge started from A, and there has been neither a gain nor an expenditure of energy.

In the case of a closed electric circuit in which an E.M.F. is being generated, the work done in carrying a unit charge once round the circuit is not zero, but is equal to the E.M.F. generated in the circuit.

The terms E.M.F. and P.D. are frequently used loosely as equivalent expressions, but they should at all times be sharply distinguished. If the definitions are borne in mind there is no possibility of confusion arising.

Definition of the C.G.S. Unit of P.D.—In 1831 Faraday discovered that if a conductor be moved in a magnetic field in such a way as to cut the lines of force, a P.D. is produced between the ends of the conductor, the magnitude of the P.D. depending directly on the *rate at which the lines of force are cut.*

The C.G.S. unit of P.D. is defined on this basis as being equal to *the P.D. produced in a conductor which is cutting lines of magnetic force at a uniform rate of one line per sec.*

This unit is such that if a unit charge (electromagnetic) be carried from one end of the conductor to the other, 1 unit of work is done, so that alternatively the C.G.S. unit of P.D. might be defined as the P.D. existing between two points when 1 erg of work is done in transferring 1 C.G.S unit quantity from the one point to the other.

The C.G.S. unit of E.M.F. is equal to the C.G.S. unit of P.D., and is similarly defined.

Practical Realization of the C.G.S. Unit of P.D.—No direct means exists for the practical realization of the C.G.S. electromagnetic unit of P.D. The units of current and resistance can be so realized, and consequently the unit of P.D. can, as will be seen later, be realized as the product of these units. The following indirect method[1] has been employed. It involves the measurement of a magnetic moment.[2]

Let a permanent magnet of known magnetic moment be mounted inside a solenoid so as to be capable of rotation about an axis passing through its middle point and perpendicular to the axis of the solenoid and that of the magnet. If the magnet be rotated, its lines will cut the various turns of the solenoid, inducing in the latter an alternating E.M.F.

[1] This method was employed by C. Limb (*Journal de Phys.*, v., pp. 61–70, 1896) to determine in absolute measure the E.M.F.s of a Clark and a standard Daniell cell.

[2] The determination of the magnetic moment of a magnet is easily carried out by a method due to Gauss; the horizontal intensity of the earth's field being determined at the same time.

(i.e. one whose direction is periodically reversed). For a certain position of the rotating magnet, the rate at which its lines cut the solenoid becomes a maximum. This maximum rate of cutting, or maximum E.M.F., may be calculated from the known moment of the magnet, its speed of rotation, and the size and shape of, and number of turns in, the solenoid.

Such a standard of E.M.F. would not, however, be very convenient, and in practice the standards of E.M.F. take the form of primary cells whose E.M.F. at a given temperature may be relied on to have a *definite* value which does not change in the course of time. The E.M.F. of such a constant or standard cell may be compared, by suitable experimental arrangements, with the maximum value of the alternating E.M.F. induced in the solenoid by the rotating magnet, and thus the value of the E.M.F. of the cell may be obtained in absolute units.

Practical Unit of P.D.—The C.G.S. unit of P.D. is too small to be convenient for practical use, and the *volt*, having a value of 10^8 in terms of the C.G.S. unit, has been adopted.

The volt is represented with sufficient accuracy for practical purposes by $\frac{1}{1.0184}$ of the E.M.F. of a standard Weston cell at a temperature of 20° C.

Formerly the Clark cell was specified, and the volt was taken as $\frac{1}{1.434}$ of the E.M.F. of this cell at a temperature of 15° C.

Fundamental E.M.F. Theorem.—When the number of lines of magnetic force linked with an electric circuit is changed, an E.M.F. is induced in the circuit.

This E.M.F. lasts only so long as the number of lines is varying, and disappears as soon as that number becomes constant. Imagine that into a closed circuit which originally contains no E.M.F. there are introduced N lines during t secs., and that the average current I is due to an average induced E.M.F. of amount E. The mechanical energy expended in introducing N lines into a circuit in which the current is I is given by N I (see Chapter II). But since the introduction of the N lines has taken t secs., the average power is $\frac{N I}{t}$. This power is transformed into electrical power, and must be equivalent to this latter.

As will be seen in the next paragraph, the electrical power is given by the product E I.

$$\text{Hence } E I = \frac{N I}{t},$$

$$\text{or } E = \frac{N}{t}.$$

In other words, *the E.M.F. induced in a circuit by variations in the number of magnetic lines passing through the circuit is equal to the rate at which the number of lines is changing.*

In cases where the change in the number of lines is produced by the displacement of a circuit in a magnetic field in such a manner that one of the conductors forming the circuit is made to cut the magnetic lines,

while the motion—if any—of the remainder of the circuit does not result in any cutting of lines, the rate at which the number of lines is changing is equal to that at which the *active* conductor is cutting lines. We may then regard the active conductor as being the seat of an E.M.F., and say that the E.M.F. induced in the conductor is equal to the rate at which it is cutting lines.

Lenz's Law.—*The direction of the E.M.F. induced by a change in the magnetic flux linked with a circuit is always such as to tend to oppose the change.*

Thus, if the change consists in an *increase* of the magnetic flux linkages the E.M.F. will be in such a direction as to produce a current (if the circuit is closed) which gives a magnetic flux *in the opposite direction to the original flux*, thus tending to prevent the increase in flux linkages.

This law provides us with a simple means of predetermining the direction of the induced E.M.F. in a coil or solenoid when the flux linkages with it are changed. In the case of a dynamo it is simpler to consider each conductor separately, and not the circuit of which it forms part.

Right-hand Rule.—The direction of the induced E.M.F. is then very simply determined with the aid of Dr. Fleming's Right-hand Rule, which applies to the case of a conductor moving parallel to itself and cutting a magnetic flux at right angles.

This rule corresponds to that already explained in Chapter II in connection with the direction in which a conductor conveying a current tends to move across a field. In the present case, however, the *right* hand is used, the forefinger pointing along the field, and the thumb along the direction of motion; the middle finger (which is at right angles to the other two) then gives the direction of the induced E.M.F.

Activity or Power in an Electric Circuit.—Let a current of I C.G.S. units flow in a circuit, and let A and B be any two points in the circuit, between which there is a P.D. of V, C.G.S. units. In accordance with the definition of unit electric quantity, we may say that I units of quantity are transferred in every second from A to B. But by the definition of P.D., the transference of *each* unit from A to B involves the expenditure of V ergs of work. Thus the total work per second, or the power, in the part of the circuit lying between the points A and B, is V I ergs per second. We therefore see that

$$\text{Activity or Power} = \text{P.D.} \times \text{current.}$$

If, instead of considering a portion of the circuit, we consider the entire closed circuit, then the work done on every unit in carrying it completely round the circuit gives us the E.M.F., which we shall denote by E. Hence the power in the entire circuit is E I.

If the P.D. be expressed in volts, and the current in amperes, then the product P.D. × current will give us the power in *volt-amperes* or *watts.*

The watt (the practical unit of power) has already been defined as a rate of working of 10^7 ergs per sec. This is in agreement with the result

just obtained, for 1 ampere = 10^{-1} C.G.S. unit and 1 volt = 10^8 C.G.S. units.

$$\text{Hence 1 watt} = 10^8 \times 10^{-1} \text{ C.G.S. units}$$
$$= 10^7 \text{ ergs per sec.}$$

Other Units of Power.—The unit of power in the British system is the horse-power, and is in such common use that it may be advisable here to give its relation to the metric unit.

$$1 \text{ H.P.} = 550 \text{ ft.-lbs. per sec.};$$

and since 1 ft.-lb. = 1.356×10^7 ergs,

$$1 \text{ H.P.} = 550 \times 1.356 \times 10^7 \text{ ergs per sec.}$$
$$= 746 \times 10^7 \text{ ergs per sec.}$$
$$\text{Or } 1 \text{ H.P.} = 746 \text{ watts.}$$

The watt is in many cases found to be an inconveniently small unit of power, and is then replaced by the *kilowatt* (usually written K.W.).

$$1 \text{ kilowatt} = 1000 \text{ watts}$$
$$= \frac{1000}{746} \text{ H.P.}$$
$$= 1.34 \text{ H.P.}$$

Unit of Energy.—The practical unit of work or energy is the *joule*, and is equal to 10^7 ergs. For supply purposes a larger unit is employed called the *Board of Trade unit* (frequently "unit" simply), and is equal to *1 kilowatt-hour* or 1000 watt-hours.

CHAPTER IV

RESISTANCE AND ITS MEASUREMENT

Definition of Resistance.—Let an E.M.F. of value E exist in a closed electric circuit. If the resulting current flowing in the circuit has the value I, then the *resistance* of the circuit is defined to be the ratio $\frac{E}{I}$; or, denoting the resistance by R,

$$R = \frac{E}{I}.$$

The activity in the circuit is given by E I, or, since $R = \frac{E}{I}$, by I^2R.

This power is employed entirely in producing heat in the circuit, since no other transformation of energy is supposed to be taking place.[1] Hence I^2R is equal to the rate at which heat is being produced in the circuit. From this we may alternatively define the resistance of a circuit as being that factor which, multiplied by the current squared, gives the rate at which heat is being produced in the circuit.

[1] E is assumed to be the *only* E.M.F. existing in the circuit.

It is scarcely necessary to point out that the resistance of a part of a circuit, i.e. of a conductor, is similarly defined. Let V be the P.D. existing between the ends of the conductor. The ratio $\frac{V}{I}$ is defined to be the resistance of the conductor, where I is the current produced by the P.D. V.

Ohm's Law.—Ohm's Law is a statement of the relation between the E.M.F. existing in a circuit, the current resulting from that E.M.F., and the resistance of the circuit; and expressed in the form of an equation is—

$$I = \frac{E}{R}.$$

Ohm's Law does *not* state that the resistance is *constant*. Ohm never intended that his law should be limited to the case of circuits or conductors having a constant resistance. It is a well-known fact that the resistance of a conductor does vary from time to time. The resistance may vary with temperature, with the current density,[1] with the magnetic field in which the conductor is placed,[2] with mechanical stress, and with "ageing". All these variations may, perhaps, be traced to a molecular change in the conductor. The resistance of most conductors, provided the temperature is unchanged, does not vary appreciably even over considerable periods. This has led to the erroneous reading of Ohm's Law referred to above. Strictly speaking, Ohm's Law simply states that if *at any instant* R is the resistance of a circuit and E is the total E.M.F., then *at that time*, the current I is equal to $\frac{E}{R}$.

C.G.S. Unit of Resistance.—The C.G.S. unit of resistance is defined to be the resistance of a conductor in which C.G.S. unit current flows when C.G.S. unit P.D. exists between the ends of the conductor.

This unit is based upon the definition of resistance, and is expressed in terms of the units of current and P.D. which have already been derived from the fundamental units of length, mass, and time.

Practical Realization of the C.G.S. Unit of Resistance.—It has already been remarked that the C.G.S. unit of P.D. cannot be determined *directly*. A current, and as we shall see, a resistance, may be measured in terms of the fundamental units; and the unit of P.D. may then be determined in terms of the units of current and resistance. The determination of a resistance in absolute units is thus one of the most important physical measurements. The method[3] employed is as follows:—

A copper disc is placed coaxially with a coil of wire, and is fitted with rubbing contacts near its centre and at the circumference. The resistance R whose value is to be determined is joined in circuit with the coil, and a current I is sent through them. This produces a fall of potential over the unknown resistance of amount R I. The disc is now rotated until its E.M.F. is found to balance the fall of potential over the unknown resist-

[1] As, for example, in the case of iron and of moist insulators.
[2] The resistance of a bismuth conductor is affected by magnetic fields.
[3] This method is due to Lorentz.

ance. Now E, the E.M.F. of the disc, is proportional to the current I producing the field of the coil in which the disc is rotating, and to the speed of rotation ω. Hence we may write $E = ki\omega$, where k is a constant whose value may be calculated from the dimensions and relative positions of the coil and disc. We thus get $ki\omega = RI$, or $R = k\omega$ when the speed has been so adjusted as to produce balance between E and RI.

The resistance is thus measured in terms of a velocity, or, in other words, in terms of a length and a time.

The Practical Unit of Resistance.—The C.G.S. unit of resistance is inconveniently small for practical purposes, and consequently a practical unit called the *ohm*, and having the value 10^9 in terms of the C.G.S. unit, has been adopted. The ohm may be represented with sufficient accuracy for practical purposes by the resistance offered to an unvarying electric current by a column of mercury, at the temperature of melting ice, having a mass of 14.4521 grammes, of constant cross-sectional area, and having a length of 106.3 centimetres.

Standards of Resistance.—Standards of resistance for ordinary use usually consist of coils of silk-covered wire specially wound so as to avoid inductive effects. In order to obviate the use of very long or very thin wires it is necessary to choose a material having a high resistivity. The material should be such that temperature has practically no effect on its specific resistance. Certain alloys best satisfy these conditions. Platinoid and manganin are materials very frequently used, but they share with all solid conductors the disadvantage of a liability to slow molecular change which involves a change in resistance.

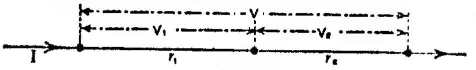


Fig. 15

This change is small enough to be negligible for all ordinary purposes, and such standards, owing to their compactness and portability, are in universal use for testing purposes. Pure mercury having a definite molecular structure is not liable to a change of resistance with time, and has for this reason been laid down as the material for the standard in the official definition of the practical unit of resistance.

Series and Parallel Arrangements of Resistances.—Let two resistances, r_1 and r_2, be connected end to end, or *in series* with each other, as in fig. 15, and let a current I be sent through them. Let V, V_1, and V_2 stand respectively for the P.D.s across the two resistances, and across the first and second taken separately. Then from the definition of P.D. it follows that $V = V_1 + V_2$; but by the definition of resistance, $V_1 = r_1 I$, and $V_2 = r_2 I$. Hence $V = (r_1 + r_2) I$, or $\frac{V}{I} = r_1 + r_2$. Again, by the definition of resistance, $\frac{V}{I}$ is the total resistance of the two resistances connected in series. If this be denoted by R, then we have $R = r_1 + r_2$, i.e. the total resistance is the *sum* of the two resistances. Similar reasoning would apply to any number of resistances joined in series, so that in general

The total resistance of any number of resistances connected in series with each other is equal to the sum of the separate resistances.

From this it follows that the resistance of a conductor of uniform cross-section is *proportional to the length* of the conductor, and that the fall of potential over it when conveying a given current is also proportional to its length.

Fig. 16

Consider next a number of conductors, whose resistances are r_1, r_2, r_3, r_4, placed side by side, as in fig. 16, and connected between two common points A and B. Such an arrangement is called a *parallel* connection of conductors. If R = joint resistance of the system of conductors, V = P.D. between A and B, and I = total current, then $R = \frac{V}{I}$. But if I_1, I_2, I_3, I_4 denote the currents in the various conductors, then $I = I_1 + I_2 + I_3 + I_4$, and $I_1 = \frac{V}{r_1}$; $I_2 = \frac{V}{r_2}$; $I_3 = \frac{V}{r_3}$; $I_4 = \frac{V}{r_4}$, so that $I = V\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4}\right)$, whence

$$R = \frac{V}{I} = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4}}.$$

The reasoning may be extended to any number of conductors; hence the following general rule:—

The joint resistance of a number of conductors connected in parallel is equal to the reciprocal of the sum of the reciprocals of the separate resistances.

It follows, that in the special case of "n" similar conductors having the same lengths and cross-sections and each of resistance "r", the resultant resistance is equal to $\frac{r}{n}$.

If "a" is the cross-section of each of the conductors, then the n conductors could be replaced by one conductor having a cross-section equal to $n a$.

It has already been seen that this conductor will have a resistance equal to $\frac{r}{n}$. Hence the resistance of a conductor varies *inversely as its cross-section.*

In practice, we very frequently have to deal with a parallel connection of two conductors. In such a case, one of them is said to form a shunt to, or to be shunted by, the other. Applying the rule given above, we easily find that in this special case the joint resistance is equal to the product of the two resistances, divided by their sum.

The reciprocal of the resistance of a conductor or system of conductors is termed the *conductance*, and in some problems it is more convenient to consider conductances than resistances.

Resistivity.—It has been seen from the above that as far as the dimensions are concerned, the resistance of a conductor is directly proportional to its length and inversely proportional to its cross-section. The other factors determining the resistance are the nature of the material and the molecular state of the material.

Let l = length of the conductor,
a = cross-section of the conductor,
r = resistance of the conductor.

$$\text{Then } r = \rho\frac{l}{a}.$$

Where ρ is a coefficient depending on the nature of the material and its molecular state. This coefficient is termed the *resistivity*[1] of the material, and may be defined as the resistance between opposite faces of a unit cube of the material. The value of the resistivity of any given material is not constant. In tables of resistivity it is usual to give values of ρ for each well-marked molecular state of the material, e.g. in the case of copper values will be given for pure electrolytic copper, hard-drawn copper wire, &c. In addition the resistivity usually varies constantly with a change in temperature of the material. Hence it becomes necessary in a table of resistivities to state the temperature to which the figures refer. Since the resistance between the faces of a centimetre cube of a metal is very small, it is usually more convenient to express the resistivity in millionths of an ohm, or microhms, per centimetre cube.

The following table gives the resistivities and temperature coefficients of some of the commonly used metals and alloys. The values of the resistivities are for a temperature of 0° C. unless otherwise stated:—

Name of Metal or Alloy.	Resistivity in Microhms per Centimetre Cube.	Approximate Values of the Temperature Coefficient.	Approximate Resistance relatively to Copper.
Aluminium (annealed)	2.886	.00390	1.825
Copper (annealed)	1.582	.00388	1.0
Copper (hard-drawn)	1.620	.00400	1.02
Iron (annealed)	9.614	.00463	6.08
Lead (pressed)	19.42	.00387	12.6
Mercury	94.10	.00072	59.5
Tantalum (hard-drawn)	16.5[2]	—	10.42
Tungsten (hard-drawn)	5.5	.0051[3]	3.92
Constantan (copper-nickel alloy)	44.3	practically zero	28.0
German silver (copper-nickel-zinc alloy)	20.78	.00044	13.12
Manganin (84 Cu + 12 Mn + 4 Ni) ...	42.0	practically zero	26.0
Platinoid (copper-nickel-zinc alloy + 1 % tungsten)	32.7	.00021	20.7
Reostene (steel alloy)	76.7	.00104	44.5

[1] The term "specific resistance" is very commonly used in place of resistivity.

[2] Resistivity at 15° C. The resistivity at the temperature of the glowing filament in a tantalum lamp is about 85 microhms per centimetre cube.

[3] This value only holds for temperatures between 0° C. and 170° C. The resistivity at the temperature of the glowing filament in a tungsten lamp is about 12.5 times that at 15° C.

Temperature Coefficient.—The constant "a" is termed the *temperature coefficient*, and may be defined as being the change of resistance *per ohm at* 0° *C.* per degree centigrade change of temperature. Thus let r_0 = resistance at 0° C., r_t = resistance at $t°$ C.

$$\text{Then } a = \frac{r_t - r_0}{r_0 t},$$

$$\therefore r_t = r_0(1 + at) \ldots\ldots\ldots (1)$$

Now let the temperature be changed to $t'°$ C. Then

$$r_{t'} = r_0(1 + at') \ldots\ldots\ldots (2)$$

Dividing (2) by (1)

$$\frac{r_{t'}}{r_t} = \frac{1 + at'}{1 + at},$$

$$\text{or } r_{t'} = r_t\left(\frac{1 + at'}{1 + at}\right),$$

and, as a is small, $r_{t'} = r_t\{1 + a(t' - t)\}$ approximately.

As a general rule, a is positive for good conductors (metals), and negative for bad conductors (electrolytes and insulators). In the case of certain metallic alloys, a is so small that within a certain range of temperature the resistivity may be regarded as independent of temperature.

Such alloys are particularly suitable for the construction of shunts and series resistances for measuring instruments which are required to be independent of temperature changes.

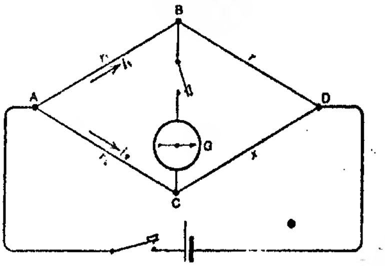


Fig. 17

Measurement of Resistance.—The methods of resistance measurement are exceedingly numerous and varied. In the following paragraphs an outline of some of the more important methods will be given, and for convenience they will be divided into three classes as follows:—

Methods suitable for measuring (*a*) resistances of moderate value, (*b*) very low resistances, (*c*) very high resistances.

Resistances of Moderate Value: The Wheatstone Bridge.—For the measurement of moderately high resistances, the most satisfactory method is that known as the Wheatstone's Bridge. The usual diagrammatic way of representing this arrangement is shown in fig. 17, where r_1, r_2, and r are three resistances having known values, and x is the unknown resistance. A battery and key is connected between A and D, and a galvanometer and key between B and C. In carrying out the measurement, r is varied until the galvanometer shows no deflection when both battery and galvanometer keys are closed. Balance is then said to have been obtained, and there is no P.D. between B and C. In order that this condition may be satisfied, it is obvious that the fall of potential from

A to B must equal that from A to C, and the fall from B to D that from C to D. These conditions are expressed by the equations

$$r_1 I_1 = r_2 I_2$$
$$r I_1 = x I_2,$$

where I_1, I_2 stand for the currents along ABD and ACD respectively. Dividing the first equation by the second, we get for the condition of balance

$$x = \frac{r_2}{r_1} r,$$

which gives us x in terms of the known resistances. In order to avoid spurious and misleading deflections of the galvanometer, *the battery key is always pressed before the galvanometer key, and released after.*

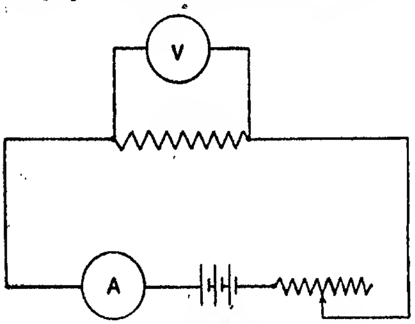


Fig. 18

There are numerous forms of the Wheatstone Bridge, such as the Post Office Box, the Slide Metre Bridge, &c., but since the principle is the same for all it will suffice to mention their outstanding features. In the Post Office type the resistances consist of coils of platinum silver, German silver, platinoid, or manganin silk-covered wire wound on bobbins of boxwood and thickly coated with paraffin-wax. These bobbins are attached to the lower surface of a horizontal slab of ebonite, which is polished on its upper surface, and the ends of the coils are connected to a set of heavy brass contact blocks arranged on the upper surface of the ebonite. Neighbouring contact blocks may be connected by means of a plug, whereby the coil between them is short-circuited. It is a matter of the highest importance to have the plugs thoroughly clean and fitting well into the conical spaces intended to receive them.

The "ratio" resistances r_1 and r_2 in the Metre Bridge consist of a uniform stretched wire 1 metre in length, and B is a sliding contact. A balance is obtained by varying the position of this contact. Since the wire is uniform, l_1 the length of wire between A and B, and l_2 the length of wire between B and D, will be proportional to r_1 and r_2 respectively. Hence

$$x = \frac{l_2}{l_1} r,$$

when a balance has been obtained.

Very Low Resistances: Drop of Potential Methods.—The following practical method, which is in constant use in workshops for the determination of the resistance of armatures, series field coils, and other low resistances, may very conveniently be employed where a moderate degree

of accuracy is required. A current is passed through the resistance R to be measured (see fig. 18), and the potential difference across the resistance and the current flowing through it are measured by suitable instruments.

Let I be the current in amperes and V the P.D. in volts, then

$$R = \frac{V}{I} \text{ ohms.}$$

There are other drop of potential methods, in which the fall of potential over the unknown resistance is compared by means of a galvanometer with that over a known low resistance connected in series with the resistance to be measured.

Kelvin Double Bridge.—This is a modification of the Wheatstone Bridge, in which the error due to resistance at the contacts is eliminated.

The general arrangement is shown in fig. 19.

B, battery. G, galvanometer. K, key.

a, b, c, and d, fixed resistances large compared with resistance s of block connecting points q and m.

R, variable standard low resistance.

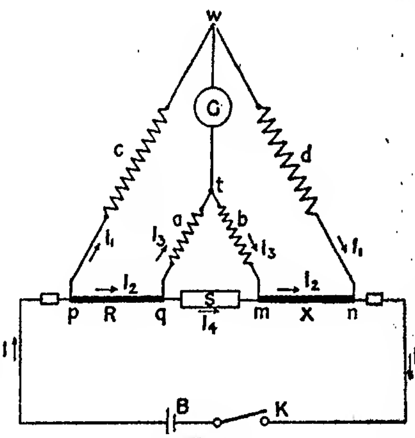


Fig. 19

X, unknown low resistance to be measured.

The ratio of resistance $\frac{c}{d}$ is made as nearly as possible equal to $\frac{a}{b}$.

A balance is obtained by varying R.

Let I, I_1, I_2, I_3, and I_4 be the currents in the various branches, as indicated in fig. 19. Let v_p, v_w, &c., be the potentials at the corresponding points p, w, &c. At balance—

$$v_w = v_t, \text{ or } v_p - v_w = v_p - v_t.$$

$$\text{Also, } v_w - v_n = v_t - v_n.$$

$$\therefore I_1 c = I_2 R + I_3 a,$$

$$I_1 d = I_2 X + I_3 b.$$

$$\therefore \frac{c}{d} = \frac{I_2 R + I_3 a}{I_2 X + I_3 b}.$$

$$\text{And if } \frac{a}{b} = \frac{c}{d},$$

$$\frac{a}{b} = \frac{I_2 R + I_3 a}{I_2 X + I_3 b},$$

$$I_2 a X + I_3 a b = I_2 b R + I_3 a b,$$

$$X = R\frac{b}{a} = R\frac{d}{c}.$$

It may also be shown, that provided that the resistance s of the copper connecting block is negligible compared with the resistances a and b, that X is equal to $R\frac{d}{c}$, even if $\frac{a}{b}$ is not absolutely equal to $\frac{c}{d}$.

This method is a very accurate one for the determination of a very low resistance, such as that of a length of trolley wire, heavy cable, or copper bar.

Very High Resistances.—The substitution method in one form or another is that most commonly used for measuring very high resistances, such as the insulation resistance of electrical machinery, cables, &c.

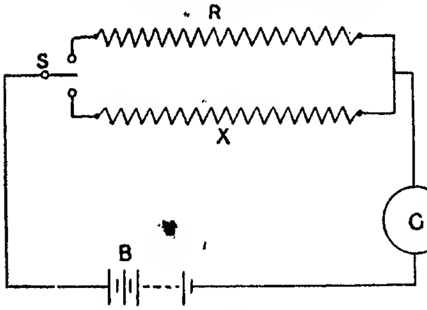


Fig. 20

A standard high resistance R and the resistance X to be determined are arranged as shown in fig. 20, so that each may be connected, in turn, in series with a galvanometer G and a battery B.

The two-way switch S is first moved so that the standard resistance R is brought into circuit, and the deflection d_1 of the galvanometer is noted. The deflection d_2 of the galvanometer when the unknown resistance is in circuit is next taken. The value of X is then readily calculable.

Let E = E.M.F. in the circuit.
I_1 = galvanometer current when R is in circuit.
I_2 = galvanometer current when X is in circuit.
b = battery resistance.
g = galvanometer resistance.

$$\text{Then, } I_1 = \frac{E}{R + g + b} = k d_1$$

$$I_2 = \frac{E}{X + g + b} = k d_2,$$

where k is a constant.

From the above equations—

$$\frac{d_1}{d_2} = \frac{X + g + b}{R + g \times b}$$

$$\text{and } X = \frac{d_1}{d_2}(R + g + b) - (g + b).$$

Since X and R are of very high value, b may be at once neglected, and if, as is usually the case, the galvanometer resistance is low, g may also be neglected. The equation then reduces to the simple form

$$X = \frac{d_1}{d_2} R.$$

It should be noted that in carrying out any measurements of resistances of very high value it is imperative to have all the instruments, switches, and connecting wires thoroughly well insulated.

Cable-makers' Test.—The test made on cables before they leave the works is a practical application of the above method. The cable to be tested is immersed in brine, and the ends are brought out and bared. The cores are connected together, and the insulation resistance between them and a metal plate immersed in the brine is measured. The apparatus is connected as shown in fig. 21. In order to avoid errors due to a leakage current flowing over the wet surface of the cable insulation, a

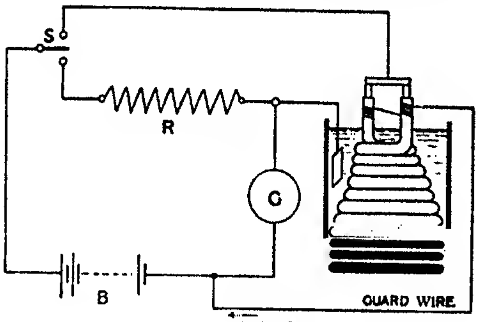


Fig. 21

Price's Guard Wire is used. The guard wire is a thin copper wire wound tightly round the cable insulation a few inches below the bared ends, and connected to a point on the battery side of the galvanometer. The leakage current is thus prevented from passing through the galvanometer and affecting the deflection recorded when the insulation resistance is in circuit.

Board of Trade Test for Tramways.—A somewhat different method is employed in the daily Board of Trade insulation test made on tramway networks.

A high resistance voltmeter is connected first between the positive bus-bar and earth,

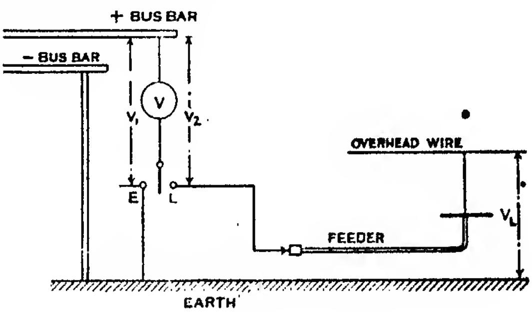


Fig. 22

and then between the positive bus-bar and the station end of each feeder. The general arrangement of the apparatus is shown in fig. 22.

Let V_1 = voltmeter reading with switch on contact E.

V_2 = voltmeter reading with switch on contact L.

V_L = potential difference between line and earth.

g = resistance of voltmeter.

I_2 = current flowing through voltmeter when switch is on [contact L.

Then $V_L = V_1 - V_2$.

But $V_L = I_2 X$,

where X is the insulation resistance of the feeder, distributors, and overhead wire combined.

$$\therefore X = \frac{V_1 - V_2}{I_2},$$

$$\text{or, } X = \left(\frac{V_1 - V_2}{V_2}\right)g.$$

Silvertown Portable Testing Set.—This testing set, which is a good example of a class of apparatus suitable for measuring high as well as low and moderate resistances with a fair degree of accuracy, has a range from 0.01 ohm upwards.

The general appearance of the instrument is illustrated in fig. 23, and fig. 24 gives a diagram of the connections. For resistances up to 9900 ohms the Wheatstone's Bridge method is used. The "ratio arms", corresponding to r_1 and r_2 of fig. 17, are represented by the coils marked 10, 100, and 1000 in the lower part of the diagram of connections, fig. 24, and the adjustable arm (r in fig. 17) is represented by the two sets of coils marked "tens" and "units". The coils in the ratio arms are thrown into circuit by *taking out* the corresponding plugs; but in the case of the adjustable arm, one plug is provided for each set of circular contact blocks, and the amount of resistance introduced is given by the number opposite the plug. If either of these two plugs be withdrawn, a complete break results. The unknown resistance is connected to the two terminals marked "bridge terminals" in the diagram. The battery is connected to the two sockets marked "bridge" by means of flexible wires ending in plugs. No battery key is provided, the insertion or withdrawal of either battery plug when the other is in, making or breaking the battery circuit. The key seen at the left-hand side is the galvanometer key.

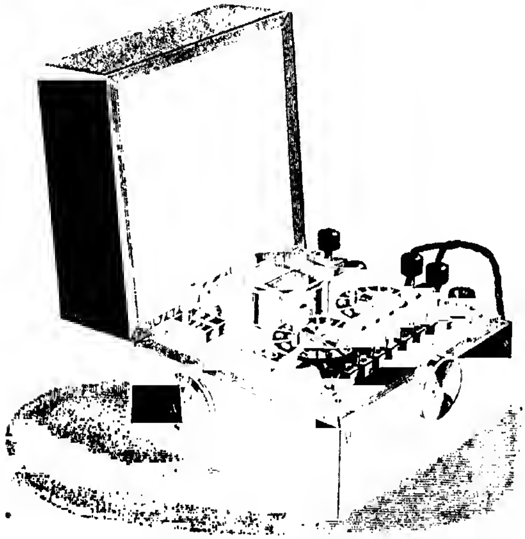


Fig. 23.—Silvertown Testing Set

If the resistance to be measured exceeds 9900 ohms, the direct-deflection method is used in place of the Wheatstone's Bridge. The battery plugs are transferred to the sockets marked "insulⁿ", and the ends of the

unknown resistance are connected to the two upper terminals marked "insul^n" and "earth". Two readings are now taken: one with a plug inserted into the hole marked "insul^n", which introduces the unknown resistance into circuit; the other with the plug in the hole marked "50,000 ohms", this disconnects the unknown resistance and for it substitutes 50,000 ohms. The galvanometer scale is marked so that the readings are proportional to the currents. It is in general necessary to shunt the galvanometer when taking the second reading, and it may further be neces-

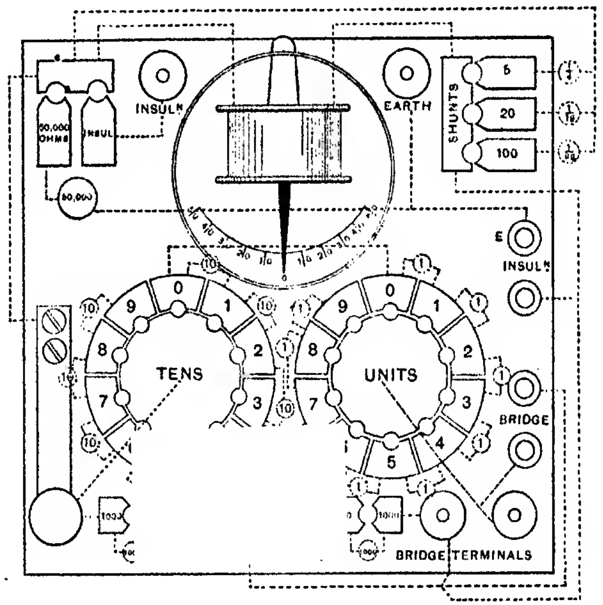


Fig. 24

sary to reduce the E.M.F. by using a smaller number of cells in the testing battery.

When using the instrument it must be levelled and set so that the pointer attached to the needle stands at zero; this position is found by trial, the instrument being turned about a vertical axis. By means of the little permanent magnet let into the case of the instrument (clearly seen in fig. 23) and capable of rotation about a horizontal axis, the sensitiveness of the galvanometer may be varied.

A portable battery of "dry cells" for use with this instrument is supplied by the makers.

Kelvin Testing Set.—This testing set, which is intended for the accurate measurement of insulation resistances, works on the principle of the substitution method. The galvanometer resistance is utilized as a standard

high resistance, and the need for a separate standard is thus done away with.

The general arrangement of the set is shown in fig. 25.

The shunts S are provided so that the galvanometer deflection can be controlled to suit the circumstances of the test. Since the employment of a shunt reduces the resultant value of the galvanometer resistance, suitable compensating resistances C R are provided, so that the total resistance in the galvanometer circuit is the same, no matter which shunt is being used.

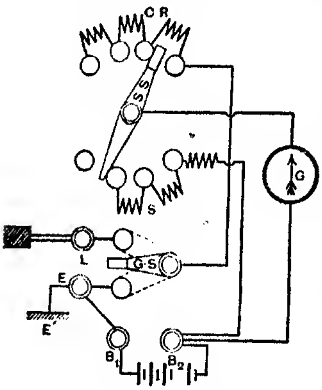


Fig. 25.—Kelvin Testing Set

If d_1 is the deflection on the galvanometer when the switch is on E, and n_1 is the multiplying power of the shunt in use, then the current through the galvanometer

$$= k d_1 = \frac{1}{n_1} I_1 = \frac{E}{g n_1}, \ldots\ldots\ldots\ldots (a)$$

where E is the E.M.F. of the battery.

If d_2 is the deflection on the galvanometer when the switch G S is on L, and n_2 is the multiplying power of the shunt then in use, then the current through the galvanometer

$$= k d_2 = \frac{1}{n_2} I_2 = \frac{E}{(g + X)n_2}, \ldots\ldots (b)$$

where X is the insulation resistance being measured.

Dividing (a) by (b)—

$$\frac{d_1}{d_2} = \frac{g + X}{g} \cdot \frac{n_2}{n_1}.$$

Simplifying we have—

$$X = \frac{d_1 n_1}{d_2 n_2} g - g.$$

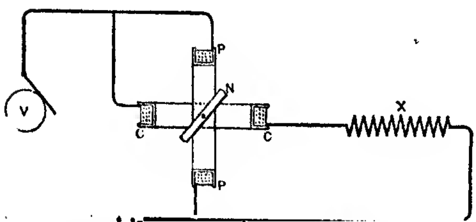


Fig. 26.—Diagram showing Principle of Ohmmeter

V, Source of pressure. CC, Current coil in series with external resistance. PP, Pressure coil directly connected to pressure. N, Magnetized needle. X, Resistance under test.

Evershed's Ohmmeter.—The idea of an instrument capable of giving the values of resistances directly in ohms was originally suggested by Professors Ayrton and Perry. The general principle of such an instrument is illustrated in fig. 26:—Imagine a magnetic needle placed in a field which is the resultant of a constant field and a variable field of constant

direction. Then as the variable field (of constant direction) assumes all possible values between a zero and a certain maximum value, the position of the needle, which is always along the resultant field, will vary continuously between certain limits depending on the relative directions of the two fields. Since the direction of the resultant remains unaltered if both fields be altered in the *same ratio*, the position of the needle will depend simply on the *ratio* of the two fields.

Suppose, next, that one of the fields is produced by a coil of constant resistance connected to a source of E.M.F., while the other is produced by a coil joined in series with a variable resistance and connected to the *same*

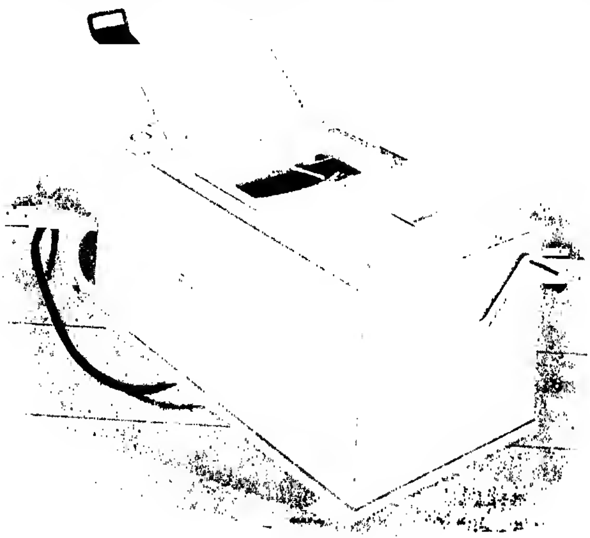


Fig. 17.—Evershed's Patent Megger

source of E.M.F. As this variable resistance is indefinitely increased from a zero to a very large value, the position of the needle steadily changes, and from what has been said above it follows that the value of the E.M.F. supplying both coils does not affect the position of the needle, which is determined solely by the value of the variable resistance. By using a set of known resistances the instrument may be "calibrated" or made direct-reading, and on then substituting any unknown resistance the value of this latter may at once be read off.

The great defect of such an instrument is the comparatively small degree of accuracy attainable, since the scale extends from zero to infinity. Hence, although there is no reason why such an instrument should not be used for the approximate measurement of any range of resistances, practically it is only employed in cases where a very high degree of accuracy is not required, viz., for the measurement of very high or insulation resistances, such as that of the wiring in a building. An instrument devised for

this purpose by Mr. Evershed, and known as Evershed's "Megger", is very largely employed.

The general appearance of the instrument is shown in fig. 27. The unknown high resistance is connected across the two terminals seen to the left, and marked "line" and "earth" respectively. The handle in front is for driving the small high-voltage generator which supplies the testing voltage. The generator and measuring instrument are combined in a single case, measuring $6\frac{1}{4} \times 6\frac{1}{4} \times 12$ inches. The entire "Megger" weighs about 18 lb. The scale is fitted immediately below the top of the case, and is viewed through a plate-glass window provided with a suitable cover. The driving shaft is fitted with an automatic clutch, which begins to slip when a certain speed is reached.

The instrument is used as follows:—It is stood on a steady base, and

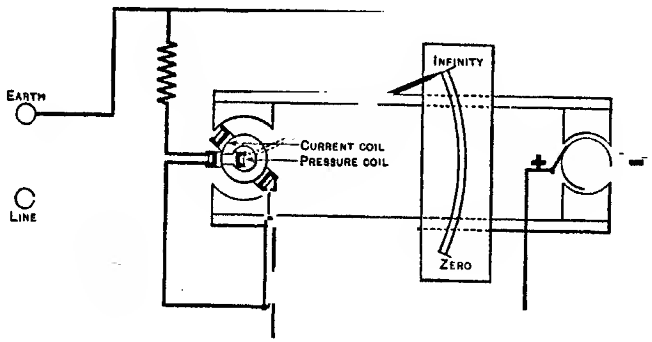


Fig. 28.—Diagram showing Principle of Megger

the driving handle is turned in a clockwise direction, the speed being gradually increased until the clutch is felt to slip. The value of the unknown high resistance is then read off on the scale.

In the original "ohmmeter" devised by Ayrton and Perry, and shown in fig. 26, two *fixed coils* and a *moving magnet* or needle were provided. The instrument was thus of the "needle" type. In Evershed's Megger this arrangement is inverted, there being a powerful *fixed magnet* and two *moving coils*. The "Megger" thus belongs to the moving-coil type of instrument so largely represented nowadays by voltmeters and ammeters for continuous currents (see fig. 35). The general arrangement of the instrument is shown in fig. 28. It will be noticed that the two permanent bar magnets are fitted with pole-pieces at each end. The pole-pieces at the left-hand end provide the instrument field, while those at the right-hand end provide the generator field. By making the same set of magnets serve this double purpose, great economy of space is effected, and a very compact form of construction is secured. The two moving coils are rigidly attached to a common axle, which carries the pointer of the instrument. The coils make a certain angle with each other. One of them, known as the *current coil*, is connected in series with the unknown resistance and the generator,

a safety resistance being also included in the circuit. The other coil, known as the *pressure coil*, is connected in series with a very high resistance and across the generator terminals. The hollow, soft-iron cylindrical core placed between the pole-pieces is slotted on one side as shown, so as to admit one half of the pressure coil. It is evident that, since this coil will tend to move so as to make its axis coincide with that of the field in which it is placed, it will take up the position shown in the figure when there is no current flowing through the current coil; this will happen when the external resistance is infinite, and the pointer will then stand at the "infinity"

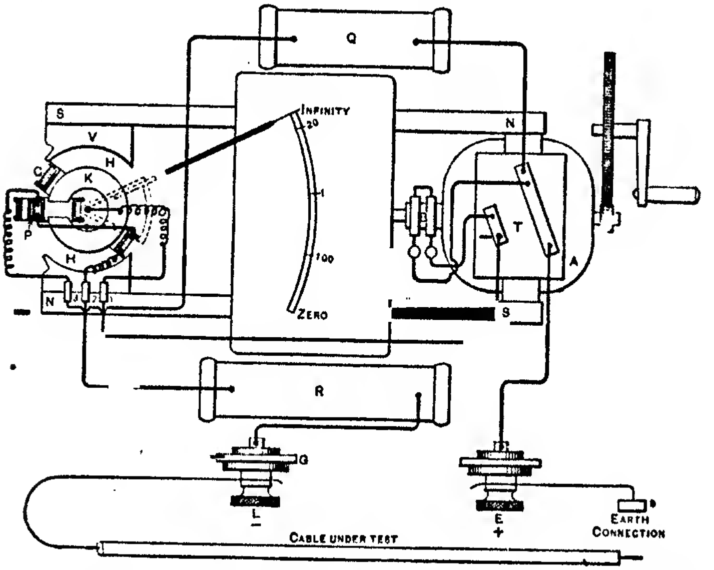


Fig. 29.—Evershed's Patent Megger. Explanatory Diagram

A, Generator armature. B, Generator commutator. C, Ohmmeter current coil. E, Earth terminal. G, Guard plate. H H, Annular air-gaps. K, Iron core. L, Line terminal. N S, Field magnets. P, Pressure coils. Q, Resistance in series with pressure coils. R, Resistance in series with current coils. T, Terminals of generator. V, Pole-pieces of Ohmmeter. 1, Common terminal of current and pressure coils. 2, Terminal of current coil. 3, Terminal of pressure coil.

division of the scale. If, however, the unknown resistance is not infinite, a current will pass through the current coil and this coil will be deflected (as in a moving-coil galvanometer, fig. 35) in a clockwise direction, the deflection being opposed by the couple acting on the pressure coil, which moves into a field of increasing intensity as the deflection increases. The coil system is quite free to move under the action of the resultant couple due to the two coils, there being no control, as the currents are led into the coils by means of extremely fine strips, whose control is negligible.

Some further details of the instrument are shown in fig. 29. In order to prevent the instrument from being affected by external fields, the

pressure coil is made astatic[1] with respect to such fields by having a small external compensating coil attached to it on one side, as shown in fig. 29. The number of turns in this compensating coil is so adjusted that the couple exerted on it by any uniform field is equal and opposite to the couple exerted by the same field on the main pressure coil.

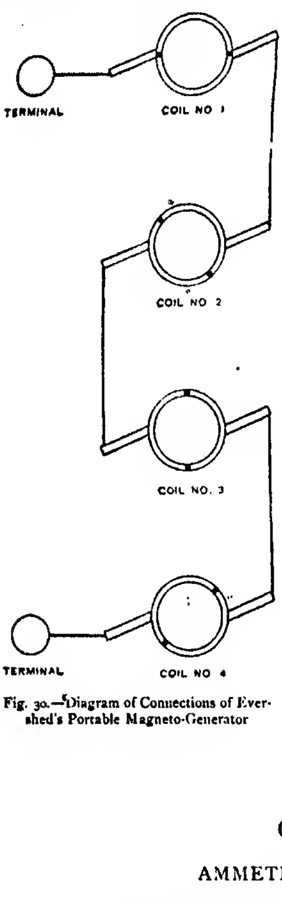


Fig. 30.—Diagram of Connections of Evershed's Portable Magneto-Generator

The connections of the generator are shown in fig. 30. Depending on the useful range of the instrument, the generator is constructed to give voltages of 100, 250, 500, and 1000 at the speed when slipping begins to take place. A high-testing voltage is essential, as a defect in the insulation under test may not be detected with a low-testing voltage. The armature of the generator consists of several independent coils spaced at equal angular distances from each other, and each coil is provided with a two-part commutator. The several commutators are then connected in series, as shown in fig. 30. The E.M.F. developed by such a generator is not absolutely steady, but fluctuates between certain limits. Double-reduction toothed gearing is used for driving the generator, as shown in fig. 29. The normal speed of the driving handle is 100 revolutions per minute.

Instruments of this type are made having a useful range up to 2000 megohms.

CHAPTER V

AMMETERS AND VOLTMETERS

An ammeter is an instrument which, by means of a pointer moving over a graduated scale, gives a continuous indication of the intensity of the current flowing in the circuit in which it is connected.

A voltmeter is an instrument which, in a similar way, gives a continuous indication of the P.D. between the two points to which it is connected.

Ammeters and voltmeters in the great majority of cases do not differ in principle, and indeed, by suitably modifying the resistance and the method of connecting the instrument to the circuit, it is possible to use one and the same instrument as an ammeter or as a voltmeter.

By far the greater number of ammeters and voltmeters depend for

[1] i.e., constructed so as to be unaffected by any *uniform* field.

their action on the production of a magnetic field by a current flowing through the instrument. Another class make use of the expansion and contraction of a fine wire through which a current is passing. The principle of electrostatic attraction is utilized in what are called electrostatic voltmeters.

It is essential that both ammeters and voltmeters shall affect as little as possible the conditions existing in the circuit before their introduction.

In fig. 31 a simple circuit is shown with an ammeter A, and voltmeter V, connected so as to measure respectively the current flowing in the circuit, and the P.D. between the points C and B.

Clearly in order that the ammeter shall affect the current flowing in the circuit as little as possible, it must have as *low* a resistance as possible. On the other hand, the voltmeter must have as *high* a resistance as possible, in order that the total resistance of the circuit shall be changed as little as possible.

It will be seen, therefore, that

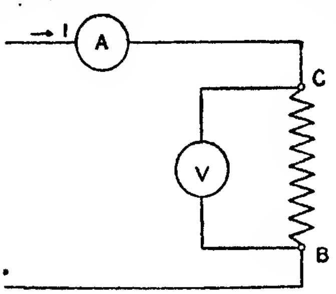


Fig. 31

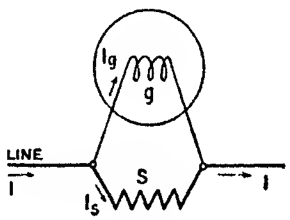


Fig. 32

the essential characteristic of an ammeter is *low resistance*, and of a voltmeter *high resistance*.

Variations in Range.—It is often convenient to be able to alter the range of an instrument. Thus an instrument which when used alone will measure up to $\frac{1}{10}$ ampere, may be required to measure, say, 10, 50, or 100 amperes. This change in range is easily obtained by using ***shunts*** of suitable values. For example, in fig. 32 let the ammeter A have a resistance of g ohms, and let S equal the resistance of the shunt to be used with the ammeter. If I_g is the current which, flowing in the ammeter alone, produces the full-scale deflection, and if V is the P.D. between the terminals of the ammeter, then I_s, the current through the shunt, will be $\frac{V}{S}$.

Also $I = I_g + I_s$, where I is the main current to be measured.

$$\therefore\ S = \frac{V}{I - I_g};\ \text{but } I_g = \frac{V}{g}.$$

$$\therefore\ S = \frac{I_g}{I - I_g} \cdot g.$$

Therefore, knowing the resistance g of the ammeter, and the current necessary to produce full-scale deflection when the ammeter is used

without a shunt, the value S of the resistance of the shunt which must be used when the range of the instrument is to be increased to I amperes, can readily be calculated.

Thus, in the case mentioned above, let us suppose that the resistance of the ammeter alone is 1 ohm, and that the resistance of the shunt increasing the range to 10 amperes is required.

$$S = \frac{\frac{1}{10}}{10 - \frac{1}{10}} \times 1 = \frac{1}{99} \text{ ohm.}$$

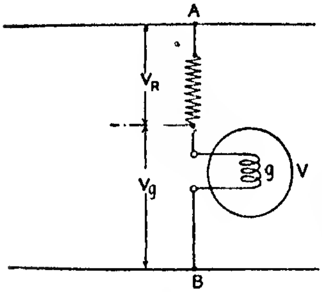


Fig. 33

The alteration in range of a voltmeter (except electrostatic voltmeters) is obtained by connecting a high resistance in *series* with the instrument.

For example, let g be the resistance of a voltmeter, and r the value of the resistance connected in series with it (see fig. 33). Let v_g be the P.D. between the galvanometer terminals which gives the full-scale deflection, and I_g the current which then flows through the voltmeter.

Then, if v_r is the P.D. across the series resistance, and v the P.D. between A and B which it is required to measure—

$$\begin{aligned} v &= v_r + v_g \\ &= I_g \,.\, r + I_g \,.\, g. \end{aligned}$$

$$\text{Or } r = \frac{v}{I_g} - g, \text{ but since } I_g = \frac{v_g}{g}$$

$$\therefore r = \left(\frac{v}{v_g} \,.\, g\right) - g.$$

For example, suppose that the voltmeter has a resistance of 1 ohm, and that its maximum reading when used alone is $\frac{1}{10}$ volt. If this voltmeter is required to read up to 100 volts, the series resistance r to be added will be—

$$r = \left(\frac{100}{\frac{1}{10}} \,.\, 1.\right) - 1 = 999 \text{ ohms.}$$

The instrument mentioned in these two examples can therefore be used—

(*a*) As an ammeter to read to 10 amps. Shunt $\frac{1}{99}$ ohm.

(*b*) As a voltmeter to read to 100 volts. Series resistance 999 ohms.

Other ranges could be obtained by providing suitable shunts and series resistances.

It may be mentioned that the use of a shunt with an ammeter also removes the serious difficulty of providing a heavy winding in the ammeter itself when it is to be used for large currents.

Some of the more familiar types of ammeters and voltmeters will now

be described, and it must be understood that unless expressly stated the descriptions apply to *both* ammeters and voltmeters.

Moving-iron Type.—This type of instrument, which from its low cost is widely used in cases where no great degree of accuracy is required, is manufactured in many different forms, all of which, however, depend for their action on the tendency of a piece of soft iron to move into the strongest part of a magnetic field in which it is placed.

In one form the instrument consists of a circular coil within which a light piece of soft iron carrying the pointer is pivoted. The pivot is placed away from the centre of the coil, so that the soft iron moves in towards the axis of the coil when a current flows.

The arrangement of the parts is shown diagrammatically in fig. 34.

Control weights are placed on a small spindle attached to the soft iron. This provides the controlling couple. The deflecting force is proportional to the pole-strength of the soft iron and to the strength of the field produced by the current flowing in the coil, provided the soft iron is at some distance from the axis of the coil. For very small values of the current both are proportional to the current, and therefore the deflecting force is for a short range practically proportional to the square of the current. For this reason the beginning of the scale is very cramped. As soon as the soft iron is saturated the deflecting force becomes practically proportional to the current itself, and the scale becomes open and more evenly divided. It is, therefore, very necessary that the soft iron should be as light as possible and easily saturated. To this end the soft iron is usually made in "rake" form.

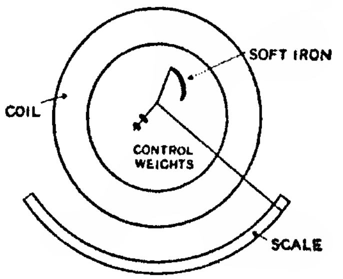


Fig. 34

If the scale is continued so far that the soft iron nearly reaches the axis of the coil, it again becomes contracted.

The direction of the deflection is independent of the direction of the current, and consequently this instrument may be used on both D.C. and A.C. circuits.

Advantages and Disadvantages of the Moving-iron Type.—

1. *Cost.*—Low.

2. *Adaptability.*—Suitable for both D.C. and A.C. circuits.

3. *Accuracy.*—Moderate.

(*a*) Ascending readings differ from descending readings, due to the hysteresis of the soft iron.

(*b*) Readings are affected by stray fields.

(*c*) Cramped scale at the lower, and also the upper end, if scale is continued so far.

4. *Operation.*—Usually not dead-beat in their action.

Moving-coil Type.—Instruments of this type are in very general use, and may be employed where a considerable degree of accuracy is desired.

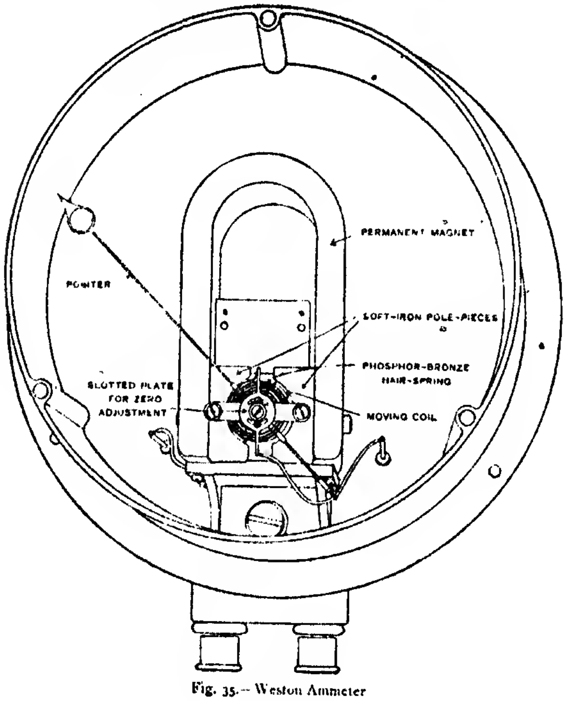


Fig. 35.—Weston Ammeter

The moving part consists of a coil of copper wire wound on a light metal frame the upper end of which carries the pointer.

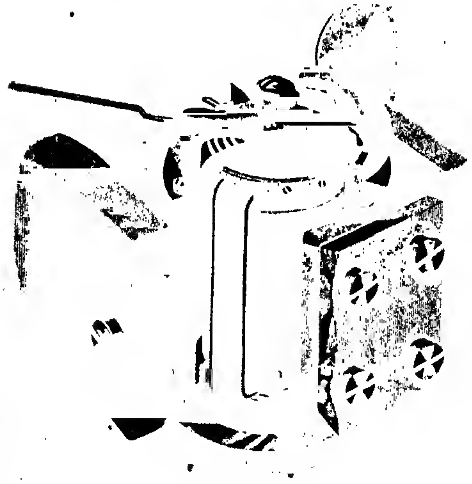


Fig. 35*a*.—Coil of Weston Ammeter

The coil moves in a narrow air-gap formed between the poles of a strong permanent magnet and a fixed soft-iron core.

The controlling couple is supplied by a pair of light spiral springs, one above and one below the coil.

The Weston ammeter is a well-known example of this type of instrument, and a switch-board instrument, with the front cover removed, is shown in fig. 35.

The coil and pole-pieces are shown in fig. 35*a*.

The shunt which, for currents of moderate value, is included in the case of the instrument is shown separately in fig. 36.

When a current flows in the coil the conductors which are parallel to the pole-faces experience a force proportional to the strength of the magnetic field in which they lie, and to the current flowing in them. This provides the deflecting couple, and if the air-gap is uniform this deflecting couple is proportional to the current as long as the coil lies well under the pole-faces.

The scale of an instrument of this type is therefore open and evenly divided.

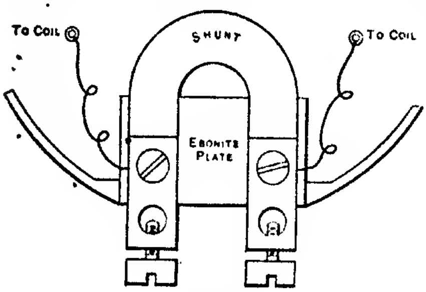


Fig 36

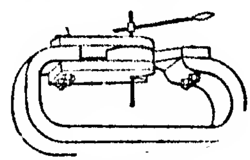


Fig. 37.—Pole-pieces, Record Long-scale Instrument.

The light metal frame carrying the coil forms a closed circuit, and the eddy currents produced in this when the coil moves produce a powerful damping effect, thus rendering the instrument dead-beat in its action. A slotted plate is usually provided carrying a projecting arm to which one end of the upper spiral spring is attached. By rotating this plate slightly the zero may be adjusted.

An interesting modification of this type of instrument has recently been introduced by the Record Electrical Company.

The essential difference is in the use of the horizontal and not the vertical conductors of the coil as the active ones.

The pole-pieces are specially shaped (as shown in fig. 37), and the air-gaps are horizontal instead of vertical.

The moving coil, shown in fig. 38, swings round the ring-shaped pole-piece (shown to the right in fig. 37), and its motion is only limited by the neck of the pole-piece, an angular motion up to 300° being thus obtained.

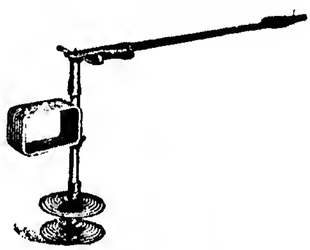


Fig. 38. Moving Coil, Record Long-scale Instrument

It is unnecessary to point out the advantages of the increased length of scale.

Another point which should be noted is that the air-gaps are in parallel, thus giving a low magnetic reluctance, a strong field in the air-gaps, and less tendency towards weakening of the permanent magnets.

A complete switch-board instrument with the case removed is shown in fig. 39.

Advantages and Disadvantages of the Moving-coil Type.—

1. *Adaptability.*—Not suitable for A.C. circuits, since the direction of deflection depends on the direction of the current.

2. *Accuracy.*—Can be made very high.

(*a*) Ascending and descending readings the same.

(*b*) Not affected by stray fields.

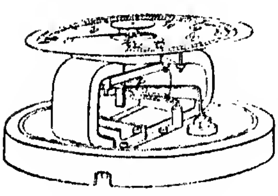


Fig. 39.—Record Switch-board Instrument

(*c*) The error due to the increase of resistance of the coil with temperature is usually reduced to a minimum by the use of a resistance, of an alloy having a zero temperature coefficient, connected in series with the moving coil.

(*d*) An error may be gradually introduced by the weakening of the permanent magnet, but this is guarded against by using a magnet built up of a number of hard-steel magnets.

3. *Operation.*—Dead-beat.

Electrodynamic Type.—This type differs from the preceding in that the field in which the moving coil is placed is produced, not by a permanent magnet, but by a fixed coil connected in series with the moving coil. The details of construction are otherwise similar to those of the moving-coil type just described.

The deflecting couple is proportional to the strength of field produced by the current flowing in the fixed coil, and to the strength of the current flowing in the moving coil. Since the same current flows in both coils the deflecting couple is proportional to the *square* of the current. The scale is consequently cramped at the beginning.

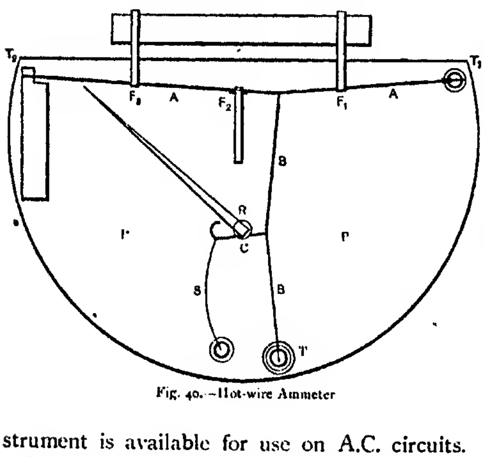


Fig. 40.—Hot-wire Ammeter

The deflection is in the same direction no matter in which direction the current flows, and therefore, the instrument is available for use on A.C. circuits.

In fact, the majority of electrodynamic instruments are used on A.C. circuits, since they there take the place of moving-coil instruments which cannot be used for alternating currents.

Advantages and Disadvantages of the Electrodynamic Type.—

1. *Adaptability.*—Suitable for both D.C. and A.C. circuits.

2. *Accuracy.*—Usually fairly high.

(*a*) Scale contracted at the beginning.

(*b*) Affected by stray fields.

(*c*) Temperature error minimized as in moving-coil type.

(*d*) Frequency error when used on A.C. circuits minimized by making the inductance of the coils very small in comparison with their resistance.

3. *Operation.*—A dead-beat action is secured, usually by means of a light aluminium piston attached by a light arm to the lower end of the pointer, and moving in an air-chamber.

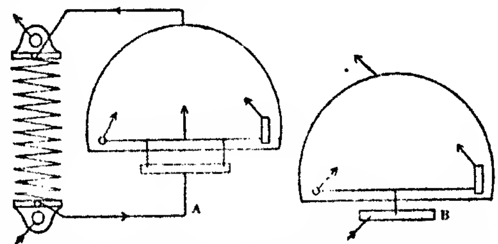


Fig. 41.—A, Diagram of Connections in an Ammeter with the Wire divided up into Four Segments. B, Connections with a Two-segment Wire

Hot-wire Type.—Numerous forms of this type of instrument have been manufactured, but that most commonly met with now is similar to that shown in figs. 40, 41, and 42.

The current to be measured flows through the wire, heats it, and causes it to expand, and the amount of expansion serves as a measure of the current flowing. When the current is a very large one a shunt is used in parallel with the wire, so that only a small fraction of the total current flows through it. It is important to remember that the fraction of the total current flowing through the hot wire is not a constant quantity, unless both the shunt and wire heat up to the same temperature and are of the same material. In most cases the shunt is arranged to become less heated than the wire, so that with the materials employed a smaller fraction of the total current flows through the wire for large currents than for small. Such an instrument is therefore only accurate for the particular shunt for which it is calibrated. A hot-wire instrument cannot safely be shunted to decrease its sensibility, nor can the shunt be removed to increase its sensibility, and the instrument used for measurement unless a calibration is performed *throughout the whole scale.*

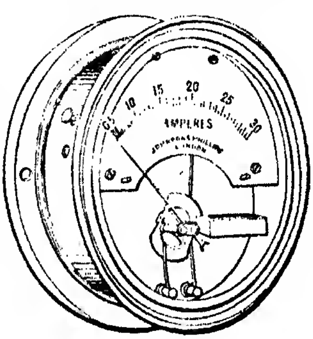


Fig. 42.—Johnson & Phillips Ammeter

One of the most popular instruments of this class is that made by Messrs. Johnson & Phillips, an instrument originally devised by Messrs. Hartmann & Braun (fig. 40). In this a platinum-silver wire A A is stretched between two blocks T_1 T_2, one of which is adjustable. To the centre of the platinum-silver wire is attached a much thinner one B B of phosphor-bronze, which is fixed at its other end to a terminal T; to the phosphor-bronze wire is attached a piece of cocoon fibre C, which is wrapped round a grooved metal roller R, to which the pointer is fixed, the other end terminating in an eyelet attachment to a

flat steel spring S. When the current flows through the wire the sag i creases, thus enabling the phosphor-bronze wire to become more deflecte The increased deflection of the latter is measured by the movement of tl thin cocoon fibre, which turns the pointer. The whole of the hot-wi movement is mounted on a compensation plate P P, whose coefficient expansion is the same as that of the wire.

The wire A A is divided into either two or four equal parts, arrange electrically in parallel with one another by means of silver-foil connecting strips, the object being to divide up the current passing through th measuring-wire and thus to diminish its effective resistance (fig. 41 With a two-segment wire the current enters at the centre and leaves the ends, as shown in the figure, while with a four-segment wire the currer enters at two points a quarter of its length from either end, and leave at the centre and ends. By this arrangement 4 or 5 amperes may be passe through the wires with a drop of potential of less than ¼ volt. A thi metal plate is fixed near the wire to protect it from any disturbance du to air draughts. The shunts used with these instruments when designe for measuring large currents are made of constantan, a substance which ha a negligible temperature coefficient, so that the indications of the instru ments are independent of temperature variation.

The instrument comes to its proper reading somewhat slowly, owing t the time taken by the wire to attain a steady temperature; for many pur poses this is rather an advantage, and in the measurement of steady alter nating currents considerable accuracy can be attained. A damping device is attached to the spindle of the needle, consisting of an aluminium disc (see fig. 42) moving between the poles of a strong magnet, which is said to assist the working of the instrument; but as the motion of the needle is slow under ordinary conditions the effect produced is not very great. When the current is suddenly switched off, however, the damping prevents the needle from swinging back too quickly, thus obviating any tendency of the pulley, to which the pointer is attached, to slip.

Advantages and Disadvantages of Hot-wire Type.—

1. *Adaptability.*—Suitable for both D.C. and A.C. circuits.
2. *Accuracy.*—Moderately high.
   (*a*) Subject to zero creep due to change in physical condition.
   (*b*) Not affected by stray fields.
3. *Operation.*
   (*a*) Takes some time to give reading.
   (*b*) Difficult to repair.
   (*c*) Must be calibrated throughout their range with the shunt with which they are to be used (ammeters only).

Induction Type.—Instruments of this class can only be used for ***alternating currents***, but for convenience they are referred to in this chapter. The principle[1] on which they work is exactly the same as that of the Induction Type Supply Meter, which will be found fully described in Chapter VI. The movement of the disc is controlled by light spiral springs.

[1] In this case, however, the phase displacement between the fluxes is usually produced by a closed copper band placed round one half of a pole-piece.

The scale is open and extremely long, and the readings are usually fairly accurate, *provided* the frequency of the circuit on which they are used is constant at the value for which the instrument has been calibrated. Since the deflecting couple is dependent on the frequency of the current passing through the instrument, this type is unsuitable for measurements on varying frequencies.

Electrostatic Voltmeters. — This is a very important class of voltmeters whose deflections are produced by the attraction exerted between bodies charged to different potentials.

When used on D.C. circuits no current flows through the instrument, and when used on an A.C. circuit only an extremely small one. This current is due to the condenser action of the instrument. In any case no power is absorbed. The accuracy of this type is usually very high, and instruments capable of measuring extremely high potential differences can be satisfactorily designed.

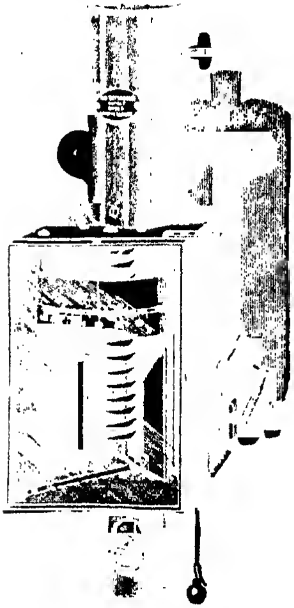


Fig. 43. — Kelvin Multicellular Voltmeter with front removed

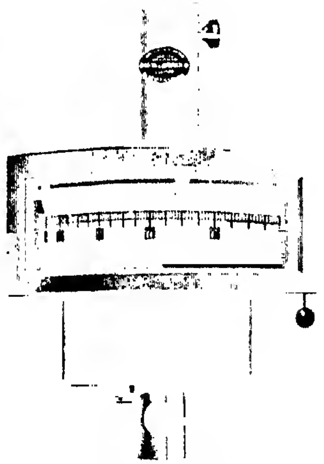


Fig. 44. Kelvin Multicellular Voltmeter

The Kelvin Multicellular Voltmeter (shown in figs. 43 and 44) is a well-known instrument for the measurement of low or medium potential differences.

In this case the moving part is a needle, made up of a number of horizontal parallel triangular plates rigidly attached to a metallic rod and suspended by an iridio-platinum wire (a small coach-spring being interposed between the vanes and the needle to prevent the latter from being damaged by vibration). The fixed part of the instrument consists of two sets of cells, which attract into them the vanes of the needle. The

cells are also triangular in shape, and are made by inserting brass plates in saw-cuts in a brass back block, thus ensuring parallelism and equidistance between them.

The arrangement of the cells and needle is well shown in the figure. Attached to the bottom end of the needle is a small horizontal plate moving in a bath of heavy oil, which effectually damps the motion of the needle and prevents undue oscillation.

For the measurement of very high P.D.s instruments having only one vane or attracted disc are used, and the controlling force is supplied by weights as in the Kelvin Volt Balance.

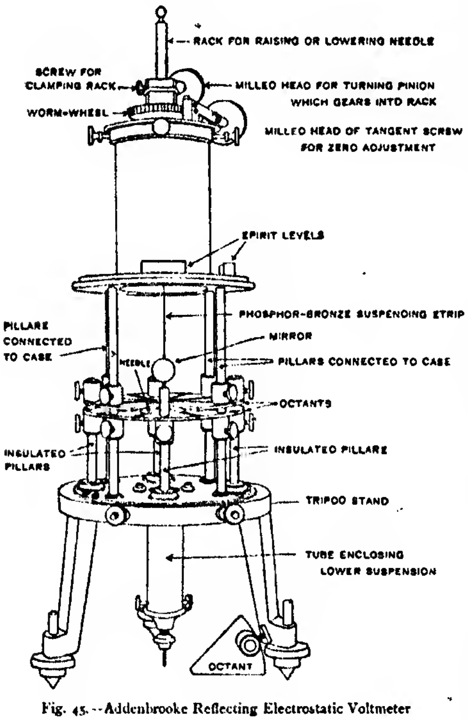


Fig. 45.—Addenbrooke Reflecting Electrostatic Voltmeter

Addenbrooke Reflecting Electrostatic Voltmeter.—The construction of a satisfactory electrostatic voltmeter capable of reading accurately small P.D.s (of the order of 1 or 2 volts), whether continuous or alternating, is a matter presenting considerable difficulties.[1] These difficulties have been successfully surmounted by Mr. Addenbrooke, who has designed the instrument shown (with the lower part of the case removed) in fig. 45. The needle is a double one, each half consisting of a cross of very light sheet brass, as shown in fig. 46, and these two crosses are displaced relatively to each other through an angle of 45°, and are insulated from each other by a disc of mica. Top and bottom suspensions of very thin phosphor-bronze strip are employed. The top suspending strip supports the weight of the needle and serves to connect the upper half of it to the case of the instrument. The height of the needle is adjustable by means of a rack-and-pinion movement (the rack is seen projecting outside the upper part of the case in fig. 45), and its position in a horizontal plane may be roughly adjusted by rotating the circular plate which supports the top suspension, and finely adjusted by means

[1] For many years Lord Kelvin's quadrant electrometer has been available for measuring electrostatically small *continuous* P.D.s; but when so arranged (the needle having an independent charge), the instrument is incapable of measuring *alternating* P.D.s.

or a tangent screw. The strip connected to the bottom half of the needle is left slack, and its lower end is soldered to the copper wire seen projecting (in fig. 45) through the tube which encloses the strip. This tube is *insulated* from the case.

The fixed plates consist of two circular brass plates divided into octants. One of these octants, removed from its usual position, is seen lying close to the right-hand levelling-screw in fig. 45; at the middle of its curved outer edge is a hollow cylindrical projection with a clamping-screw, which enables it to be rigidly clamped to the supporting pillar. Each pillar carries two octants, one being below and the other above the needle. The positions of the octants relatively to the needle are indicated by the dotted lines in fig. 46. By sliding the octants up or down the pillars, the distance of the octants from the needle, and with it the sensitiveness of the instrument, may be varied. One set of four alternate pillars is insulated and connected to the bottom half of the needle and the insulated terminal of the instrument, while the remaining set of four alternate pillars is connected to the case, and through it to the upper half of the needle and the uninsulated terminal.

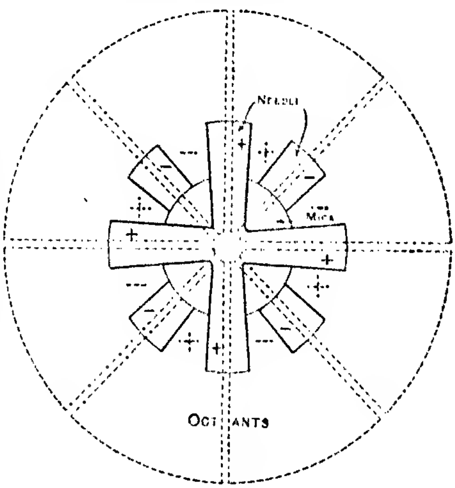


Fig. 46

If we suppose that the terminals of the instrument are connected to a source of E.M.F., then the needles and octants become charged as shown in fig. 46, and it is at once evident that a deflection of the needle will result.

The instrument is shown in fig. 45 with the lower part of the case removed. This is provided with four mica windows arranged at the extremities of two mutually perpendicular diameters, and by this means the octants, needle, and mirror may be observed without having to remove the case, and the ray of light is allowed to enter and leave the instrument by one of the mica windows.

The usual lamp-and-scale method of reading the deflections is used in connection with this instrument, and the mirror is clearly seen, in fig. 45, above the octants.

All electrostatic instruments have to be screened from external electrostatic disturbances, and for this purpose they are nearly completely enclosed in a metallic case. Some opening must, however, be left for reading the scale, and this part of the case is made of glass. It has been found that

a very slight amount of friction, such as that obtained by wiping the glass with a handkerchief, is capable of electrifying the glass to an extent which is sufficient to seriously affect the readings of the instrument.[1] In order to eliminate this source of disturbance, Prof. Ayrton and Mr. Mather have devised a transparent *conducting* varnish with which the glass may be coated, and thereby rendered as efficient a screen as one made of metal.

Standard Instruments.—Some form of standard instrument is commonly used in calibrating ammeters and voltmeters. Lord Kelvin's balances are among the most reliable and accurate electrical measuring instruments made, and are particularly suitable for simple and accurate calibrations. The balances are constructed on the electrodynamic principle, and by having two movable coils carrying currents in opposite

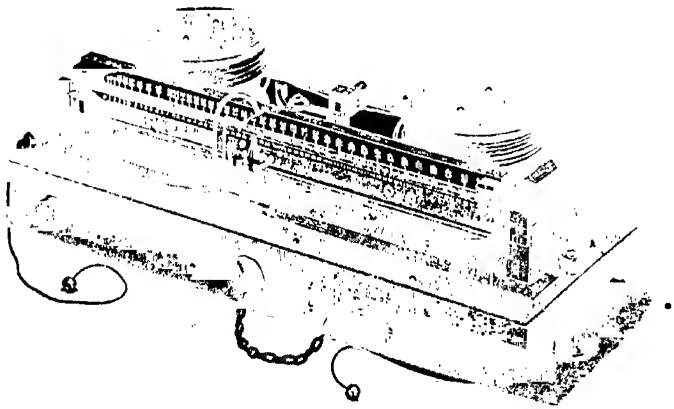


Fig. 47.—Kelvin's Deciampere Balance

directions an astatic movable system is obtained. A number of forms of the balance are manufactured, suitable for current, P.D., and power measurements.

The following description of a Kelvin Current Balance will make clear the main features of these instruments: The two movable coils are mounted at the ends of a balance beam, and above and below each movable coil is a fixed coil. The direction of the currents is shown in fig. 48, and it will be seen that each pair of fixed coils produces a radial horizontal field in which is placed the movable coil. The connections are such that the left-hand coil tends to move down while the right-hand one tends to move up.

As in other electrodynamic current-measuring instruments the deflecting couple acting on the movable system is proportional to the *square* of the current. This deflecting couple is balanced by sliding a weight along the finely-divided scale attached to the balance beam. Since the balancing

[1] This difficulty is by no means confined to electrostatic instruments: some other delicate instruments are equally liable to be affected by electrification of their glass covers.

couple varies as the displacement of the weight, it follows that the current is proportional to the *square root* of the displacement. The fixed or inspectional scale seen above the finely-divided scale in fig. 47 has divisions proportional to the square roots of the corresponding divisions on the movable scale, and the value of the current is obtained by multiplying the fixed-scale readings by a certain constant.

One of the chief features of these instruments is the ingenious manner in which large currents may be conveyed into and out of the movable system of coils without interfering with the flexibility of the suspensions. The method adopted is shown diagrammatically in fig. 48. The weight of the balance beam is borne by a double set of suspending "ligaments", which consist of a number of very fine copper wires laid side by side and soldered to suitably insulated contact-pieces. The current enters the movable coils by one set of ligaments, and leaves by the other. Hundreds of fine copper wires are required in the case of instruments intended for the measurement of very large currents.

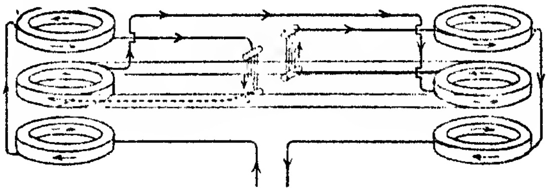


Fig. 48

The scale of a Kelvin balance is rendered very open over the greater portion of the range of the instrument by the simple device of providing three or four different sets of sliding weights which are simple multiples of each other. A different constant will then correspond to each sliding weight. In order to maintain equilibrium of the balance beam in the zero position of each weight, a set of counterpoise weights is provided. These latter are made cylindrical, and are supported in the triangular trough seen to the right of fig. 47, a cross-pin which passes through the counterpoise fitting into a hole at the bottom of the trough and thus giving the weight a perfectly definite position with respect to the beam. The sliding weights fit over a carriage (clearly shown in fig. 47), which may be moved along the beam by pulling one or other of the two strings attached to a self-releasing pendant and passing outside the glass case which is placed over the instrument when in use.

The Kelvin balance is an example of a *zero instrument*, i.e. an instrument requiring an adjustment before a reading can be taken. In an instrument intended for ordinary commercial work the necessity for such an adjustment would be considered a very serious disadvantage. Hence all ordinary commercial instruments are direct-reading *deflectional* instruments; i.e. they indicate the value of the current by the position of a pointer moving over a scale marked in amperes, no preliminary adjustment or reference to tables or multiplication by a constant bei[...]

An important feature of the electrodynamic class of instruments (which they share with instruments of the hot-wire type) is that they are capable of giving correct readings on alternate-current circuits without requiring recalibration.

Calibration of Ammeters.—An ammeter may be *calibrated*, i.e. the true values of its readings ascertained, by one of the following methods:—

1. By the use of a voltameter.
2. By comparison with a standard instrument.
3. By the potentiometer method.

Use of Copper Voltameter.—Although in the legal definition of the ampere special reference is made to the silver voltameter, and a schedule is appended which contains detailed instructions regarding the use of such a voltameter, we shall here consider only the copper voltameter, as this is more suitable for the measurement of large currents. The silver voltameter can be conveniently used for currents up to 1 ampere; its cost would be prohibitive for larger currents.

Although the electrochemical equivalent of an ion is an absolute constant of nature, it is found that the values obtained by experiment in the case of copper deposited from a solution of copper sulphate vary appreciably under different conditions of working. Such variations are not real, but only *apparent*, and are due to the solvent action of the electrolyte on the freshly-deposited copper. This solvent action depends on the density of the solution and on the temperature, and in order to obtain reliable results with a copper voltameter the following precautions should be observed:—

The solution is prepared by dissolving pure recrystallized sulphate of copper in clean ordinary tap-water until a density of 1.18 is reached, and then adding 1 per cent by volume of strong sulphuric acid. It should be replaced by a freshly-prepared solution when the aggregate time during which it has been in use amounts to ten hours.

The area of the cathode plate or plates should be such that there are not less than 50 sq. cms. per ampere of current, and the area of the anode plates should always be in excess of that of the cathodes.

The deposition is allowed to go on for about one hour, the current being maintained constant by means of a suitable rheostat.[1] The cathode plates are taken out of the solution immediately after the current has been switched off, and are plunged into a vessel filled with water which has been acidulated with a few drops of sulphuric acid. They are then thoroughly washed and dried, first by the application of warm white blotting-paper and then by being held in front of a clear fire or over a spirit-lamp, care being taken not to allow them to get appreciably hot.

The values of the *apparent* electrochemical equivalent of copper are given in the following table, prepared by Prof. T. Gray:—

[1] Rheostats consisting of troughs of mercury of variable length are very convenient for this purpose.

Area of Cathode in sq. cms. per ampere.	Values of Apparent Electrochemical Equivalent of Copper.				
	2° C.	12° C.	23° C.	28° C.	35° C.
50	.0003288	.0003287	.0003286	.0003286	.0003282
100	.0003288	.0003284	.0003283	.0003281	.0003274
150	.0003287	.0003281	.0003280	.0003278	.0003267
200	.0003285	.0003279	.0003277	.0003274	.0003259
250	.0003283	.0003278	.0003275	.0003268	.0003252
300	.0003282	.0003278	.0003272	.0003262	.0003245

The copper-voltameter method of calibrating an ammeter is especially suitable in the case of instruments obeying a known law, such as the Siemens electrodynamometer and Kelvin balance. In the case of other instruments, a single determination with the voltameter only enables us to verify the accuracy of the instrument for one particular scale reading. The voltameter method, which is the one actually employed in the standardization of Lord Kelvin's balances, is thus unsuitable for calibrating commercial instruments over their entire scale. For this purpose the next method is more convenient.

Calibration of Ammeters by Comparison with a Standard Instrument.—The instruments used as standards for this purpose are generally Lord Kelvin's current balances. The instrument to be calibrated is connected in series with the Kelvin balance, a suitable rheostat, and a number of secondary cells, and a series of simultaneous readings is obtained with both instruments. A calibration curve may then be plotted, instrument readings being measured horizontally and their correct values vertically; or an "error curve" may be plotted instead, whose abscissæ (horizontal distances) represent instrument readings, and ordinates the amount to be added to, or subtracted from, these readings in order to obtain the correct values of the current as given by the standard instrument.

Potentiometer Method of Calibrating Ammeters.—In this method the ammeter is joined in series with a known standard resistance (preferably an exact submultiple of 10), and the value of the P.D. across this resistance is measured by means of an instrument known as a "potentiometer" (described below). The P.D. divided by the value of the standard resistance gives the true value of current corresponding to the reading of the ammeter.

In the calibration of ammeters it is important to have all the connecting wires or cables arranged so that there is no direct magnetic effect produced on the instruments by current flowing along the connecting wires. The latter should be preferably twisted so as to form a twin conductor, all large loops or coils being carefully avoided. Neglect of this precaution may give rise to serious error.

Calibration of Voltmeters.—Voltmeters may be calibrated by comparison with a standard instrument, or by the potentiometer method. It is needless to remark that in comparing the readings of two voltmeters these

instruments should be connected in *parallel*, and not in series as would be the case with ammeters. A Kelvin Centi-ampere Balance connected in series with a high resistance is frequently used as a standard instrument.

The Potentiometer.—The principle underlying the potentiometer method was originally introduced by Poggendorff, and is illustrated in fig. 49. It is essentially a method for determining the ratio of two E.M.F.s or P.D.s. Let E be a source of constant E.M.F. (a battery of secondary cells) which maintains a constant current in the conductor A B, and let this conductor be so arranged that contact may be established at any point C of it (by means of a slider), the value of the resistance A C being accurately known. If while the current is flowing along A B we arrange a branch circuit between the points A and C, including a galvanometer G in this circuit, then the P.D. which exists between the points A and C will also produce a current in the branch circuit from A to C, and this current will be indicated by the galvanometer. Suppose next that an E.M.F. E_s is introduced into the branch circuit, the direction of this E.M.F. being from C to A, i.e. in opposition to the P.D. existing between those two points. If we suppose that this E.M.F. is steadily increased, then the effect will be to steadily decrease the current in the branch circuit, and for a certain value of E_s—when it becomes just equal to the P.D. between A and C—the current will vanish, and the galvanometer will show no deflection. We then say that E_s *balances* the P.D. across A C. If the value of E_s be still further increased, a current will flow from C to A, the galvanometer deflection being reversed.

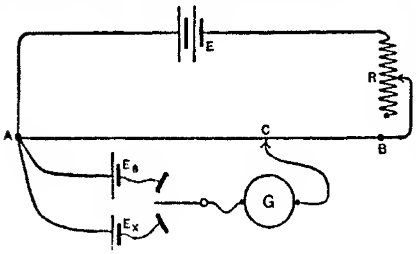


Fig. 49

Instead of supposing that E_s is varied while the P.D. across A C remains constant, let us now take E_s to have a definite fixed value, and to represent the known E.M.F. of a standard cell. Balance may in this case be obtained by varying the P.D. across A C. This may be done in two ways: either by shifting the point of connection C along the wire A B while the current in A B remains constant, or else by keeping C fixed and altering the current along A B by means of an adjustable resistance R placed outside A B. Let us suppose that by either of these two methods balance has been obtained: then we know the value of the P.D. across A C, for this must equal the E.M.F. E_s of the standard cell. Again, if r_s be the resistance of A C corresponding to a state of balance, then the current along A C (or along A B) is given by E_s/r_s. Let, next, an unknown E.M.F. or P.D. of amount E_x be substituted for E_s[1] (this may be quickly done by means of the two-way switch shown in the diagram), and let balance be once more obtained *by shifting the point* C, *the current along* A B *remaining constant.* If r_x is the resistance of A C corre-

[1] Care being taken to connect it up the right way—in opposition to the P.D. across A.C.

sponding to balance in this second case, then E_x/r_x gives the current along A B, and since this has the same value as before, we must have $E_x/r_x = E_s/r_s$, whence $E_x = \frac{r_x}{r_s}E_s$. We thus have a method of obtaining an unknown E.M.F. in terms of a known one, and the arrangement shown diagrammatically in fig. 49 for doing this is known as a *potentiometer*.

A simple but crude and not very accurate form of potentiometer is one in which A B is represented by a bare wire of high-resistivity material stretched along a graduated scale, the diameter of the wire being as uniform as possible, so that the resistance of any portion of it may be assumed to be proportional to its length, and hence the ratio r_x/r_s equal to that of the corresponding two lengths of the potentiometer wire; these lengths may be read off directly on the scale. The point C would correspond to a knife-edge contact attached to a suitable slider.

The main objection to such an arrangement is the want of uniformity in the wire, and its liability to become accidentally damaged or destroyed. For these reasons, in all accurate forms of potentiometer,[1] the slide-wire either forms only a small fraction of the total resistance corresponding to A B, or else is entirely absent.

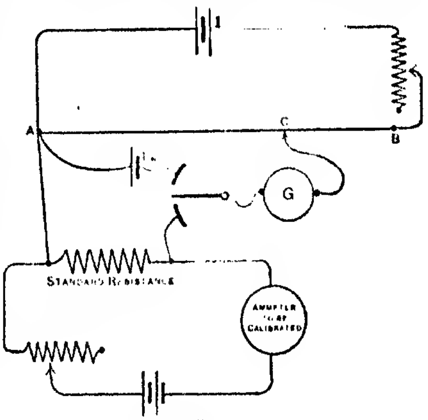


Fig. 50

A good though not very convenient form of potentiometer may always be improvised by using two resistance-boxes, one to represent A C, the other C B, and, balance having been obtained with the standard cell and the value of A C noted, again obtaining balance with the unknown E.M.F. or P.D., by varying the resistances A C and C B in such a manner as to keep their sum constant. This is done by plugging an equivalent resistance in C B when a certain resistance in A B has been unplugged, and vice versa.

It will be noticed that when balance has been obtained, the standard cell is not required to give any *current*. If it is allowed to send large currents, its E.M.F. is lowered, and an error introduced into the measurement. In order to prevent the possibility of large currents passing through the standard cell, it is advisable, during the early stages of the measurement, to introduce a large resistance into the galvanometer and cell circuit, and then to cut this out as balance is approached.

The potentiometer itself as usually constructed is suitable for the measurement of only relatively small E.M.F.s—not exceeding about 1.5

[1] The best-known forms are those manufactured by Messrs. Crompton & Co., Ltd., Messrs. Elliott Brothers, and Messrs. Nalder Bros. & Co.

volt. In combination with a standard resistance it may be employed for the calibration of ammeters as already mentioned, the standard resistance taking the place of E_x in fig. 49 (E_x would be represented by the P.D. across the standard resistance). The complete diagram of connections is given in fig. 50.

The range of the instrument may be enormously extended, so that it may be used for the measurement of very high P.D.s, by the simple device of using a small, accurately *known* fraction, of the unknown high P.D., and balancing this on the potentiometer.

Subdivision of P.D. by Means of Resistance — Calibration of Voltmeters.—A known fraction of an unknown P.D. is easily obtained by introducing between the two points across which the P.D. exists a resistance of known value, and using a known fraction of the total resistance. The P.D. across this section of the resistance then corresponds to the same fraction of the total P.D. This arrangement enables us to calibrate high-reading voltmeters by means of the potentiometer.

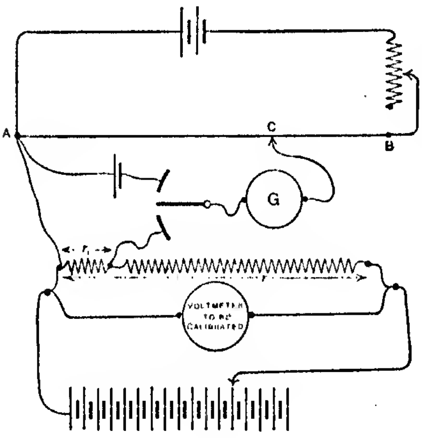


Fig. 51

The diagram of connections is shown in fig. 51. A resistance r is connected in parallel with the voltmeter, so that the P.D. across this resistance is the same as that which exists between the voltmeter terminals. The P.D. across r_1, which is a known fraction of r, is then balanced on the potentiometer; if the value corresponding to balance is E_x volts, then the true value of the voltmeter reading is $E_x\, r/r_1$.

Recording Ammeters and Voltmeters.—In many cases it is desirable to have a continuous record of the current in a circuit, or of the P.D across its terminals, and in such cases *recording* instruments are employed. The electrical and magnetic arrangements of such an instrument present no peculiarities, and are similar to those used in ordinary indicating instruments, but the simple pointer is replaced by a tracing point or pen which traces out a record on a sheet of paper moved at a constant rate by clockwork, in a direction which is at right angles to the motion of the tracing pen.

Short-scale Instruments.—Electrical energy is generally supplied at a constant P.D., and many of the voltmeters found in central stations are intended to enable the switch-board attendants or engine-drivers to maintain a constant P.D. between certain points. It is evident that under these conditions only a limited portion of the scale is used, readings which are either much higher or much lower than the normal P.D. not being required.

It is an obvious advantage to make the working part of the scale as open as possible, and to suppress the remainder. This may be done in a variety of ways. For instance, in the case of a moving-coil instrument the field in which the coil moves may be made of variable strength by making the air-gap of variable length, the shortest length, and hence most intense field, corresponding to the normal reading; a given change in the current through the coil at the normal reading will then produce a much larger displacement of the coil across the field than at a reading either lower or higher than the normal.

Although short-scale instruments are mainly represented by voltmeters, the construction is equally applicable to ammeters, should such be required.

CHAPTER VI

ELECTRICITY SUPPLY METERS

A form of electrical measuring instrument which is used in larger numbers than any other is the instrument which is connected to the wiring of a consumer of electrical energy for the purpose of measuring the total amount of energy supplied to him. From the electrical point of view such instruments may be divided into two large classes, known as *quantity-meters* (coulomb meters, or integrating ammeters) and *energy*-meters (watt-hour meters, or integrating wattmeters).

Electrical energy is supposed to be supplied to a consumer at a constant, or approximately constant, P.D. Now, if the P.D. be assumed constant, it is evident that the power will be simply proportional to the current, and hence the energy to the quantity of electricity (in coulombs or ampere-hours) supplied. On the assumption of constancy of P.D. an instrument capable of measuring the quantity of electricity supplied may be used as an energy-meter. Many forms of such *quantity*-meters are in use.

A more satisfactory form of meter is, however, one capable of measuring correctly the actual energy, and not merely the quantity.[1] Such a meter is called an energy-meter.

The mechanism of an electric supply meter is so arranged as to make the meter read directly in Board of Trade units (kilowatt-hours), either on a series of dials—resembling those used in a gas-meter—or on some other suitably arranged scale.

In many modern instruments the counting mechanism is similar to that made so familiar by the cyclometer. The advantage of this mechanism is that the readings can be more quickly taken and without risk of error.

The fundamental law which a thoroughly satisfactory meter must fulfil is that the rate of registration at any instant should be proportional to the current in the case of quantity-meters, and to the power in the case of energy-meters; and the great difficulty which manufacturers have

[1] If the P.D. at which a consumer is supplied falls below the specified value, a quantity-meter will register an amount which is in excess of the actual energy received by the consumer.

always had to face is the construction, at a reasonable cost, of meters capable of obeying this law with sufficient accuracy.

For convenience in description the meters will be divided into the following classes:—

I. *Quantity-meters.*
1. Electrolytic meters.
2. Permanent-magnet motor meters.

II. *Energy-meters.*
1. Clock meters.
2. Motor meters.
(*a*) With commutator.
(*b*) Induction motor type.

The principle of the purely A.C. meters of the induction motor type will be more fully understood after reading the section on A.C. measurements.

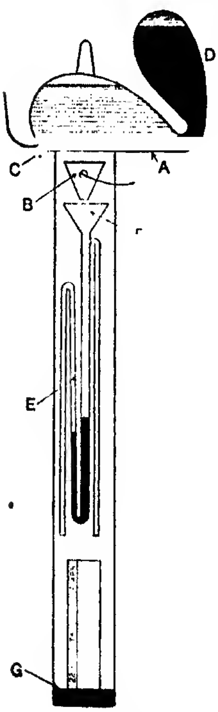


Fig. 52.—Wright Electrolytic Meter

Numerous forms of electrolytic quantity-meters have been devised. Edison used copper and, at a later stage, zinc voltameters; by weighing the cathode plates, after the meter had been in use for some time, the amount of copper or zinc deposited could be determined, and from this the quantity of electricity supplied calculated. Such meters, however, are not direct-reading, and involve the troublesome operations of collecting and weighing the plates.

Two direct-reading electrolytic quantity-meters are in use at the present time. One of these, the Bastian meter, consists of a glass vessel fitted with platinum electrodes which reach down to the bottom of the vessel. The vessel is filled with acidulated water, and a layer of oil is floated on the top of the water to prevent loss by evaporation. The current passing through the meter decomposes the water, and the quantity of electricity supplied is directly proportional to the amount of water decomposed, and may be read off on a suitably arranged vertical scale.

The main defect of the Bastian meter is its large counter-E.M.F. (about 1.5 volt) and consequent large absorption of power; hence it is only suitable for small currents.

In the Wright electrolytic meter, illustrated in figs. 52 and 53,[1] a double iodide of mercury and potassium is used as the electrolyte, and the counter-E.M.F. is so small as to be negligible. D is a reservoir filled with mercury, and connected with a flat glass vessel whose lower part is provided with an annular groove AC

[1] From the *Journal of the Institution of Electrical Engineers*, by permission of the Council of the Institution. In order to prevent any mercury from being shaken down the measuring-tube from the annular anode by mechanical vibration, a guard cylinder of platinum gauze is fixed into the glass lip which forms the inner boundary of the annulus.

containing mercury. This circle of mercury forms the anode, and as the mercury is used up by going into solution, its level is maintained by the flow, at intervals, of additional mercury from the reservoir D, the place of this mercury being taken by the electrolyte, which forces its way past the narrow neck connecting the two vessels when the mercury level in A C falls slightly. The entire vertical glass tube seen below A C is filled with the electrolyte. The conical cathode B is made of iridium-foil, and the mercury which collects on it drops into the funnel F at the top of the siphon-tube E, and gradually fills this tube. When the tube has been completely filled (which

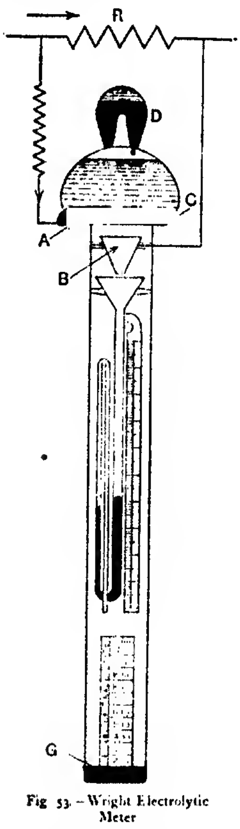


Fig 53.—Wright Electrolytic Meter

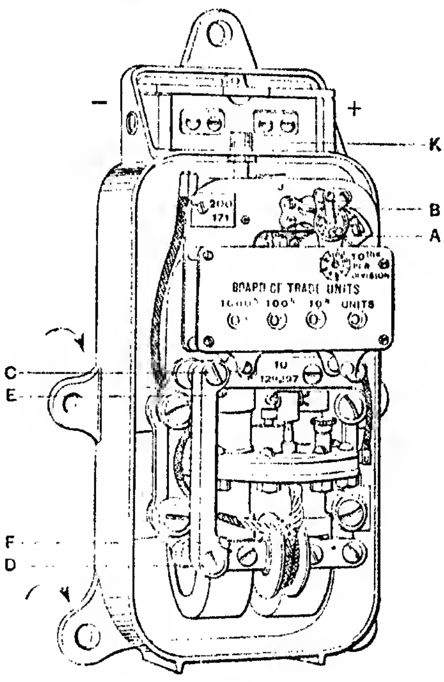


Fig. 54.—Interior of Ferranti C.C. Meter

corresponds to 100 units), the mercury automatically siphons out and falls to the bottom of the outer tube, as shown at G in the figures. The current which produces electrolysis is only a small fraction of the total current, the bulk of which passes through the manganin shunt R. An iron-wire resistance is connected in series with the voltameter, the value being so adjusted as to make the total resistance of the voltameter circuit independent of temperature changes. When all the mercury contained in the reservoir D has been used up, the meter is easily reset by tipping it up and allowing the mercury to flow back into G.

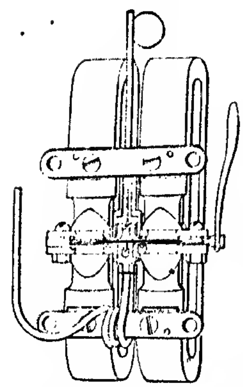


Fig. 55.—Motor Element, Ferranti Meter

Permanent-magnet Motor Meters.—This class includes a number of well-known types. The examples which we have selected for a detailed description are the Ferranti, and the Chamberlain & Hookham meters. Both of these are quantity-meters.

Ferranti Meter.—The general appearance of the Ferranti meter is illustrated in fig. 54, which shows a meter with the cover and the terminal door removed. The instrument consists essentially of a copper disc which rotates in a mercury bath between the poles of a double permanent magnet, the limbs being connected by cross-bars above and below the pole-pieces, as shown in fig. 55.

The mercury chamber is closed by an insulating ring through the right-hand side of which a lead passes and makes contact with the mercury.

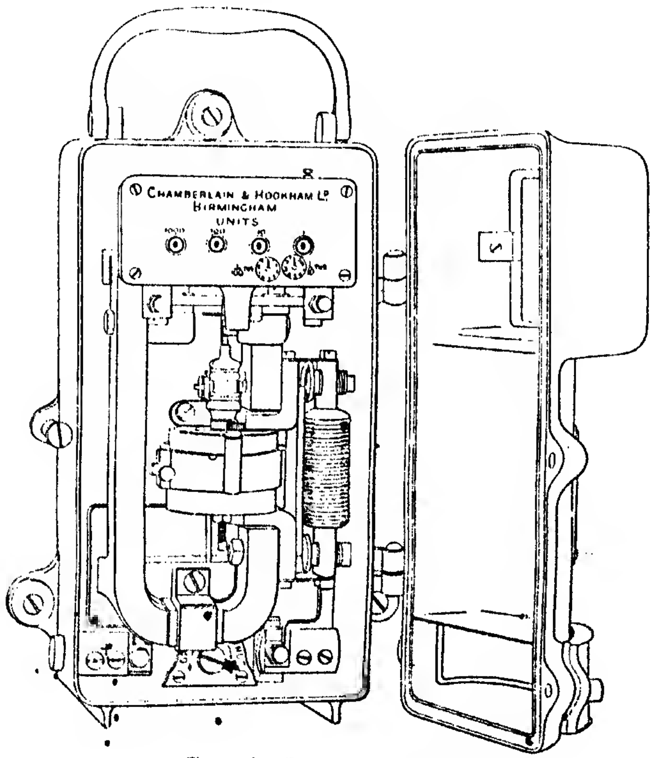


Fig. 56.—Chamberlain & Hookham Meter

A second lead makes contact with the mercury at the bottom of the chamber, and is continued in the form of a coil round the lower cross-bar. The current passes through the copper disc from the side connection to the connection at the bottom of the chamber.

This current flowing from the edge to the centre of the disc is at right angles to the magnetic flux between the right-hand pair of poles, and the disc is thus subjected to a driving torque proportional to the current. The controlling torque is supplied by the interaction of the magnetic flux between both pairs of poles with the eddy currents produced in the copper disc, and is proportional to the speed of the disc.

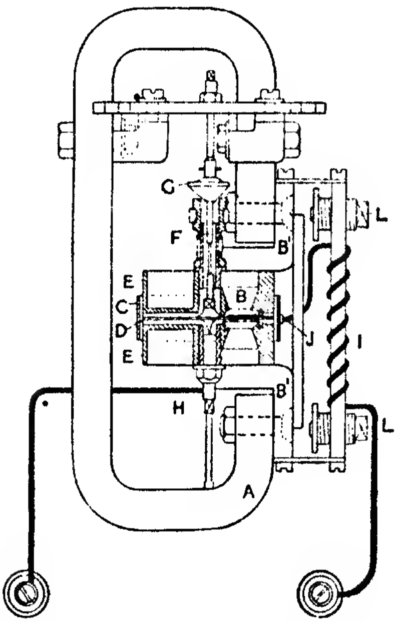


Fig. 57.—Chamberlain & Hookham D.C. Meter Motor Element

The speed at which the disc runs is thus proportional to the current passing through the meter. An additional retarding torque is supplied by the fluid friction of the mercury, and since this increases slightly as the speed rises it has to be compensated for. This is accomplished by the coil on the lower cross-bar, which, by increasing the magnetic flux between the right-hand pair of poles and diminishing it between the left-hand pair, increases the driving torque and at the same time diminishes the retarding torque at the higher speeds.

An iron bar secured by iron screws connects the left-hand ends of the cross-bars. This provides a means of adjusting the speed of the meter. For example, if the meter is found to be reading low, the bar is tightened up by means of the screws. The flux between the left-hand pair of poles is thereby reduced, and consequently the retarding torque is diminished, allowing the meter to run faster. By slackening the screws the meter may be made to run slower.

The counting mechanism is of either the cyclometer or dial type as required.

For currents exceeding 25 amperes the meter is provided with a shunt contained within the case.

Chamberlain & Hookham Meter.—The most recent type of Chamberlain & Hookham House Service D.C. Meter is shown in figs. 56 and 57, and it will be seen that the instrument is very similar to that described above, the principal difference being in the use of a single pair of poles.

For switch-board and other large meters the older pattern shown partly

in section in fig. 58 is retained. A A is a powerful permanent magnet to whose poles are attached the soft-iron pole-pieces B B, separated from each other in the middle by the insertion of a brass distance-piece C. Below these pole-pieces is placed a piece of soft iron D D, so that some of the magnetic lines pass downwards on one side and upwards on the other across the gap which separates D D from B B. In this gap is placed the horizontal copper disc N, which forms the "motor disc", the remainder of the space being filled up with mercury. The remaining magnetic lines pass up two cylindrical pole-pieces E on one side, across iron bridge-pieces G, and then down two pole-pieces E on the other side. In so doing the lines have to cross the air-gaps interposed in their path half-way up each pole-piece. Into these air-gaps is introduced the "brake disc" O, which is made of a low-temperature-coefficient alloy, and which is mounted on the same vertical spindle as the motor disc. This spindle is geared with the counting mechanism.

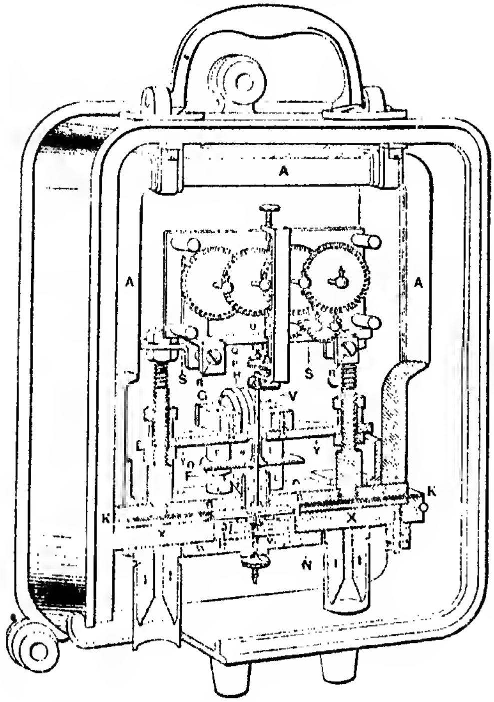


Fig. 58.—Chamberlain & Hookham Meter

The current flows along the left-hand copper strip K, enters the mercury, and then flows through the motor disc in a diametral direction, leaving it by the right-hand copper strip K. The magnetic field in which the disc is placed causes the latter to rotate—the speed of rotation being, even at full load, slow, and hence the friction of the mercury inconsiderable. Since the field is constant, the driving torque acting on the motor disc will be simply proportional to the current. The retarding torque is provided by the currents induced in the brake disc on account of its motion across a non-uniform magnetic field, and, since the intensity of this field remains constant, the retarding torque is simply proportional to the speed. Hence the speed varies in direct proportion to the current.

The compensating coil wound round the upper bridge-piece and carrying the main current serves to correct for the effect of the fluid friction of the mercury by weakening the field in which the brake disc is placed.

Clock Meters—The Aron Meter.—This meter, of which the general appearance is shown in fig. 59, depends for its action on the force exerted between current-carrying coils placed near one another. The heavy-wire fixed coils are connected in series and carry the main current. The fine-wire coils are attached to the lower ends of two similar pendulums.

These coils are connected in series, and each has a large non-inductive resistance in series with it. The whole is connected *across* the mains, so that the current carried by the pendulum coils is proportional to the P.D. between the mains. The coils are so wound that while the force between one fixed coil and the corresponding pendulum coil is one of *attraction*, between the other pair the force is one of *repulsion*. This corresponds in effect to an increase in the gravitational force acting on one pendulum, and to a decrease in that acting on the other pendulum.

This causes one pendulum to oscillate more rapidly than the other. The pendulums are driven by clock mechanisms which are coupled by the differential gear shown in fig. 60. These clock mechanisms are adjusted so as to go at exactly the

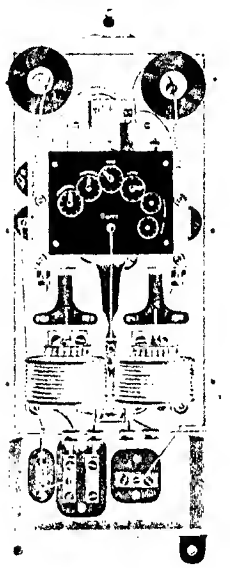


Fig. 59.—General View of Aron Meter with case removed

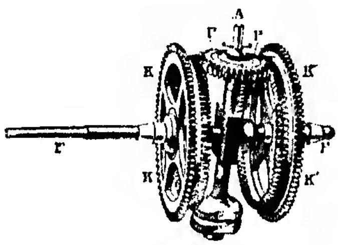


Fig. 60.—Differential Gear of Aron Meter

same rate when the meter is disconnected from the mains, and the difference in their rates when the meter is connected to the mains is proportional to the power being supplied. The two driving-wheels of the differential gear revolve but will not move bodily; but as soon as one wheel moves faster than the other the central wheel will move at a speed equal to the difference in rate at which the two driving-wheels are revolving. The translational movement of the central wheel therefore measures the amount of gain of one clock on the other, and the support for this wheel is geared to a counting mechanism which records its motion. The train of wheels which connects the central wheel to the counters is so arranged that the instrument records directly the number of kilowatt-hours supplied through it.

In order that the two clocks may be driven by one spring a similar

differential gear is used to connect them to the driving mechanism shown in fig. 61. The winding is done electrically, and under normal conditions the winding gear acts at about every half-minute.

In the earlier patterns of this meter long pendulums were used, and even then, although the clocks were synchronized to begin with, it was found that often in the course of time a record (either positive or negative) accumulated on the dials although no current had passed through the meter. In the more recent patterns this difficulty has been entirely got rid of by a simple device which renders *exact* synchronism unnecessary. This permits of the use of short pendulums, and therefore reduces the size of the meter. The device referred to consists of an extra wheel between the differential gear and the counting mechanism, which is thrown into and

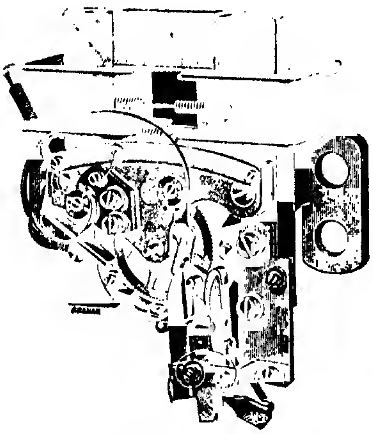


Fig. 61. · Winding Mechanism, Aron Meter

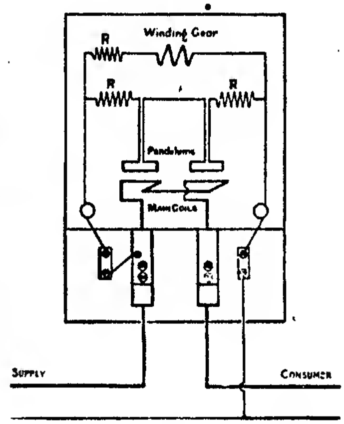


Fig. 62.

out of gear at regular intervals, thus reversing the latter periodically. The record accumulated during one interval is therefore neutralized by that of the succeeding interval. This alone would effect the erroneous and legitimate records alike, so that a commutator has to be provided which reverses the current through the pendulum coils at the same instant as the counting mechanism is reversed. By this means the legitimate record is continually accumulated in the same direction, while the erroneous one is continually destroyed. The reversing-gear and commutator are operated simultaneously every ten minutes by an intermediate spring actuated by the main power spring.

The meter can be used with equal success on D.C. and A.C. circuits, and is in any case a very accurate instrument. The connections for a two-wire D.C. circuit are shown in fig. 62.

When used on an A.C. circuit the instrument itself is unaffected by frequency changes, but the electrical winding-gear has to be designed to suit the normal frequency of the circuit on which the meter is to be used.

By a simple re-arrangement of the internal connections, as shown in

fig. 63, the meter may be used on a three-phase circuit, the principle being that of the "two wattmeter method" (see Chapter V, Article II).

Motor Meters — Thomson Meter.—The commutator type of motor meter, originally suggested by Professors Ayrton and Perry, has become very popular. It is suitable for either D.C. or A.C. circuits. A well-known example in this country is that devised by Professor Elihu Thomson (fig. 64). It consists essentially of a small motor in which the field coils F carry the main current, and the armature A the shunt current. There is no iron in the field-magnet system, so that the magnetic field produced by the current is always directly proportional to it. The force f acting on the armature conductors at any instant is dependent on the strength of field H, and the strength of the shunt current I_s. Hence $f \propto I_s \times$ H, and H being proportional to I, the main current, f is proportional to $I_s I$.

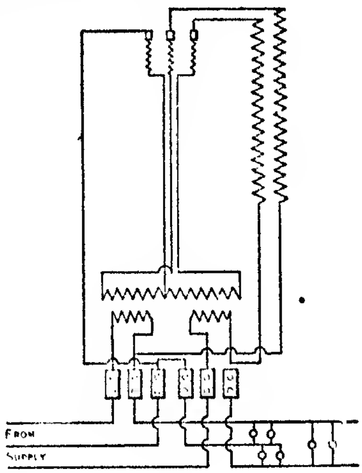


Fig. 63

Now $I_s = \frac{E}{R}$, if the inductance of the shunt circuit is negligible, and R is a constant, we have torque $\propto$ I E at any instant. If there were no resisting torque the motor would increase in speed until the back E.M.F. it produced was equal to the E.M.F. applied. In order that the number of revolutions of the meter may register energy, the speed at which the meter is running must be proportional to the power supplied through it. This is effected if a retarding torque is applied which is *proportional to the speed*. When such an arrangement is adopted the speed will increase until the driving and retarding torques are equal, and consequently, the speed, being proportional to the retarding torque, will be proportional to the driving torque, that is, to the power supplied.

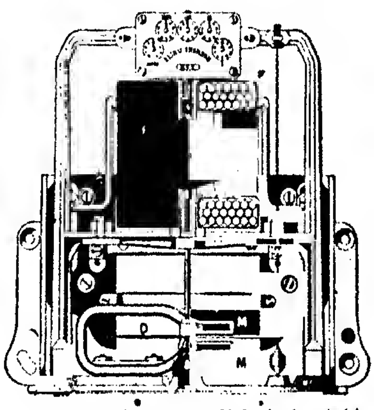


Fig. 64.—Part Section of Thomson Meter, showing principle of construction

This retarding torque is produced, usually, by attaching to the arbor of the armature a flat copper sheet D (fig. 64), which moves between the

poles of a strong permanent magnet. Eddy currents are induced in the copper by its motion, and these eddy currents react on the magnetic field, producing a retarding torque, which is always exactly proportional to the speed at which the plate is moving. In order that the meter may record accurately, many details have to be looked to; the chief difficulty arises from the necessary friction at the bearings. To reduce this to the smallest possible amount, the bottom of the arbor, which is made of hard steel, is pivoted on a sapphire jewel, and the top supported in a cup of hard steel. The spindle is steadied at both ends by collar bearings, loosely fitting. The friction of the brushes on the commutator (which is usually of eight parts) is eliminated in the latest type by using silver strip, which is arranged to press edgewise on the commutator surface.

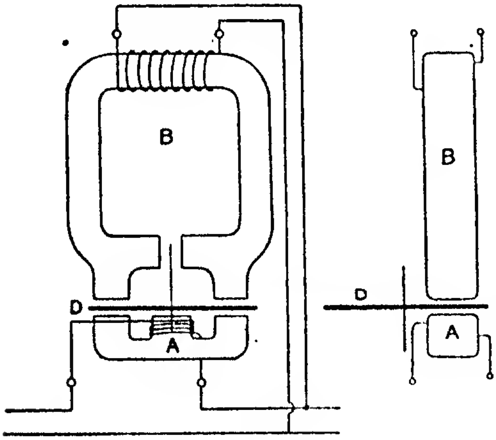


Fig. 66

Owing to the inductance of the shunt winding it is impossible to get *exact* phase equality between the shunt current and the applied P.D., no matter how large a non-inductive resistance is connected in series with the shunt winding. This leads to a limitation of the accuracy of the instrument when used on A.C. circuits in which the power-factor is low. For example, in certain extreme cases, where the meter has been used to measure the power expended in the dielectric of a cable (the current leading by nearly 90° on the applied potential difference), the meter has been found to go backwards, thus showing that the phase difference between the main current and shunt current was greater than 90°, and that therefore the instrument was no longer recording power. Except in these cases the meters are, however, very reliable.

Westinghouse Meter.—This meter is very similar to the one just described, and the general construction is well illustrated in fig. 65.

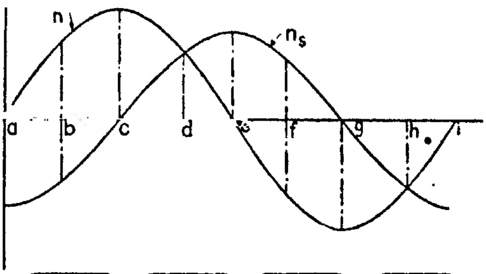
A small adjustable coil (shown to the right of the right-hand fixed coil in fig. 65) is connected to the shunt circuit, and supplies a small torque to compensate for friction, and to make the starting-current as small as possible. The meter is normally fitted with a roller-cyclometer counting mechanism, and is designed for use on D.C. circuits.

It may be mentioned that the Westinghouse Company have recently been making experiments with a view to employing impregnated paper in place of ivory as an insulation in the commutator of these instruments.

The commutator which has been used up to the present consists of three silver segments mounted on an ivory tube with ivory end-rings.

Induction Motor Type.—The driving torque in meters of this class is obtained by the interaction of a gliding magnetic field (produced by two component fields 90° out of phase with one another) with the eddy currents produced by it in a metallic disc. The principle is thus similar to that of the induction motor, and meters of this type can only be used on A.C. circuits. The following rough explanation will give an idea of the action.

In fig. 66, which shows diagrammatically one arrangement of the motor element, B is the shunt magnet excited by a coil connected *in parallel* with the mains, and A is the series magnet excited by a coil connected *in series* with the mains.



The metallic disc D, which is made as light as possible, is placed so that the pole-pieces of the magnets cover a strip parallel to a diameter.

The disc has a jewel pivot at the lower end, and the upper end of the spindle is connected by a worm gear to the recording mechanism. The controlling torque is supplied by the eddy currents produced in the disc by a permanent magnet (not shown in the figure) placed opposite to the driving magnets.

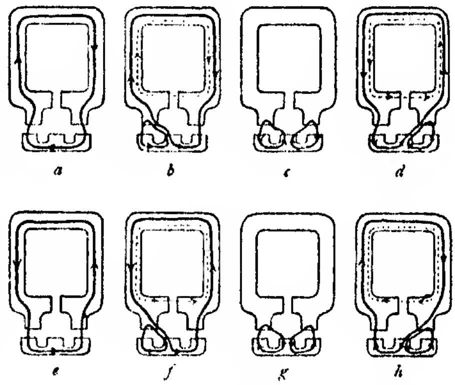


Fig. 67

The current i_s in the highly inductive shunt circuit lags behind the current i in the series circuit by an angle which may by suitable design be made practically 90°.

The production of the gliding magnetic field will be understood from the figs. 67, *a* to 67, *h*. The fluxes n and n_s, produced by the series and shunt magnets respectively, are assumed to be exactly 90° out of phase and to vary as shown in fig. 67. The figs. 67, *a*, 67, *b*, &c., correspond to the fluxes at times *a*, *b*, &c., in fig. 67, and show in a very rough fashion the paths and directions of the lines of force at these instants.

It will be seen that the resultant flux moves from the left hand across the pole-pieces to the right-hand side of the magnets during the first half of the cycle, and is followed by a flux of the same magnitude but of

opposite sign which moves in the same direction during the second half of the cycle.

A flux is, therefore, continually gliding from left to right, and cutting the strip of disc under the pole-pieces at right angles. The eddy currents thus produced react with the flux and continually urge the strip forward from left to right, thus causing the disc to rotate.

The phase difference between n and n_s is dependent on the phase difference between the current in the mains and the P.D. across them. The resultant flux will clearly move more and more slowly from left to right as the mains current and P.D. become more and more out of phase with one another, and will eventually become stationary when the current and P.D. are 90° out of phase, the driving force on the disc, therefore, also falling to zero. By suitably designing the magnets the driving torque may be made practically proportional to $I I_s \cos \phi$[1] over the whole range of load, and therefore proportional to the true power supplied by the mains, since I_s is proportional to the P.D. across the mains.

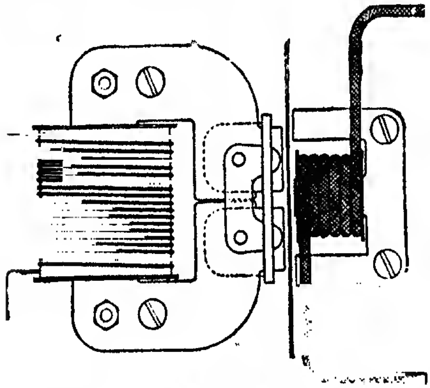


Fig 69.—Line Diagram of Motor Element, Ferranti A.C. Meter

The controlling torque, being due to eddy-currents, is proportional to the speed. The amount recorded by the meter is, therefore, proportional to the true energy supplied by the mains. Meters of this class can be manufactured at a low cost but are liable to error when running on low loads and at low power-factors.

In the more elaborate patterns means are provided to compensate for errors due to these conditions.

The Ferranti Single-phase A.C. Meter, shown in figs. 68 and 69, is a good example of the class just described. The construction will be readily seen from the figures.

Adjustments for low loads and power-factors are fitted to the motor element.

Either cyclometer or dial type recording mechanisms as required are supplied.

The Westinghouse A.C. Meter, shown in fig. 70, is another well-known meter of the induction-motor type, and is of very similar construction.

The Electrical Company's meter, shown in fig. 71, has a different form of magnet system, and the shunt and series coils are differently arranged. A single C-shaped magnet core, having a set of three pole-pieces below and a single pole-piece above, is used. The shunt coil is wound on the lower part of the middle pole-piece of the bottom set. The series coil is

[1] Where $\cos \phi$ is the power-factor of the load on the mains.

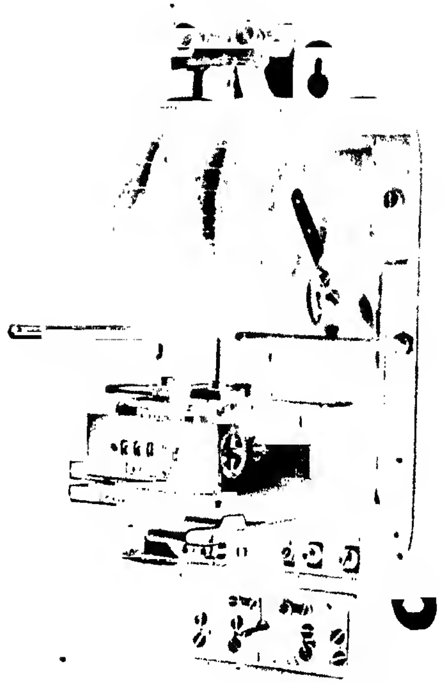


Fig. 68. — Westinghouse D.C. Watt-Hour Meter

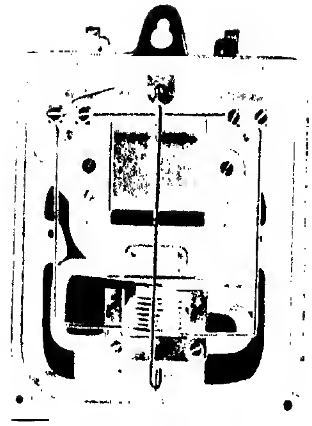


Fig. 69. — Ferranti Single Phase A.C. Meter

Fig. 70. — Westinghouse Single Phase A.C. Meter

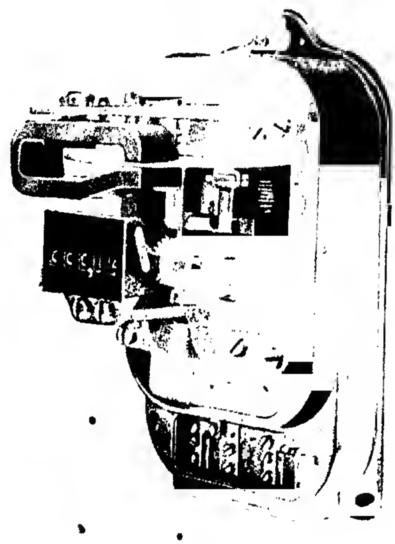


Fig. 71. — Electrical Company's Single Phase Meter

divided, and half is wound on each side pole-piece near the tip. Iron laminations, providing leakage paths, connect the side pole-pieces to the middle one, the laminations passing just above the shunt coil and just below the series coils. Arrangements for adjusting the meter for low and for inductive loads are provided.

Modifications of all the above patterns, suitable for use on polyphase circuits, are also manufactured.

CHAPTER VII

MAGNETIC MEASUREMENTS

Magnetic Force and Magnetic Induction — Susceptibility and Permeability.—We have already, in Chapter I, considered some of the more important magnetic units. It will now be necessary to introduce a number of additional units which are indispensable in connection with the testing of the magnetic qualities of iron and steel.

One of the terms which we have frequently had occasion to use is magnetic force, and we have defined this as the force which would act on a unit pole if placed at the point considered, the assumption being tacitly made that the point lies in an air-space. Let us now suppose that this restriction is removed, and that we consider a point inside a mass of iron. If we attempt to determine the force acting on a unit pole by scooping out a cavity in the region immediately surrounding the point, and introducing our unit pole into this region, then we find that the force depends on the shape of the cavity, and varies from a very small to a very large value, according to the shape. Unless, therefore, we specially restrict the meaning of the term "magnetic force", this term ceases to have any definite meaning. It has been assigned a perfectly definite meaning by specifying the manner in which the cavity surrounding the given point is to be shaped. We agree, namely, to restrict the term to denote that particular value of the force acting on the unit pole which is obtained by forming the walls of the cavity so that they are everywhere *along the direction of magnetization* of the iron. The main effect of this arrangement is the absence of magnetic polarity on the bounding surfaces of the mass of iron which immediately surrounds the point—in other words, the effect due to the magnetization of the mass of iron immediately surrounding the point is eliminated. The magnetic force so obtained is usually denoted by **H**.

It is usual, when the magnetization of iron is brought about by electrical means, to regard the magnetization as being due to a *magnetizing force* equal to the magnetic force produced. **H** is therefore frequently referred to as the magnetizing force.

Let us now take the other extreme, and suppose the cavity so shaped that the development of magnetic polarity on its bounding surfaces is as great as possible. This will be obtained by making the walls every-

where *perpendicular to the direction of magnetization.* Let the magnetic polarity per unit of area of the wall be **I**.[1] Then the ratio **I**/**H**, generally denoted by κ, is defined to be the *susceptibility* of the material. We thus have

$$\kappa = \frac{\mathbf{I}}{\mathbf{H}}, \text{ or } \mathbf{I} = \kappa \mathbf{H}.$$

Let us now introduce our unit pole into this second cavity, and determine the force which acts on it. This force is defined to be the *magnetic induction* at the point considered, and is denoted by **B**. The ratio **B**/**H**, denoted by μ, is called the *permeability* of the material. Thus

$$\mu = \frac{\mathbf{B}}{\mathbf{H}}, \text{ and } \mathbf{B} = \mu \mathbf{H}.$$

Instead of saying that the magnetic induction has the value **B** at a given point, we sometimes say that there are **B** lines of magnetic induction per unit of area of a plane drawn at right angles to the direction of magnetization at that point.

If we take any surface bounded by a closed curve, divide it up into a large number of small elements of area, multiply each little area by the value of the magnetic induction in a direction perpendicular to the area,[2] and then take the sum of all the products so formed, then this sum gives us what is termed the *total number of magnetic lines* crossing the surface, or the *total magnetic flux* through it. For this reason **B** is sometimes spoken of as the magnetic *flux density*, since it corresponds to the flux per unit of area.

We may now obtain a relation connecting **H**, **I**, and **B**. The force which acts on the unit pole in the second case (walls perpendicular to direction of magnetization), and which we denote by **B**, may be regarded as made up of two amounts: (1) the force **H**, which exists independently of magnetic polarity in the immediate neighbourhood of the point; (2) the additional amount due to the development of polarity on the walls of the cavity in the second case. In order to find this additional amount, we notice that the pole-strength per unit of area being **I**, and the lines of force going straight across the cavity, the number of these lines per unit of area is $4\pi\mathbf{I}$, since each unit pole gives rise to 4π lines. But the number of lines of force per unit of area is the same as the force acting on the unit pole. Hence, this latter amounts to $4\pi\mathbf{I}$. Thus the total force acting on the unit pole **B** is the sum of **H**, the force existing quite apart from the presence of any magnetic material in the neighbourhood of the point considered, and $4\pi\mathbf{I}$, which is the additional amount contributed by the development of magnetic polarity on the walls of the cavity. We thus have

$$\mathbf{B} = \mathbf{H} + 4\pi\mathbf{I}.$$

[1] This, it will be remembered (see Chapter I), is identical with intensity of magnetization, or magnetic moment per unit volume.

[2] It must be remembered that magnetic induction is a quantity possessing *direction* as well as magnitude.

From this we at once deduce the relation

$$\mu = \frac{B}{H} = 1 + 4\pi\frac{I}{H},$$

$$\text{or } \mu = 1 + 4\pi\kappa.$$

Stress between Two Magnetized Surfaces.—Imagine two plane magnetized surfaces separated by a short air-gap, the intensity of magnetization being **I** and the magnetic induction **B**. Then a unit pole introduced into the gap would experience a force of **B** dynes. This force may be regarded as due partly to the north and partly to the south pole-face, the amount contributed by each being one-half of the total, i.e. $B/2$. Now imagine the unit pole being part of one of the surfaces. Then this surface will exert no force on the pole in a direction at right angles to the surface, so that the total force on a unit pole which is part of either surface is that due to the other surface only, viz. $B/2$. But the pole-strength of either surface per sq. cm. is **I**. Hence the pull per sq. cm. of the surface is $\frac{1}{2}BI$. If **H** be negligible in comparison with **B**, then $I = \frac{B}{4\pi}$, and the stress between the two magnetized surfaces, per sq. cm. of area, becomes

$$\frac{B^2}{8\pi}.$$

Relation connecting B and H. Magnetization Curves.—If we take a piece of iron which is in a non-magnetic condition to start with, and apply to it a known magnetic force **H** whose value is steadily increased, measuring **B** (by any of the methods to be explained presently) at the same time; then, on plotting a curve connecting values of **B** and **H**, we find that this curve, as shown in fig. 72, may be described roughly as consisting of three approximately straight-line portions united by curves. Such a curve is called a **B-H** or *magnetization curve*. The upper bend in the curve is spoken of as the *knee*, and for any point lying well beyond the knee, and corresponding to the region in which a relatively large increase of **H** produces only a relatively small increase in **B**, the iron is said to be *saturated*.

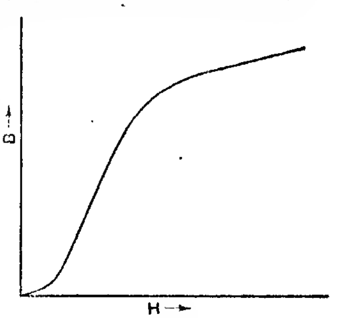


Fig. 72

A knowledge of the exact relation connecting **B** and **H** is of prime importance in all problems concerning the design of electromagnets, and in the following table we give the data from which the reader may plot the **B-H** curves for (*a*) cast iron; (*b*) wrought iron of good quality; (*c*) dynamo-magnet cast steel:—

H.	B.		
	Cast Iron.	Wrought Iron.	Cast Steel.
5	1900	9000	8150
10	3000	12200	12100
15	3900	13600	14000
20	4550	14450	15000
25	5100	15050	15700
30	5500	15500	16200
35	5870	15800	16500
40	6180	16100	16750
45	6450	16300	16900
50	6700	16500	17140
60	7150	16800	17450
70	7530	17000	17750
80	7900	17220	18000
90	8250	17400	18200
100	8570	17580	18400
125	9200	17930	18900

The Magnetic Circuit — Calculation of Ampere-turns required to produce Given Magnetic Flux.—Every line of magnetic induction forms a closed loop or curve, and the assemblage of the paths followed by the magnetic lines is spoken of as the *magnetic circuit*. In order to produce a given magnetic flux in a circuit a definite M.M.F. must be applied to it, and by analogy to an electric circuit the ratio $\frac{\text{M.M.F.}}{\text{magnetic flux}}$ is termed the magnetic *reluctance* of the circuit.

We shall now briefly indicate how the ampere-turns necessary to produce a given flux may be approximately calculated. Let us suppose that we proceed around a closed loop which is a line of induction, and that the value of the magnetic flux through every cross-section of the circuit is given. Then by dividing the flux by the cross-section we obtain the value of **B** in every part of the circuit. In the most general case, in passing along any line of induction we find that **B** varies continuously from point to point. In order to simplify the problem it is usual to divide the line of induction into a number of parts, and to assume that **B** has a constant value along each part. Then if the values of **H** corresponding to those of **B** be determined by reference to the **B**-**H** curves, the sum of the products obtained by multiplying each value of **H** by the corresponding length of the curve gives the M.M.F. Finally, 0.8 of the M.M.F.[1] gives the ampere-turns which must be provided.

A numerical example will make the matter clear. Let it be required to find the ampere-turns necessary to produce a flux of 40,000 C.G.S. lines across the space between the parallel plane faces of the pole-pieces P P in fig. 73, the area of each pole-face being 8 sq. cms., the pole-pieces being of

[1] See Chapter I.

cast iron, and the sum of the lengths of the portions A B and C D of the dotted line of induction amounting 10 cms. Let the length of the air-gap between the pole-pieces be .5 cm. Further, let the exciting coil be wound on the cylindrical core M, made of wrought iron, 4.5 cms. long and 4 sq. cms. in cross-section.

Of the lines produced by the exciting coil at the middle cross-section of the core M, only a certain fraction will pass across the plane faces of the pole-pieces, a large number *leaking* out into the surrounding air-space, forming closed loops of induction such as the one indicated by the chain-dotted curve in the figure. Before we can solve the problem we must know what fraction of the total lines produced in M will cross the air-gap, where they are wanted. The ratio of the maximum flux through M to the useful flux across P P is termed the *leakage coefficient* of the magnetic circuit. Let us assume that in the case considered its value is 1.3. We further assume that instead of a uniformly distributed leakage we have to deal with a leakage which takes place suddenly along the edges of the pole-pieces. This is tantamount to assuming that **B** is constant along each of the portions D A, A B, B C, and C D of the magnetic circuit.

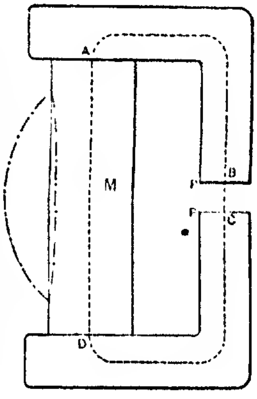


Fig. 73

Now consider each of these portions separately, and find the corresponding values of **H**. In the air-gap $\mathbf{H} = \mathbf{B} = \frac{40,000}{8} = 5000$. The flux through the core M and the pole-pieces is, on account of leakage, $1.3 \times 40,000 = 52,000$. Hence, **B** in the pole-pieces $= \frac{52,000}{8} = 6500$. Referring to the B-H curve for cast iron, we find the corresponding value of **H** to be 46. Next, in the wrought-iron core we have $\mathbf{B} = \frac{52,000}{4} = 13,000$. A reference to the curve for this material gives $\mathbf{H} = 12.5$. The total M.M.F. round the circuit is thus

$$\underbrace{5000 \times .5}_{\text{Air-gap.}} + \underbrace{46 \times 10}_{\text{Cast-iron pole-pieces.}} + \underbrace{12.5 \times 4.5}_{\text{Wrought-iron core.}} = \underbrace{3016}_{\text{Total.}} \text{ approximately.}$$

The corresponding ampere-turns are $.8 \times 3016 = 2413$ approximately.

It will be noticed that by far the largest portion of the M.M.F. is employed in maintaining the flux across the air-gap. Such is frequently the case in practice.

Calculation of Winding to produce Given Number of Ampere-Turns.—A coil may be required to produce a given number of ampere-turns either with a given exciting current or with a given P.D. across its terminals. In the former case the number of turns is at once obtained

by dividing the ampere-turns by the amperes. Thus, in the numerical example just considered, if the exciting current is given as 10 amperes, the turns which must be wound on the coil are 241. The *size* of the wire under these conditions does not, obviously, affect the result, and is determined solely by considerations regarding the permissible rise of temperature.

Let us next suppose that V, the P.D. at which the excitation is to be produced, is given. In order to calculate the winding we have to make an assumption regarding the probable *depth* of winding, and from this to find the *mean* length of a turn.[1] Let the length of turn so determined be l, and let the size of wire be such that the resistance of this length is r. If we denote the current by i, and the number of turns by S, then, since the total resistance of the coil is Sr, we have $i = \frac{V}{Sr}$, or $r = \frac{V}{Si}$; i.e.,

$$\text{resistance of mean turn} = \frac{\text{P.D.}}{\text{ampere-turns}}.$$

Knowing the length l of the turn, we find the resistance of the wire per unit of length, r/l; a reference to wire tables then gives the *size* of wire required.

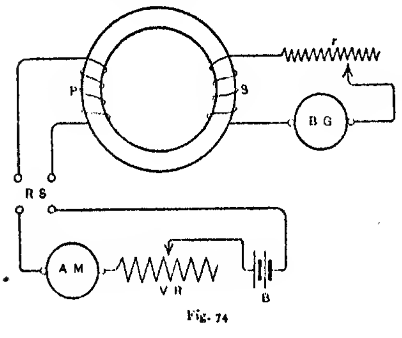


Fig. 74

It will be noticed that in this second case (when the P.D. is given) the ampere-turns depend on the *size* of wire used, and are (so long as the length of the mean turn is not appreciably altered) independent of the number of turns, or the amount of wire used; this latter is determined solely by considerations regarding heating.

Experimental Determination of B–H Curve—Ballistic Method.—By far the most satisfactory method of finding the relation which connects **B** and **H** is to use the sample of the material to be tested in the form of a ring, such as the one shown in fig. 74. A convenient size is one having an external diameter of about 6 inches, a radial depth of ½ inch, and an axial length of 1 inch, the section being rectangular (it would be an advantage to arrange the area of the cross-section to correspond to an exact number of sq. cms.). This ring is wound with some 100 or 200 turns of fine silk-covered copper wire, carefully insulated; this winding, which forms the secondary or exploring coil, need not be uniformly distributed, but may be concentrated over a small portion of the ring. Another, *uniformly distributed* winding, consisting of several layers, and forming the primary or magnetizing coil, is then wound round the ring.

The arrangement of connections is shown in fig. 74, where P and S are

[1] In electromagnets of moderate size, the depth of winding lies between 1 and 3 inches.

the primary and secondary coils,[1] R S a reversing switch, A M an ammeter for measuring the exciting current, V R a variable resistance, and B the battery (of secondary cells) supplying the current. The secondary coil is joined in series with a resistance r of convenient amount and a *ballistic* galvanometer B G—i.e. a galvanometer having a long period of vibration and subject to as little damping as possible.[2]

The problem is to find **H**, and the corresponding value of **B**. The former is at once found by the method already explained in Chapter III. If i = exciting current in amperes, S_1 = number of primary turns, and l = mean circumference of ring, then the mean value of **H** over the cross-section of the ring is given by

$$\mathbf{H} = \frac{1.257\ S_1 i}{l}.$$

In order to find **B**, we observe the throw of the galvanometer obtained on reversing the primary current. If a = cross-section of ring in sq. cms., then the change in the magnetic flux through each turn of the secondary coil on reversal amounts to $2\mathbf{B}a$. If we suppose that the reversal takes t secs., then the mean rate of change of flux—which is numerically equal to the mean induced E.M.F. in C.G.S. units—amounts to $2\mathbf{B}a/t$, and the total mean E.M.F. (in C.G.S. units) in the secondary coil is $2\mathbf{B}S_2a/t$, where S_2 = number of turns in secondary coil. If now we suppose that the total resistance of the secondary circuit (including secondary coil, galvanometer, and the extra resistance r) amounts to r_2, then the mean value of the induced secondary current is $\frac{2\mathbf{B}S_2a}{tr_2}$, and as this current lasts t secs., the total quantity q discharged through the galvanometer amounts to

$$q = \frac{2\mathbf{B}S_2a}{r_2},$$

whence

$$\mathbf{B} = \frac{r_2}{2S_2a}q \quad \ldots\ldots\ldots(1)$$

All the quantities on the right-hand side are known except q. The final step, then, is to find the connection between q and the corresponding throw of the galvanometer, i.e. to calibrate the galvanometer.

There are several ways of doing this. The most satisfactory is to use a *standard solenoid* (some 4 feet long and 3 inches in diameter), uniformly wound and having a small exploring or secondary coil *inside* it at the middle. When all the readings have been taken with the sample ring, the standard solenoid is substituted for it, and the throws obtained on the galvanometer scale by reversing a number of gradually increasing primary currents are noted. The corresponding quantities may now, however, be easily calculated. For since **H** has the same numerical value as **B** in this

[1] For the sake of clearness in the diagram, P is shown concentrated over a portion of the ring; in reality it is *uniformly* distributed.

[2] The most convenient form of ballistic galvanometer for carrying out such tests is a moving-coil galvanometer of the *undamped* type (i.e. without any metallic frame supporting the coil).

case (there being no iron core), the flux through each turn of the secondary coil is given by

$$\frac{1.257\, S_1'\, i_1\, a_2}{l_1},$$

where S_1' = turns in solenoid, i_1 = current in amperes, a_2 = area in sq. cms., of one turn of secondary coil, l_1 = length, in cms., of solenoid (see Chapter III).

By reasoning similar to that employed above in connection with the ring, we find for the quantity discharged through the galvanometer when a primary current of i_1 amperes is reversed the value

$$\frac{2 \times 1.257\, S_1'\, S_2'\, i_1\, a_2}{l_1\, r_2},$$

where S_2' = number of turns in secondary coil of solenoid. Everything in this expression being known, we can calculate the quantity.

A curve may now be plotted giving the relation connecting galvanometer throw with quantity, and this *calibration curve* may be used for determining the values of q corresponding to the various throws obtained in the experiment with the sample ring. Formula (1) may then be used for calculating **B**.[1]

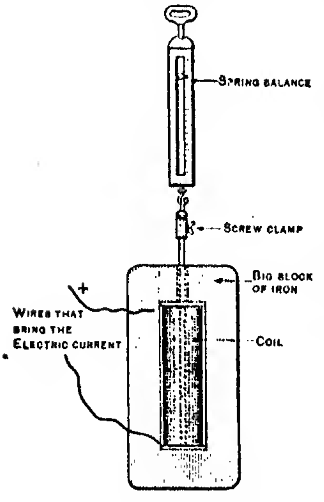


Fig. 75.—Thompson's Permeameter[2]

Traction Methods—S. P. Thompson's Permeameter.—The ballistic-ring method just described, although requiring a considerable amount of time and experimental skill, is the most reliable method as yet devised for the magnetic testing of iron or steel. A rough *workshop* method, in which the value of **B** is calculated from the pull required to separate two magnetized surfaces originally in contact, has been devised by Professor S. P. Thompson, and will now be briefly described.

The instrument is shown in fig. 75, and is termed by its inventor a "permeameter". The specimen is in the form of a cylindrical rod, which passes through one side of a massive rectangular block of iron, technically termed a "yoke", into a magnetizing solenoid, the lower faced end of the rod butting against the faced inner surface of the yoke. The cross-sectional area of the yoke being very much larger than that of the rod, **B** inside the yoke will have a small value, and **H** in the yoke will be very small in com-

[1] It must be particularly noted that all the quantities on the right-hand side of (1) are in C.G.S. units. So that if the resistance of the secondary circuit is, e.g., given in ohms, then before substituting in the formula it must be multiplied by 10^9.

[2] From Prof. Ewing's *Magnetic Induction in Iron and Other Metals*. By permission.

parison with its value inside the rod. We may therefore neglect the work done in carrying a unit pole along a line of induction inside the yoke, and assume that the M.M.F. consists entirely of the work done in conveying the unit pole inside the rod from end to end of the solenoid. On this assumption, $\mathbf{H} = 1.257\,S\,i/l$, where S = turns in solenoid, i = current in amperes, l = distance between inner faces of yoke.

The pull required to detach the rod from the yoke having been found, we divide this pull by the area of contact (or area of cross-section of the rod), thus obtaining the stress per unit of area. If this stress be expressed in dynes per sq. cm., then, as we have already shown, its value is $\mathbf{B}^2/8\pi$, and we have a means of calculating **B**.

Residual Magnetism—Coercive Force—Hysteresis.—When a piece of iron is subjected to the action of a magnetic force which is subsequently withdrawn, the induction does not disappear, and in general a very large fraction of it is retained by the specimen. The curve connecting **B** and **H** is, in fact, for descending values of **H** totally different from the curve for ascending values. This is shown in fig. 76, where O P B is the ascending, and B C the descending branch of the curve. The induction O C which remains in the specimen is termed *residual induction*, and its value depends, for a given specimen, on the maximum induction previously reached by it. Thus, if the maximum induction had been carried up to the point D in fig. 76, the corresponding residual induction would have been represented by O E. The property possessed by the magnetic materials of retaining a certain amount of magnetization after the withdrawal of the magnetic force is sometimes spoken of as *retentiveness*. In order to wipe out the residual induction, a definite magnetic force O F must be applied in the reverse direction, and this force—which will, of course, vary with the residual induction—is termed the *coercive force* for that induction.

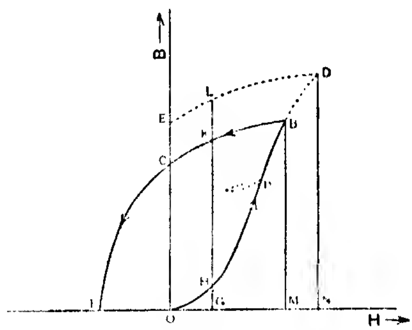


Fig. 76

If in fig. 76 we consider any value of **H**—such as O G—then corresponding to this value of **H** we get a value of **B** which is represented by G H if the specimen is thoroughly demagnetized to start with, and **H** is simply increased from zero to O G. But we get the very much larger value G K if **H** is first increased to O M, and then brought down to O G; and the still larger value G L if it is increased to O N, and then reduced to O G. In fact, corresponding to any value of **H** we have an *infinite number* of values of **B**, depending on how the particular value of **H** has been arrived at. There is thus no definite connection between these two quantities, unless we specify the manner in which the changes in **H** and **B** take place. When we speak of the "**B–H** curve", we always mean the particular curve

obtained when the specimen is thoroughly demagnetized to start with,[1] and when **H** is *steadily increased*, and never allowed first to increase and then to decrease, or vice versa.

If, for example, when the point P (fig. 76) is reached we first decrease **H** and then restore it to its original value, the curve connecting **B** and **H** will trace out the loop shown by the dotted line, and when **H** has regained its original value, **B** will have increased by a small amount.

The facts which we have been considering may be summed up by saying that a magnetic material always tends to retain its magnetic state—the changes in the magnetic force being always ahead, as it were (not as regards time, but as regards phase), of those in the induction. The term *hysteresis*, introduced by Professor Ewing, is used to describe this species of magnetic inertia.

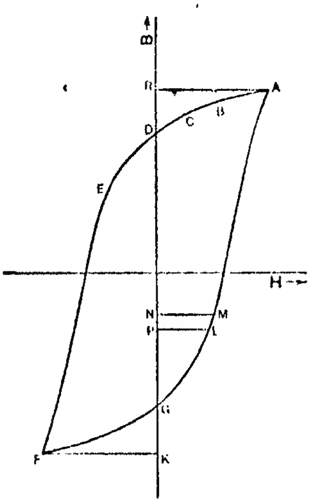


Fig. 77

Hysteresis Loops and their Experimental Determination.—If a piece of iron be subjected to a series of cyclical changes in the magnetic force, consisting of repeated applications of it in opposite directions, the magnetic force increasing continuously from a zero to a fixed maximum value, then the curve connecting **B** and **H** will take the form of a closed loop, as shown in fig. 77, which consists of an ascending and a descending branch, the two branches enclosing a certain area. Such a closed loop, representing a complete cycle of magnetic operations, is termed a *hysteresis loop*.

The experimental determination of the hysteresis loop corresponding to a given maximum value of **B** or **H** is a matter of extreme importance, and we shall now explain how this determination may be carried out.

The loop being symmetrical, it will be completely determined by finding either of its two branches. The most convenient method is always to start from the corner or peak of the loop, A in fig. 77, and to observe the throws of a ballistic galvanometer obtained on passing from A to a number of points B, C, D, &c., on the descending branch, the calculation of the changes in **B** and **H** being carried out as already explained in connection with the **B**-**H** curve.[2] In order to pass from A to B, for example, all that is necessary is to suddenly reduce the current by an amount corresponding to the value of **H** at that point; and this may be easily done by suddenly unplugging resistance in the primary circuit. By varying this

[1] Such demagnetization is effected by subjecting the specimen to the action of a magnetic force which undergoes rapid reversals while its amount slowly diminishes and ultimately vanishes.

[2] The specimen is supposed to be in the form of a ring, with primary and secondary coils arranged as explained above.

resistance, a series of points between A and D may be obtained. In order to reach a point (such as E) which lies lower down, to the left of the B-axis, a *reversal* of current is necessary, but the reversed current must have a value much smaller than the maximum corresponding to the points A or F. The operation of passing from A to E involves, then, a sudden increase in the resistance of the primary circuit, accompanied by a reversal.

In order to be able to effect these changes conveniently, a reversing switch may be used which is connected to a variable resistance R as shown in fig. 78—an arrangement due to Professor Ewing. RS is the reversing switch, the dotted lines showing the change of connections due to throwing over the switch. It will be noticed that when the resistance R is plugged up, throwing over the switch from right to left produces a simple reversal; but if some of the resistance is unplugged, the introduction of this into the primary circuit accompanies the act of reversal.

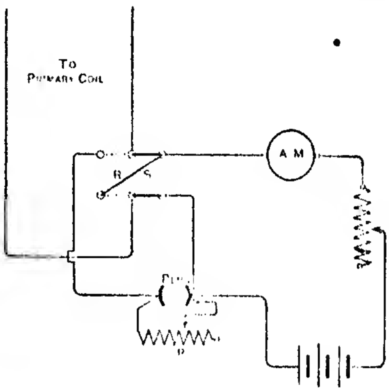


Fig. 78

Between every two readings, R is plugged up, and the switch worked to and fro a number of times, so as to produce a series of simple reversals and thereby restore the iron to a steady cyclic state.

While taking readings corresponding to points lying between A and D in fig. 77, the switch is kept in the left-hand position, the reduction of current being obtained by suddenly unplugging any desired amount of resistance. For points lying beyond D, such as E, the switch would be in the right-hand position, and the point E would be reached by throwing it over to the left.

Energy dissipated by Hysteresis - Steinmetz's Law.—When a mass of iron is carried through a rapid succession of magnetic cycles, it is found to grow hot. The development of heat is due to the dissipation of part of the energy spent in producing the magnetization, and it may be shown that the amount of energy dissipated in every cubic cm. of the iron during a complete magnetic cycle is directly proportional to the area of the corresponding hysteresis loop.

Referring to fig. 77, let us suppose that at a certain stage the point L has been reached, and that H increases from PL to NM, the increase being supposed small, and the mean value of the current, *in C.G.S units*, during this change being i. Let b = NP be the corresponding change in B. If a = cross-section of the ring in sq. cms., then the total change of magnetic flux through each turn of the primary coil is ba, and if this takes place during t secs., the average induced E.M.F. is Sba/t, where S = number of turns in primary coil. Since the current is increasing, the induced E.M.F. opposes it, and the power spent in maintaining the

current against this E.M.F. is $iSba/t$; the power being exerted during t secs., the total *energy* spent in producing the change of flux is $iSba$. Now if l = mean circumference of ring, then the value of H corresponding to i is $H = \frac{4\pi S i}{l}$, so that $Si = \frac{1}{4\pi} H l$. Substituting this value of Si in the expression just obtained for the energy, we find that (fig. 77)

$$\text{energy spent in passing from L to M} = \frac{1}{4\pi} b H l a.$$

But since la = total volume of iron in cubic cms., we see that in passing from L to M we communicate to every cubic cm. of the iron an amount of energy equal to $\frac{1}{4\pi} b H = \frac{1}{4\pi} \times$ area of strip P L M N

Hence if we pass from G to A, the total energy given to the iron will be $\frac{1}{4\pi}$ times the area G A R.

Now in going back from A to D, the current is decreasing; the induced E.M.F. will help to maintain it, and the ring in giving up its magnetism will act as a generator of energy. The amount of the stored energy so returned (by each cubic cm. of the iron) is given by $\frac{1}{4\pi}$ time the area R A D. The remainder, $\frac{1}{4\pi}$ time the area G A D, is not given up. Similarly, as the cycle is completed by passing from D through E and F to G, a further amount of stored energy, represented by $\frac{1}{4\pi}$ time the area D F G, is not returned. Hence during a complete cycle there is an amount of the stored energy, per cubic cm. of the iron, represented by $\frac{1}{4\pi}$ *time the area of the hysteresis loop*,[1] which is not returned, and which is used up in producing heat.

We thus see that $\frac{1}{4\pi}$ of the area of a hysteresis loop gives the energy (in ergs) dissipated in every unit volume of the material subjected to the magnetic cycle. The area of the loop will, of course, depend on the maximum value of the induction. As a result of a very large number of measurements, Steinmetz found that if B = maximum induction, then the energy dissipated, in ergs per cubic cm. per cycle, could be represented by the formula

$$\eta B^{1.6},$$

where η is a constant for a given material, called the *hysteretic coefficient*, and has a value varying from .001 to .003 for soft iron.

The result that for a given material the hysteresis loss varies as the 1.6th power of the maximum induction is known as *Steinmetz's Law*. This law only holds within a certain range of induction, and is not correct for either very low or very high inductions.

[1] This energy will be expressed in C.G.S. units, i.e. ergs.

Ewing's Hysteresis Tester.—If we attempt to rotate a well-laminated mass of iron in a magnetic field, then although, on account of the lamination, there will be no appreciable resistance due to induced eddy currents, yet a considerable resistance will be encountered by reason of the dissipation of energy by hysteresis. The resistance to the motion will increase in proportion to the energy dissipated per cubic cm. per cycle.

A highly ingenious yet simple instrument for the comparative testing of the hysteresis loss in specimens of sheet iron or steel has been constructed on this principle by Professor Ewing, and is shown in fig. 79. Between the poles of a permanent magnet supported on a horizontal knife-edge and provided with a pointer which moves over a scale is rotated the specimen to be tested, which takes the form of a few strips of the sheet-metal held in a suitable clamp. The greater the hysteresis loss the larger will be the angle through which the pointer is deflected.

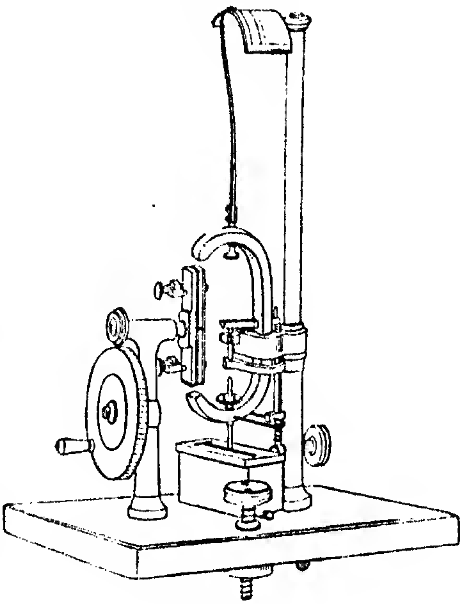


Fig. 79.—Ewing's Hysteresis Tester[1]

CHAPTER VIII

PRIMARY BATTERIES

Simple Voltaic Cell and its Defects.—Before the introduction of the dynamo, the only practicable method of producing electric currents was by means of primary cells. Since the advent of the dynamo, however, primary cells have become much less important, and in many cases where until recently they were largely employed, they are now being superseded by secondary cells. There is little doubt, however, that for certain purposes—such as electric bells, &c.—they will always continue to be used. Further, we have the important class known as standard cells, which form the commonly employed standards of E.M.F., and have for this

[1] From Professor Ewing's *Magnetic Induction in Iron and Other Metals*. By permission.

reason been very carefully studied. In view of these facts, no account of the subject of electric measurements would be complete without a reference to primary cells.

A typical form of rudimentary or simple voltaic cell consists of a plate of copper and one of zinc in dilute sulphuric acid. One great objection to this form is the waste of zinc taking place during the time that the cell is not in use, since ordinary commercial zinc is readily attacked by sulphuric acid. In order to remedy this defect the practice of amalgamating the zinc, or coating it with a thin layer of mercury, was introduced in 1828 by Kemp. The coating of amalgam is found to afford a very effective protection to the zinc. The readiness with which ordinary zinc is attacked by the acid is due to impurities in the zinc; these impurities when in contact with the acid form little voltaic cells with the zinc, and since the circuit of such cells is closed through the contact of the impurity with the zinc, a local current flows from the impurity to the zinc through the point of contact, and from the zinc back to the impurity through the acid, causing the zinc to go into solution. For this reason the waste of zinc which goes on when there is no current being taken out of the cell is generally referred to as "local action".

There is, however, another very serious defect in the simple voltaic cell under consideration. As soon as the cell is allowed to send a current, the sulphuric acid is split up by it into its constituent ions—H_2 and SO_4—the former appearing at the copper plate, the latter at the zinc plate, which it attacks, forming $Zn\,SO_4$. The hydrogen which appears at the copper plate partly escapes, but some of it adheres to the plate, and gives rise to an effect known as "polarization", which consists in a lowering of the E.M.F. of the cell. A plate of copper coated with hydrogen bubbles does not, in fact, behave like an ordinary copper plate. A simple experiment is sufficient to show the difference. If into a vessel filled with dilute sulphuric acid we introduce a plate of zinc and two newly cleaned plates of copper, then on connecting a galvanometer between the two copper plates, no appreciable current will, in general, be found to flow, showing that there is no potential difference between them. But if one of the copper plates be connected for a time to the zinc plate through an external resistance, and a deposition of small bubbles of hydrogen be thereby produced, then on introducing a galvanometer between the two copper plates a current will be found to flow round the external circuit from the unused plate to the one coated with hydrogen. The hydrogen, then, introduces an opposing E.M.F. into the circuit of the cell, and as a consequence there is a rapid fall in the current.

The effect of polarization would not be so serious were it constant, but it varies continually and irregularly as the hydrogen bubbles become detached from the copper plate.

In addition, the hydrogen has a mechanical effect, in that it reduces the active area of the copper plate.

In order to obviate this defect some means must be taken to prevent the deposition of hydrogen on the positive plate.

The various forms of primary cell differ from one another mainly in

the means adopted for counteracting polarization. We shall here describe only the more important types of primary cell.

The Daniell Cell and its Modifications.—In this form of cell, which is shown in fig 80, polarization is prevented by the use of two electrolytes —copper sulphate and sulphuric acid—separated from each other by a porous pot (of unglazed earthenware). The cylindrical plate of zinc and dilute sulphuric acid[1] are contained in the porous pot, while the external containing vessel is filled with a saturated solution of copper sulphate, the copper plate surrounding the porous pot.

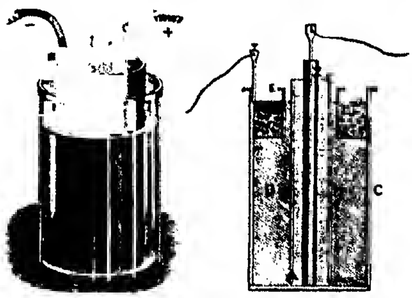


Fig. 80.—Daniell Cell

When a current passes through the cell, SO_4 appears at the zinc plate, and causes it to go into solution. The H_2 which is split off from the last molecule of H_2SO_4 before the $CuSO_4$ is reached attacks this latter, re-forming H_2SO_4, and the action is transmitted through a chain of $CuSO_4$ molecules, until the last molecule is reached, whose Cu is deposited in the form of metallic copper on the copper plate. The final result, then, is the solution of zinc and the deposition of copper on the copper plate. At the same time the outer liquid gets poorer in $CuSO_4$, and in order to supply the deficiency it is usual to have a supply of $CuSO_4$ crystals in the outer vessel, so as to maintain the solution saturated.

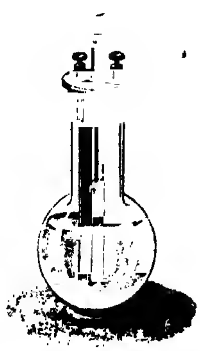


Fig. 81.—Bichromate Cell

The substance, then, which prevents polarization —or, as it is technically termed, the *depolarizer*—in the Daniell cell is a liquid—copper sulphate solution.

Numerous modifications of this cell have been devised. One of the most important of these is the *Minotto* cell. It consists of a cylindrical glass jar, with a disc of copper (to which is riveted a connecting wire) at the bottom of the jar, a layer of crystals of $CuSO_4$ above the disc, then a layer of blotting-paper or cloth, followed by a thick layer of sawdust or clean river sand, another thickness of blotting-paper, and finally a disc of zinc with a column supporting the terminal. The cell is, to start with, filled with water slightly above the level of the disc of zinc. This cell is largely used in India for telegraph circuits.

The Bichromate Cell.—This is shown in fig. 81. Arrangements must be provided for lifting the zinc out of the liquid when the cell is not in use, in order to prevent the formation of an insoluble chromium salt on the surface of the zinc.

[1] This gradually becomes contaminated with $ZnSO_4$ during use.

The Leclanché Cell and its Modifications.—One form of Leclanché cell is shown in fig. 82.

The zinc is in the form of a cylindrical rod, and the other plate consists of a block of carbon. The liquid used is a solution of sal-ammoniac, and the depolarizer is manganese dioxide ($Mn\,O_2$).

When the cell is in use the zinc dissolves, forming $Zn\,Cl_2$, and the $N\,H_4$—*which is the cation of an* $N\,H_4\,Cl$ *molecule*—*is acted on by the* $Mn\,O_2$, *according to the equation*

$$2\,N\,H_4 + 2\,Mn\,O_2 = Mn_2\,O_3 + H_2O + N\,H_3.$$

Various other forms of Leclanché cell have been devised, and the so-called "dry cells" are simple modifications of it.

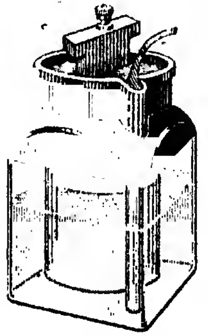


Fig. 82.—Leclanché Cell

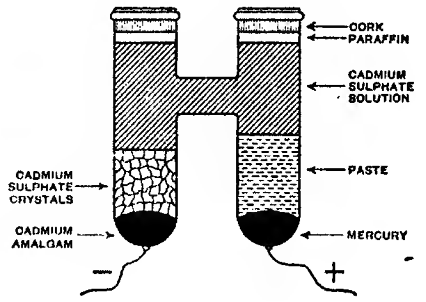


Fig. 83.—Cadmium Cell

The Leclanché cell, if used for intermittent work, is the least troublesome of all primary cells, and will work for months without requiring any attention. It is very largely used for bell and telephone work and testing purposes.

Standard Cells—The Cadmium or Weston Cell.—This cell, which has superseded the Clark cell in the definition of the practical unit of P.D. and E.M.F., is usually made up in the **H** form, as shown in fig. 83.

The negative element consists of a cadmium amalgam—1 part of cadmium to 6 parts of mercury. The positive element is formed of pure mercury, over which is placed a paste of mercurous sulphate. The remainder of the cell is filled up with a saturated solution of cadmium sulphate, crystals of cadmium sulphate being placed on the top of the negative element to ensure that the solution is always saturated. Connection is made to positive and negative elements by platinum wires sealed through the glass of the side tubes at their lower ends. The tubes are closed by a layer of paraffin wax, a cork, and then sealed with sealing-wax.

The E.M.F. of this cell is 1.0184 volts at 20° C., and only changes .00005 volt per degree centigrade change of temperature.

This gives the cell, for use as a standard of E.M.F., a great advantage over the Clark cell, which has a larger variation of E.M.F. with temperature.

The Clark Cell.—The construction of this cell, which is still commonly used as a standard of E.M.F., will be understood by reference to fig. 84.

The containing vessel consists of a glass tube, and contact is made with the mercury by means of a platinum wire sealed into a glass tube, as shown. Above the mercury is a paste consisting of mercurous sulphate and a saturated solution of zinc sulphate. The insoluble mercurous sulphate, which acts as the depolarizer, settles down on the top of the mercury, leaving a layer of clear solution of zinc sulphate above it. In order to maintain the solution saturated at all temperatures, a few crystals of the salt are added, and these may be generally seen resting on the top of the mercurous sulphate.

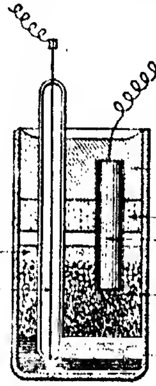


Fig. 84.—Clark Cell[1]

The cell constructed as described possesses several disadvantages, and numerous modifications of it have been suggested and used. In order to render it more portable, Dr. Muirhead dispenses with the large mass of mercury at the bottom of the cell, and for it substitutes a flattened spiral of platinum wire, which has been amalgamated by heating to redness and plunging into mercury; the small amount of mercury retained by the spiral is quite sufficient, and this arrangement renders the cell much more convenient to handle.

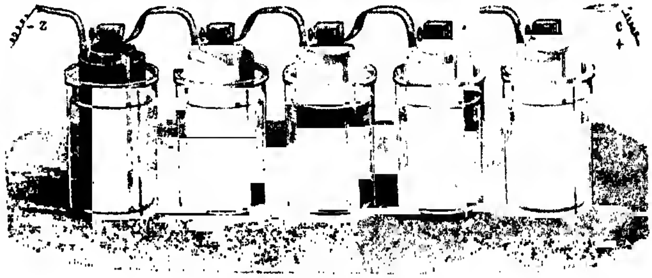


Fig. 85.—Bunsen's Battery

The E.M.F. of a Clark cell at 15° C. is 1.434 volts, and the E.M.F. at any other temperature t° C. is given by

$$1.434 \{ 1 - .00077 (t - 15) \}.$$

Grouping of Cells in a Battery.—Suppose that we have a number

[1] From *Primary Batteries*, by W. R. Cooper. ("The Electrician" Printing and Publishing Co., Ltd.) By permission.

n of precisely similar cells, each having an electromotive force E and internal resistance r, and that we connect them in series as in fig. 85, with a conductor of resistance R joining their poles.

The whole electromotive force of the circuit will be nE, and the whole resistance will be $nr + \mathrm{R}$; hence the strength of the current will be

$$\mathrm{I} = \frac{n\mathrm{E}}{nr + \mathrm{R}}.$$

This equation shows that, if the external resistance R is much greater than the resistance nr in the battery itself, any change in the number of cells will produce a nearly proportional change in the current; but that when the external resistance is much less than that of one cell, as is the case when the poles are connected by a short thick wire, a change in the

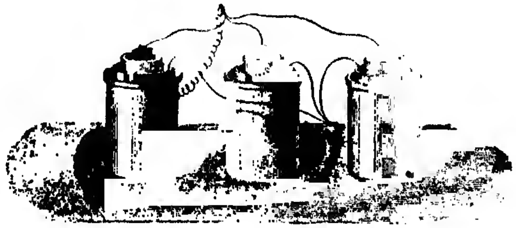


Fig. 86 Cells with Similar Plates connected

number of cells affects the numerator and denominator almost alike, and produces no sensible change in the current. It is impossible by connecting any number of similar cells *in series* to obtain a current exceeding $\frac{\mathrm{E}}{r}$, which is precisely the current which one of the cells would give alone if its plates were well connected by a short thick wire.

It is possible, however, by a different arrangement of the cells to obtain a current about n times as strong as this, namely, by connecting all the positive plates to one end of a conductor and all the negative plates to the other end, as in fig. 86. The arrangement is equivalent to a single cell with plates three times as large superficially and at the same distance apart. The electromotive force with n cells so arranged is simply E, but the total resistance is only $\frac{r}{n} + \mathrm{R}$, so that the current is

$$\mathrm{I} = \frac{\mathrm{E}}{\frac{r}{n} + \mathrm{R}} = \frac{n\mathrm{E}}{r + n\mathrm{R}}.$$

Cells arranged in this way are said to be *in parallel*.

From the above equation it is apparent that if R is negligible compared with r the current is proportional to the number of cells in parallel. On the other hand, if R is very large compared with r, an increase in the number of cells will make no perceptible change in the current.

In practice the cells in a battery are usually arranged in a way which is a combination of the two arrangements.

Thus q groups, each consisting of p cells in *series*, are connected in *parallel*.

The total E.M.F. is then pE, and the total resistance is $\frac{pr}{q} + R$.

The current is therefore

$$I = \frac{pE}{\frac{p}{q}r + R} = \frac{E}{\frac{r}{q} + \frac{R}{p}}.$$

Since $pq = n$, the total number of cells, the product of $\frac{r}{q}$ and $\frac{R}{p}$ is *constant*.

Consequently the sum of $\frac{r}{q}$ and $\frac{R}{p}$ is a minimum when $\frac{r}{q} = \frac{R}{p}$.

Therefore I is a *maximum* when $\frac{r}{q} = \frac{R}{p}$. For maximum current then p and q have to be so chosen that $R = \frac{p}{q}r$,[1] or the arrangement has to be such that the internal resistance of the battery is as nearly as possible equal to the external resistance through which the current has to be passed.

[1] It is, of course, impossible in many cases to arrange for exact equality, since p and q are necessarily integers.

2. Alternating-Current Measurements

CHAPTER I

THE REPRESENTATION OF SIMPLE ALTERNATING CURRENTS

Introductory.—An alternating current is one which, instead of flowing continuously in one direction round a circuit, periodically reverses its direction. The time which elapses between successive reachings of zero value in the *same* direction is defined to be the *period* of the alternating current, and the current is said to have completed one *cycle* during this time. The number of complete cycles performed per second is called the *frequency*. Clearly an alternating current may vary in any manner, but for practical purposes it is sufficient to consider only those currents in

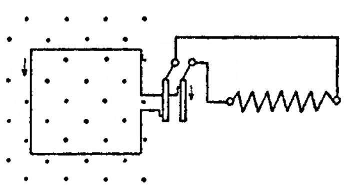


Fig. 1a

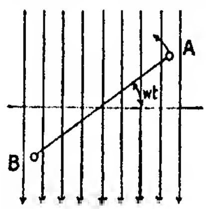


Fig. 1b

which the successive halves of any cycle are similar. Although the preceding remarks have been confined to currents it must be understood that they apply equally well to E.M.F.s, P.D.s, and magnetic fluxes.

Production of an Alternating E.M.F.—A simple alternating E.M.F. may be produced by rotating a loop of wire with a uniform angular velocity in a uniform magnetic field, as shown in fig. 1*a*.

Let the loop be rotated in a counter-clockwise direction (fig. 1*b*), and let time be measured from an instant when the loop is in a horizontal plane with the conductor A to the right.

Let A = area enclosed by the loop.
H = strength of the magnetic field.
n = flux embraced by the loop at any time t.
$N_{max.}$ = maximum flux embraced by the loop.
e = E.M.F. generated in the loop at any time t.
ω = angular velocity of the loop.

At any time t the angle through which the loop has rotated will be ωt, and therefore

$$n = \text{H A} \cos \omega t.$$

Since the E.M.F. generated in the loop is equal to the rate of change of the magnetic flux linked with the loop, and is opposed to the direction of change,

$$e = -\frac{d}{dt}(\text{H A} \cos \omega t)$$
$$= \text{H A} \omega \sin \omega t.$$
$$\text{But H A} = \text{N}_{\text{max.}},$$
$$\therefore e = \text{N}_{\text{max.}} \omega \sin \omega t \text{............C.G.S. units.}$$

From this equation it will be seen that the E.M.F. is a simple sine function of the time, and may be represented as shown in fig. 2.

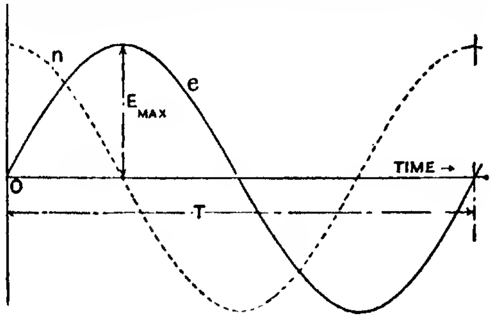


Fig. 2

Since during the time the loop rotates through an angle 2π, the E.M.F. goes through one complete cycle, the period T of the E.M.F. is equal to $\frac{2\pi}{\omega}$.

$$\text{Or if } f = \text{frequency}$$
$$f = \frac{\omega}{2\pi}.$$

The equation for the E.M.F. may now be rewritten

$$e = 2\pi f \text{N}_{\text{max.}} \sin \omega t \text{............C.G.S. units.}$$

The conditions will not be altered if a magnetic flux *varying* according to the equation $n = \text{N}_{\text{max.}} \cos \omega t$ is interlinked with a *stationary* loop; and, as before, the E.M.F. generated in the loop will be

$$e = 2\pi f \text{N}_{\text{max.}} \sin \omega t \text{............C.G.S. units.}$$

This is a simple case corresponding to the production of an E.M.F. in a static transformer, while the former case corresponds to the generation of the E.M.F. in a rotational machine.

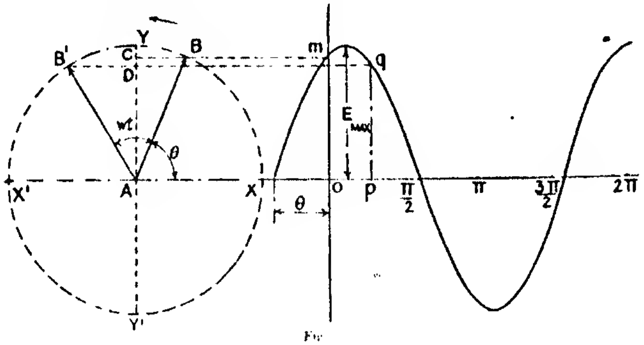
In either case, if the loop consists of S turns so concentrated that they

all cut the same flux and at the same time, the E.M.F. is multiplied S times and the equation becomes

$$e = 2\pi f S N_{max.} \sin \omega t \text{............C.G.S. units.}$$

The E.M.F.s and currents in practical machines are seldom simple sine functions, but contain harmonics of the fundamental. For the present, however, our attention will be confined to pure sine functions.

Vector Representation.—Such functions may be very conveniently represented by a vector, whose length represents to some scale the maximum value of the function, and which rotates with a constant angular velocity equal to $\frac{2\pi}{T}$, where T is the period of the function.



For instance, let A B in fig. 3 represent the maximum value $E_{max.}$ of an E.M.F. acting in a circuit.

Let the E.M.F. vary according to the equation $e = E_{max.} \sin(\omega t + \theta)$.

The vector A B rotates about the fixed end A with a constant angular velocity ω. The direction of rotation is a matter of indifference, but for the sake of uniformity the International Committee for Electrical Symbols has laid down that the direction of rotation shall be *counter-clockwise*.

At any instant the projection of A B on the vertical axis Y′Y represents the value of the E.M.F. e, to the same scale as A B represents $E_{max.}$

At the instant shown in the above figure, A C is the projection of A B on the vertical axis.

$$\text{And } O m = A C = A B \sin\theta = E_{max.} \sin\theta.$$

This refers to the instant when $t = O$. At a time t secs. later the vector A B will have moved into the position A B′, and A D is now the projection on the vertical axis.

$$\text{And } pq = A D = A B \sin(\omega t + \theta) = E_{max.} \sin(\omega t + \theta).$$

The ordinates of the curve drawn through such points as m, q, &c., will thus represent the instantaneous values of the E.M.F., the maximum value of which is represented by A B.

It is usually more convenient to express the abscissæ in terms of angles[1] rather than times, and this has been done in the above figure.

The value of e is zero when ωt is equal to $-\theta$, $(\pi - \theta)$, $(2\pi - \theta)$, &c. The curve of e is, in effect, a sine curve displaced to the left along the axis of abscissæ by an amount θ from zero.

The value of a vector diagram becomes apparent when a circuit having several variables of the same kind or of different kinds is considered. For example, let two E.M.F.s, of which $E_{1\,max.}$ and $E_{2\,max.}$ are the maximum values, exist in a circuit, and let their instantaneous values be given by the equations

$$e_1 = E_{1\,max.} \sin \omega t \ldots\ldots\ldots\ldots\ldots\ldots (a)$$

$$e_2 = E_{2\,max.} \sin\left(\omega t + \frac{\pi}{2}\right) \ldots\ldots\ldots\ldots (b)$$

Let also the instantaneous value i of the current flowing in the circuit be given by

$$i = I_{max.} \sin \omega t \ldots\ldots\ldots\ldots\ldots\ldots (c)$$

The magnitude of the maximum value of the resultant E.M.F. in the

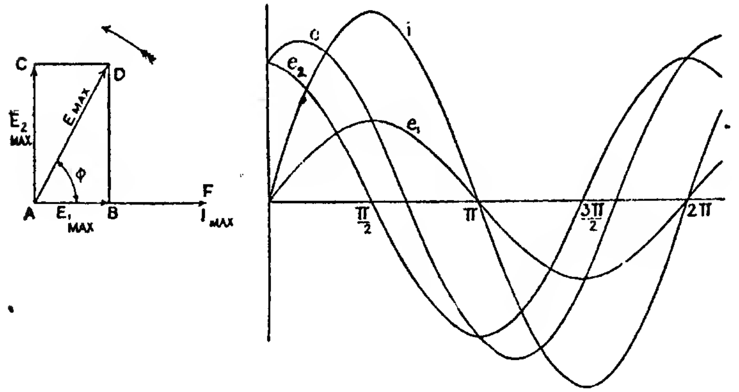


Fig. 4

circuit, and the angle of phase difference between the current and the resultant E.M.F. can readily be found by means of a vector diagram.

In fig. 4 let the vector AB represent $E_{1\,max.}$ From equation (b) it will be seen that $E_{2\,max.}$ is $\frac{\pi}{2}$ ahead of $E_{1\,max.}$

Therefore AC drawn at right angles to AB and of length representing the magnitude of $E_{2\,max.}$ will represent $E_{2\,max.}$ in maghitude and phase. $E_{max.}$, the maximum value of the resultant E.M.F., will be represented by the vector AD obtained by completing the parallelogram ABDC, for the projection of AD on the vertical axis is at every instant equal to the sum

[1] Since T is equal to the number of seconds taken to complete one cycle, it is a simple matter to convert "angles" into "times", if this is required.

of the projections of AB and AC; i.e. $e = e_1 + e_2$, where e is the instantaneous value of the resultant E.M.F.

The vector AF of length representing the magnitude of $I_{max.}$, and drawn coincident with AB (see equations (c) and (a)), will represent I_{max}, the maximum value of the current, in magnitude and phase. The angle FAD $= \phi$ is the angle of phase difference between the current and the resultant E.M.F. The sine curves on the right of fig. 4 are plotted from the rotating vectors on the left, and show the instantaneous values of the various E.M.F.s and the current.

The vector diagram may be very rapidly drawn, and gives simply and effectively the information required. As will be seen later, the vector diagram is also a valuable adjunct where for any reason an analytical method is used instead of a graphical one.

Mean and Effective Values. — The mean value of a sine wave current taken over any number of *complete periods* is obviously zero.

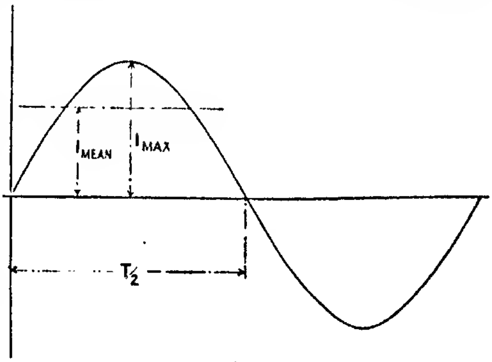


Fig. 5

The meaning of the term *mean* value as applied to an alternating current is, however, always understood to be the mean of the instantaneous values of the current taken over one half period *between successive zero values* of the current; i.e. it is the greatest mean value which can be obtained over *any* half period.

Thus, let an alternating current, as shown in fig. 5, be represented by the equation $i = I_{max.} \sin \omega t$.

Then the *mean value* is

$$I_{mean} = \int_0^{\frac{T}{2}} I_{max.} \sin \omega t \, dt \div \frac{T}{2}.$$

$$I_{mean} = \frac{2}{T} I_{max} \frac{1}{\omega} \Big[- \cos \omega t \Big]_0^{\frac{T}{2}}.$$

$$\text{Or since } \omega = \frac{2\pi}{T}$$

$$I_{mean} = \frac{2}{T} I_{max.} \frac{T}{2\pi} \Big[- \cos \frac{2\pi}{T} t \Big]_0^{\frac{T}{2}}$$

$$= \frac{2}{\pi} I_{max} = .636 \, I_{max.}$$

The mean value is not of itself important, but, as will be seen later, is involved in the form factor.

The effective value of an alternating current or pressure is, however,

of great practical importance, since the power developed depends on the effective values.

The effective value of an alternating current or an alternating E.M.F. *may be defined as equal to the value of that continuous current or E.M.F. which will produce the* ***same heating effect*** when acting in any given circuit.

Let i = instantaneous value of the alternating current
I = effective value of the alternating current.
r = resistance of the circuit.

Then from Joule's Law the mean rate of heat production is

$$\mathrm{I}^2 r = \int_0^{\mathrm{T}} i^2 r\,dt \div \mathrm{T}$$

$$= \frac{1}{\mathrm{T}}\int_0^{\mathrm{T}} \mathrm{I}^2_{\text{max.}}\, r\left(\sin^2 \frac{2\pi}{\mathrm{T}} t\right) dt$$

$$\mathrm{I}^2 r = \frac{\mathrm{I}^2_{\text{max.}}\, r}{\mathrm{T}} \int_0^{\mathrm{T}} \tfrac{1}{2}\left(1 - \cos \frac{4\pi}{\mathrm{T}} t\right) dt$$

$$= \frac{\mathrm{I}^2_{\text{max.}}\, r}{2\mathrm{T}} \left[t - \frac{\mathrm{T}}{4\pi} \sin \frac{4\pi}{\mathrm{T}} t\right]_0^{\mathrm{T}}.$$

$$\text{Or } \mathrm{I}^2 = \frac{\mathrm{I}^2_{\text{max.}}}{2}$$

$$\text{and } \mathrm{I} = \frac{\mathrm{I}_{\text{max.}}}{\sqrt{2}} = .707\ \mathrm{I}_{\text{max.}}$$

In a similar way

$$\mathrm{E} = \frac{\mathrm{E}_{\text{max.}}}{\sqrt{2}}.$$

Instead of the expression "effective value" the expressions "virtual value" or "root-mean-square value" (R.M.S. value) are frequently used—the latter owing to the fact that the effective value is obtained by taking the root of the mean of the squares of the instantaneous values.

Form Factor.—The *form factor* is defined to be the ratio of the *effective* to the *mean* value of the wave.

$$\text{Thus } \gamma = \frac{\text{effective value}}{\text{mean value}},$$

where γ = form factor.

In the case of a pure sine wave, we obtain, using the results developed above,

$$\gamma = \frac{\dfrac{\mathrm{I}_{\text{max.}}}{\sqrt{2}}}{\dfrac{2}{\pi}\,\mathrm{I}_{\text{max.}}} = \frac{\pi}{2\sqrt{2}} = 1.11.$$

The form factor is of importance in determining the effective value when the mean value is known. This may be illustrated by means of the simple example of a loop rotated in a magnetic field, and already referred to in the earlier part of this chapter. On referring to fig. 2 it will be seen that during one complete cycle the flux falls from a maxi-

mum to zero, rises from zero to a maximum, falls from a maximum to zero, and again rises from zero to a maximum. In other words, the flux undergoes *four* changes of equal amount. It should be particularly noted that this involves no assumption as to the wave form of the flux other than that it is symmetrical.

It follows, therefore, that the *mean* rate of change of flux

$$= \frac{4\,N_{max.}}{T}$$

$$\therefore\ E_{mean} = 4 f N_{max.}\ 10^{-8} \text{ volts.}$$

Since no assumption has been made as to the wave form of either the flux or the E.M.F., it follows that E_{mean} is determined by $E_{max.}$, and is *entirely independent of the wave form.*

By the use of the form factor the effective value of the E.M.F. may be readily obtained. Thus for a pure sine wave

$$\begin{aligned} E &= 1.11\ E_{mean} \\ &= 4.44 f N_{max.}\ 10^{-8} \text{ volts.} \end{aligned}$$

This is in agreement with the equation deduced above, viz.—

$$\begin{aligned} e &= 2\pi f N_{max.} \sin \omega t \text{............C.G.S. units.} \\ \text{For } E_{max} &= 2\pi f N_{max.}\ 10^{-8} \text{ volts.} \\ \text{And since } E &= \frac{E_{max.}}{\sqrt{2}} \\ E &= \sqrt{2}\,\pi f N_{max.}\ 10^{-8} \text{ volts} \\ &= 4.44 f N_{max.}\ 10^{-8} \text{ volts.} \end{aligned}$$

In the case of wave forms containing harmonics the effective value of the E.M.F. can in this way be readily calculated if the maximum flux and the form factor are known.

Notation.—For the sake of uniformity the following notation for currents, E.M.F.s, and P.D.s will be adhered to throughout this article:—

(*a*) Small letters indicate instantaneous values, thus

i = instantaneous value of the current.

(*b*) Capitals without suffix indicate effective values, thus

I = effective value of the current.

(*c*) Capitals with suffix *max.* or *mean* indicate respectively maximum or mean values, thus

$I_{max.}$ = maximum value of the current.
I_{mean} = mean value of the current.

(*d*) Capitals with a dot below indicate *vector* quantities, thus

$\underset{\cdot}{I}$ = the current considered as a vector and not as an algebraic quantity.

CHAPTER II

INDUCTANCE

When dealing with the flow of direct or continuous currents through any circuit it is sufficient to know what is the resistance of the circuit, in order to determine the strength of the current that will flow when a given potential difference is applied to the terminals. With alternating currents, however, other properties of the circuit have to be taken into account. The magnitude of the alternating current that will flow depends not only on the resistance but on the shape of the circuit, i.e. on the way in which the parts are arranged relatively to each other.

The reason for this is at once arrived at from the fact that the magnetic field set up by an alternating current, around the wire in which the current flows, varies continually as the current varies.

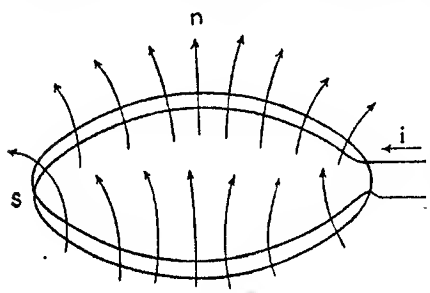


Fig. 6

Since any variation of the magnetic flux linked with a circuit gives rise to an E.M.F. proportional to the time rate of change of the interlinkages between the flux and the circuit, it follows that a factor is introduced when *alternating currents* are considered, which does not exist when *steady currents* have to be dealt with.

The number of interlinkages is dependent on the size and arrangement of the parts of the circuit. By a suitable disposition of the parts of the circuit it is possible to reduce the number of interlinkages to a minimum.

The above statements may be investigated by means of a simple example. For instance, let us consider the case of a circuit in the form of a loop consisting of s turns (see fig. 6) in which an alternating current of instantaneous value i absolute units flows.

The instantaneous value n of the flux produced by this current is, in accordance with the law of the magnetic circuit, given by $n = \frac{4\pi s i}{\rho}$, where ρ is equal to the reluctance of the magnetic circuit in which n exists, that is $\Sigma \frac{l}{\mu a}$.

Also the instantaneous value e_{si} of the E.M.F. induced by the varying flux is given by

$$e_{si} = -\frac{d}{dt}(s n).$$

Inserting the value of n just obtained,

$$e_{si} = -\frac{d}{dt}\left(\frac{4\pi s^2 i}{\rho}\right).$$

We may replace $\frac{4\pi s^2}{\rho}$ by the coefficient L, thus giving

$$e_{si} = -\frac{d(\mathrm{L}i)}{dt}.$$

Coefficient of Self-induction.—The coefficient L is known as the *coefficient of self-induction* or simply as the *inductance*, and may be defined as follows:—

"The inductance of a circuit is the number of interlinkages of the circuit with the lines of magnetic force produced by *unit* current in the circuit".

In accordance with this definition, $\mathrm{L} = \frac{sn}{i}$ absolute units, if i is in absolute units. From which we obtain, by inserting the value of n,

$$\mathrm{L} = \frac{s}{i}\left(\frac{4\pi s i}{\rho}\right) = \frac{4\pi s^2}{\rho} \text{ absolute units,}$$

which is the expression for L which we have already made use of above.

Mutual Inductance.—Mutual inductance is similar in its nature, and may be similarly defined as follows:—

"The mutual inductance of two circuits is the number of interlinkages of *the one circuit* with the lines of magnetic force produced by unit current flowing in *the other circuit*".

Inductance, whether self or mutual inductance, depends upon the reluctance of the path of the flux, and is, therefore, not necessarily constant.

Where the flux path includes magnetic material the inductance is not constant, but has a different value for every value of the current. The flux of self-inductance in practical machines is usually spoken of as the *leakage flux*. In many cases the reluctance of the leakage path may be taken to be practically constant, and the inductance may, therefore, be considered constant.

Units of Inductance.—Following the definition of inductance, the *absolute unit of inductance* may be defined as the inductance of a circuit in which the number of interlinkages of the circuit with the lines of magnetic force, produced by *one absolute unit* of current flowing in the circuit, is unity. Since a current varying *uniformly* at the rate of one absolute unit per second will give rise to a steady induced E.M.F. *numerically equal to* the inductance of the circuit, the absolute unit of inductance may alternatively be defined as the inductance of a circuit in which one absolute unit of E.M.F. is induced by a current which varies uniformly at the rate of one absolute unit per second. It is on this basis that the practical unit of inductance—the *henry*—is usually defined. Thus a circuit in which an E.M.F. of 1 *volt* is induced by a current which varies *uniformly* at the rate of 1 *ampere per second* is said to have an inductance of 1 *henry*. The henry has the value 10^9 in terms of the absolute C.G.S. unit.

Energy Stored in a Magnetic Field.—If a steady E.M.F. be applied to an inductive circuit the current will rise until it attains a final value I.

During the rise of the current, energy[1] *is being stored not in the circuit itself but in the magnetic field linked with the circuit.* In order that the current may rise, a component e_L of the applied E.M.F. must be devoted to overcoming the self-induced E.M.F. e_{si}.

$$e_L = -e_{si}.$$

The rate at which energy is being given to the circuit is $e_L i$, therefore W, the stored energy, when the current has reached its final value I will be

$$W = \int_{i=0}^{i=I} e_L i\,dt.$$

$$\text{But } e_{si} = -\frac{d(Li)}{dt} \text{ and } e_L = \frac{d(Li)}{dt},$$

$$\therefore W = \int_{i=0}^{i=I} i\left(L\frac{di}{dt}\right)dt$$

$$= \int_0^I L\,i\,di$$

$$= \tfrac{1}{2} L I^2.$$

If L is in henries and I in amperes,

$$W = \tfrac{1}{2} L I^2 \text{ joules.}$$

In highly inductive circuits, such as the field windings of machines, the stored energy is very considerable, and disastrous results may follow the sudden opening of such a circuit. The stored energy is suddenly given up by the collapse of the magnetic field, and the excessive voltage thus induced may wreck the insulation of the coils. For this reason special arrangements are made on all large machines whereby the field current is gradually and not suddenly brought to zero.

Inductive Reactance. — Let us now return to our example of a circuit consisting of a loop of s turns, and let the current be given by the equation

$$i = I_{max.} \sin \omega t.$$

The self-induced E.M.F. will then be

$$e_{si} = -\frac{d(Li)}{dt}$$

$$= -L\frac{d}{dt}(I_{max} \sin \omega t),$$

since the inductance L of the loop is constant. In order to overcome[2] e_{si} and cause the current i to flow an applied E.M.F. e_L equal and opposite to e_{si} must be provided (it being assumed that the circuit is *without resistance*).

$$\therefore e_L = L\frac{d}{dt}(I_{max.} \sin \omega t)$$

$$= \omega L I_{max.} \cos \omega t.$$

[1] This is analogous to the storage of kinetic energy in a body accelerated to a steady velocity.

[2] The difference between the self-induced E.M.F. and the E.M.F. required to *overcome it* must always be clearly borne in mind.

From these equations we see that e_L *leads* the current i by $\frac{\pi}{2}$, while e_{si} *lags behind* i by the same amount. It is more usual to think of the phase position of the current relatively to the E.M.F., and it will be seen that in a *purely inductive circuit* the current *lags* by an angle $\frac{\pi}{2}$ behind the applied E.M.F.

In fig. 7, in addition to the curves of e_{si}, e_L, and i, the curve of the product $e_L i$ has been plotted. The area included between this curve and the

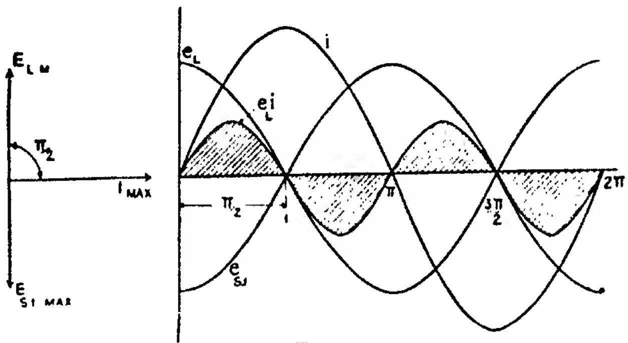


Fig. 7

horizontal axis represents the energy alternately stored in and restored by the magnetic field linked with the circuit. For the sake of clearness this area is shown shaded in the figure. It should be particularly noted that all the energy given to the circuit is subsequently restored, i.e. the positive and negative areas are equal, and that therefore the net energy supplied to the circuit is zero.

From the equation $e_L = \omega L I_{max.} \cos \omega t$, we at once obtain

$$E_{L\,max.} = \omega L I_{max.}$$

and

$$E_L = \omega L I;$$

or the effective applied E.M.F. E_L is equal to ωL times the effective current I.

The factor ωL may be replaced by a single coefficient x_L. This coefficient is called the *inductive reactance*.

$$x_L = \omega L = 2\pi f L.$$

The reactance in an inductive circuit therefore varies directly as the frequency.

A reactance has the dimensions of a resistance, and the practical unit of reactance is 1 ohm. The expression for the effective value of the applied E.M.F. in a purely inductive circuit therefore becomes

$$E = I x_L \text{ volts,}$$

where I is the effective value of the current in amperes, and x_L is the inductive reactance in ohms.

CHAPTER III

CAPACITY

When a condenser, say for example a pair of metallic plates separated by an air space, is connected to a source supplying a steady E.M.F., a current will flow momentarily until a quantity Q of electricity has been transferred to the condenser. The quantity Q stored upon the plates of the condenser per unit of P.D. between the plates is defined to be the *capacity* of the condenser.

Thus, if in the above example V is the P.D. at the terminals of the condenser, and C is its capacity,

$$Q = CV.$$

In the case of a steady applied E.M.F., the flow of current ceases as soon as the condenser has received its charge Q (see *fig. 8*).

If, however, an alternating E.M.F. is applied to the condenser, an alternating current will flow in the circuit, and will continue to do so as long as the E.M.F. is applied, for as soon as the E.M.F. has reached its maximum in the positive direction and the condenser has acquired its full charge, the E.M.F. begins to fall and the condenser gives up its charge, thus causing a current to flow in the opposite direction, which continues until the E.M.F. has reached its maximum in the negative direction, and the condenser has acquired its full charge in the opposite sense. As the E.M.F. falls to zero this charge is given up by the condenser, and a current flows in the original direction and continues until the E.M.F. has risen to a positive maximum, and the condenser has again acquired its full charge in the original sense. The cycle of events then repeats itself, the alternating E.M.F. thus causing an alternating current to flow in the circuit.

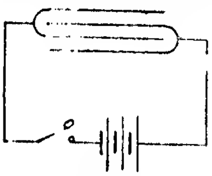


Fig. 8

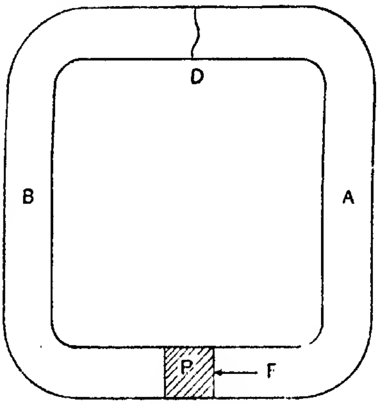


Fig. 9

The action of a condenser may be well illustrated by an hydraulic analogy.

Thus, let AB (fig. 9) be a closed pipe filled with water divided into two sections A and B by the piston P and a rubber diaphragm D.

If now a steady force F be applied to the piston from right to left, the

diaphragm D will be distorted as shown in fig. 9. A flow of water will take place up B and down A, but this will cease as soon as D has reached the distortion corresponding to the force F.

If now a force acting alternately in opposite directions be applied to P, the diaphragm will be distorted first in one direction and then in the other, an alternating flow of water taking place in A and B. The alternating flow of water produced by the alternating force applied to P corresponds to the alternating current produced by the alternating E.M.F. in the electric circuit, and the distortion of the diaphragm corresponds to the strain produced in the dielectric between the plates of the condenser.

It should especially be noted, that provided no leakage takes place through the diaphragm or the dielectric, that no transference of in the one case water, nor in the other of electricity, takes place from the one section of the circuit to the other.

The amount of distortion of the diaphragm for a given force applied to P is clearly dependent on the size and material of the diaphragm, just as the charge given to a condenser by a given E.M.F. is dependent on the area and thickness of the dielectric between the plates and the material of the dielectric. If the force applied to the piston is increased until the diaphragm breaks down, a transference of water will take place from the one side to the other. Similarly, if the E.M.F. applied to the electric circuit be increased until the dielectric of the condenser breaks down, a spark will pass from the one plate of the condenser to the other, and a transference of electricity will thus take place through the condenser from the one section of the circuit to the other.

Practical Unit of Capacity.—The practical unit of capacity is the *farad*, and is defined as the capacity of a condenser to which a charge of 1 coulomb is given by a difference of potential of 1 volt between its terminals.

This unit is inconveniently large for practical use, and the microfarad ($= 10^{-6}$ farads) is invariably employed.

Capacity Reactance.—Let us consider a circuit having zero resistance and inductance, and containing a condenser of capacity C.

Let the circuit be the seat of an E.M.F. given by the equation

$$e = E_{max.} \sin \omega t.$$

The charge q of the condenser at any instant is $q = Cv$, where v is the P.D. between the terminals of the condenser.

Since a current is equal to the rate at which electricity is conveyed by it, the current i at any instant is given by

$$i = \frac{dq}{dt}.$$

Or inserting the value of q,

$$i = \frac{d}{dt}(Cv) = \frac{d}{dt}(Ce),$$

since the circuit is without resistance and inductance.

$$\therefore i = C\frac{de}{dt}$$
$$= C\frac{d}{dt}(E_{max.} \sin \omega t)$$
$$= \omega C E_{max.} \cos \omega t.$$
$$\therefore I_{max.} = \omega C E_{max.}$$

The current therefore *leads* the E.M.F. by $\frac{\pi}{2}$, and has a maximum value equal to $\omega C E_{max.}$

In fig. 10 curves of e, i, and the product ei are shown. The area included between the last mentioned and the horizontal axis represents the

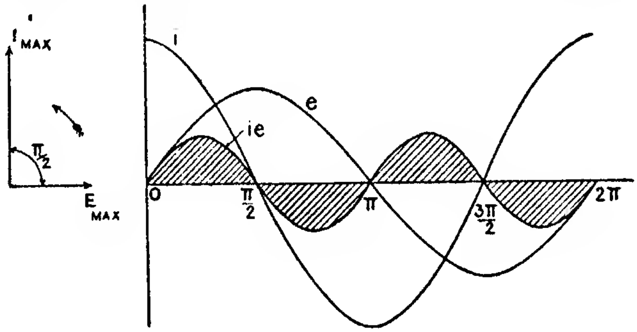


Fig. 10

energy alternately given to and restored by the condenser. As in the case of the purely inductive circuit, the net energy given to the circuit is zero. In this case the energy is stored in the *electrostatic field* produced between the plates of the condenser. In practical condensers other than those with an air dielectric, a certain amount of energy is required to supply a loss in the dielectric analogous to the hysteresis loss in a magnetic material. In such a case therefore the energy supplied exceeds the energy restored by an amount equal to the dielectric loss.

From the equation developed above

$$I = \omega C E$$
$$\text{or } E = \frac{I}{\omega C}.$$

As in the case of the inductive circuit this equation may be written in the form

$$E = I x_c,$$

where x_c is called the *capacity reactance* of the circuit, and is equal to $\frac{1}{\omega C}$.

If C is expressed in farads, then

$$x_c = \frac{1}{2\pi f C} \text{ ohms,}$$
$$\text{and } E = I x_c \text{ volts.}$$

Capacity in Electric Circuits.—It may be pointed out that, apart from the presence of condensers as such, all electric circuits have a certain capacity, although this may be so small as to be quite negligible.

For instance, a pair of overhead conductors form the plates of a condenser of which the dielectric is formed by the intervening air. Unless the line is very long, or the conductors are placed very near one another, the capacity of such an arrangement would be very small. In the case of cables the capacity is of some importance, since the conductors are near one another, and are separated by insulating material of possibly high specific inductive capacity.

In large A.C. installations the capacity is an item of considerable importance, since its presence gives rise to difficulties which have to be specially guarded against. The charging current, apart altogether from the useful load current, is in large systems considerable, and the dielectric losses already referred to contribute largely to the heating of the cables.

CHAPTER IV

IMPEDANCE

In Chapters II and III circuits in which the resistance was assumed to be zero have been dealt with. All practical circuits have a greater or lesser amount of resistance, and circuits having resistance in addition to inductance, capacity, or both, will now be considered.

Inductance and Resistance in Series.—Let us consider a circuit having a resistance of R ohms and an inductance of L henries, and let the capacity be zero. Let an alternating E.M.F. e be applied to the circuit, and let the current produced by this E.M.F. be $i = I_{max.} \sin \omega t$.

The E.M.F. e will at any instant consist of two components, e_L and e_r, which respectively overcome the self-induced E.M.F., and cause the current to flow through the resistance R.

$$\text{Thus } e = e_L + e_r$$

As we have already seen $e_L = \omega L I_{max.} \cos \omega t$.

$$\text{Also } e_r = ir$$
$$= r I_{max.} \sin \omega t.$$

The component e_L to overcome the self-induced E.M.F. leads the current by $\frac{\pi}{2}$, while e_r is in phase with the current.

The curves of the E.M.F.s and the current are shown in fig. 11, together with the corresponding vector diagram.

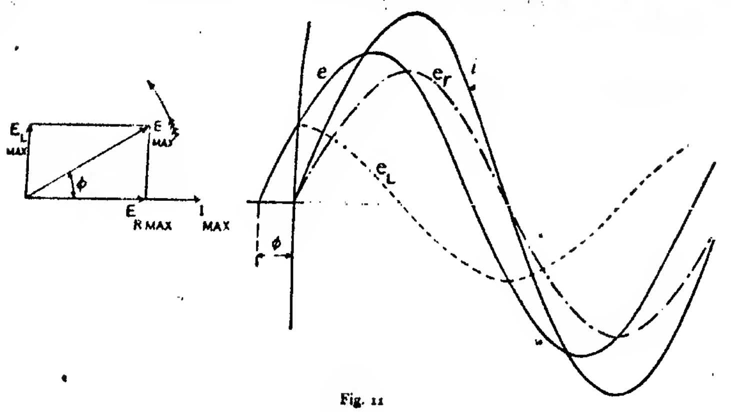


Fig. 11

It will be seen that the current *lags* by an angle ϕ (which is *less* than $\frac{\pi}{2}$) behind the applied E.M.F. From the vector diagram we see that

$$E_{max.} = \sqrt{E^2_{r\,max.} + E^2_{L\,max.}}$$

As, however, it is the effective value which is of practical importance, we may write

$$E = \sqrt{E_r^2 + E_L^2}.$$

$$\text{But, } E_r = I\,R,$$
$$\text{and } E_L = \omega L I.$$
$$\therefore\ E = I\sqrt{R^2 + \omega^2 L^2} \text{ volts.}$$

Since the tangent of the angle ϕ is equal to

$$\frac{E_{L\,max.}}{E_{r\,max.}} \left(= \frac{E_L}{E_r} \right),$$

the angle itself is given by

$$\phi = \tan^{-1}\left(\frac{\omega L I}{I R}\right)$$
$$= \tan^{-1}\left(\frac{\omega L}{R}\right).$$

This equation shows that the angle of lag *decreases* as R is *increased*, and varies from $\frac{\pi}{2}$ to 0, as R varies from 0 to ∞.

The expression $E = I\sqrt{R^2 + \omega^2 L^2}$ may be written

$$E = I z,$$

where z has the dimensions of a resistance, and is called the *impedance* of the circuit. Thus, the effective value in volts of the applied E.M.F. is equal to the effective value in amperes of the current multiplied by the impedance of the circuit in ohms.

As a practical example we may take the case of a circuit having a

resistance of 5 ohms and an inductance of .02 henry, to which a sine wave E.M.F. of effective value 100 volts at a frequency of 50 is applied.

The effective value of the current and the angle of lag can now be calculated as follows:—

$$I = \frac{E}{z},$$

$$\text{where } z = \sqrt{R^2 + \omega^2 L^2} = \sqrt{(5)^2 + (2\pi \times 50 \times .02)^2}$$

$$= \sqrt{64.5} \doteqdot 8.03 \text{ ohms}$$

$$\therefore I = \frac{100}{8.03} \doteqdot 12.33 \text{ amperes}$$

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left(\frac{2\pi}{5}\right)$$

$$= 51^\circ\ 29'.$$

Capacity and Resistance in Series.—Let us now consider a non-inductive circuit having a resistance of R ohms and a capacity of C farads. Let an alternating E.M.F. e of sine wave form be applied to the circuit, and let the current produced by this be $i = I_{max.} \sin \omega t$.

As before, the applied E.M.F. e will consist of two components, one e_c to charge the condenser, and the other e_r to cause the current to flow through the resistance,

$$e = e_r + e_c.$$

We have already seen that the component e_r is in phase with the current, and is given by

$$e_r = I_{max.} R \sin \omega t.$$

In the last chapter it was shown that e_c has a maximum value equal to $\frac{I_{max.}}{\omega C}$, and that it *lags* behind the current by an angle $\frac{\pi}{2}$.

$$\therefore e_c = \frac{I_{max.}}{\omega C} \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= -\frac{I_{max}}{\omega C} \cos \omega t.$$

In fig. 12 the curves of e, e_r, e_c, and i have been plotted, and the corresponding vector diagram is shown to the left.

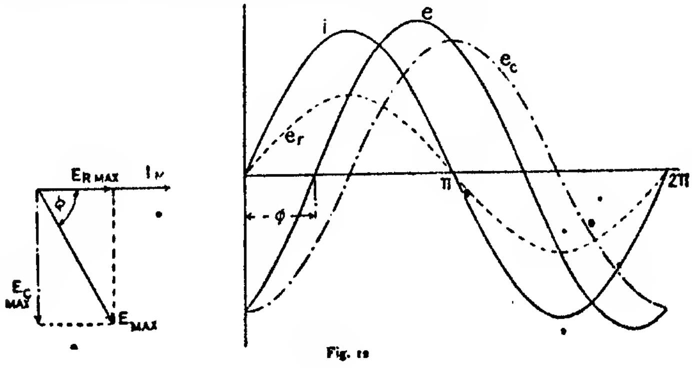


Fig. 12

From the vector diagram we see that

$$E_{max.} = \sqrt{E^2_{R\,max.} + E^2_{C\,max.}},$$

and consequently

$$E = \sqrt{E^2_R + E^2_C}.$$

$$\text{But } E_R = IR \text{ and } E_C = \frac{I}{\omega C}.$$

$$\therefore E = I\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2},$$

which may be written

$$E = I\sqrt{R^2 + x_c^2} = IZ,$$

where Z is the impedance of the circuit.

The current *leads* the applied E.M.F. by an angle ϕ *less* than $\frac{\pi}{2}$.

From the vector diagram

$$\tan\phi = \frac{E_{C\,max.}}{E_{R\,max.}} = \frac{I_{max.}}{\omega C} \times \frac{1}{I_{max.} R}$$

$$= \frac{1}{\omega C R},$$

$$\text{or } \phi = \tan^{-1}\left(\frac{1}{\omega C R}\right).$$

The angle of lead *decreases* as R *is increased*, and varies from $\frac{\pi}{2}$ to 0 as R varies from 0 to ∞.

As a practical example we will calculate the current and the angle of lead in a circuit having a resistance of 50 ohms and a capacity of 50 microfarads, when a sine wave E.M.F. of effective value 100 volts at a frequency of 50 cycles per sec. is applied to the circuit.

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$= \sqrt{(50)^2 + \left(\frac{10^6}{2\pi \times 50 \times 50}\right)^2}$$

$$\doteqdot \sqrt{2500 + 4058}$$

$$\doteqdot 81 \text{ ohms.}^1$$

$$I = \frac{E}{Z} = \frac{100}{81} \doteqdot 1.235 \text{ amperes.}$$

$$\phi = \tan^{-1}\left(\frac{1}{\omega C R}\right)$$

$$= \tan^{-1}\left(\frac{63.7}{50}\right)$$

$$= \tan^{-1}(1.274) = 51° \, 52'.$$

[1] In order to obtain the results in practical units all the quantities must be expressed in practical units. The practical unit of capacity is the *farad*, therefore the value of C in *microfarads* must be multiplied by 10^{-6}. This should be carefully noted.

Inductance, Capacity, and Resistance in Series.—From the two cases just considered it will be evident that inductance and capacity may be regarded as opposite in their effects. Inductance causes the current to *lag*, while capacity causes it to *lead*. In a circuit in which a capacity is connected in series with an inductance, the inductance component of the applied E.M.F. is 180° out of phase with the capacity component. These components, therefore, tend to neutralize one another, and, given suitable values of the inductance, capacity, and frequency, may do so completely, giving rise to the condition of electrical resonance, which will be referred to later.

In the circuit shown in fig. 13 let R be the resistance in ohms, L the inductance in henries, and C the capacity in farads, and let the current

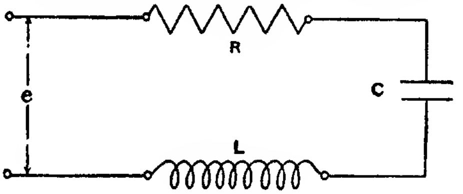


Fig. 13

resulting from the sine wave applied E.M.F. e be given by the equation $i = I_{max.} \sin \omega t$.

There will in this case be three components of the applied E.M.F., viz.:

e_L to overcome the self-induced E.M.F.,
e_c to charge the condenser,
e_r to cause the current to flow through the resistance.
$\therefore\ e = e_r + e_l + e_c$.

Now it has already been shown that e_l has a maximum value $\omega L I_{max.}$ and leads e_r by $\frac{\pi}{2}$;

$$\therefore\ e_l = \omega L I_{max.} \sin\left(\omega t + \frac{\pi}{2}\right).$$

Also e_c has a maximum value $\frac{I_{max}}{\omega C}$ and lags $\frac{\pi}{2}$ behind e_r,

$$\therefore\ e_c = \frac{I_{max.}}{\omega C} \sin\left(\omega t - \frac{\pi}{2}\right),$$

$$\text{and } e_r = I_{max.} R \sin \omega t.$$

The curves of e, e_r, e_l, e_c, and i have been plotted in fig. 14, and the corresponding vector diagram is shown to the left.

From the vector diagram it will be seen that

$$E_{max.} = \sqrt{E^2_{R\,max.} + (E_{L\,max.} - E_{C\,max.})^2}$$

$$\therefore\ E = \sqrt{E^2_R + (E_L - E_c)^2}.$$

Inserting the values of E_R, E_L, and E_C,

$$E = \sqrt{I^2 R^2 + \left(\omega L I - \frac{I}{\omega C}\right)^2}$$

$$= I\sqrt{R^2 + \left(\omega L - \frac{I}{\omega C}\right)^2}.$$

This may be written $E = I\sqrt{R^2 + x^2}$, where x is the total reactance

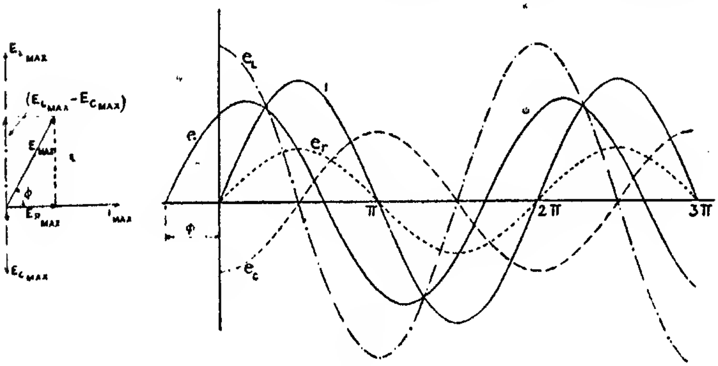


Fig. 14

of the circuit and is equal to $\left(\omega L - \frac{I}{\omega C}\right)$. The equation may be reduced to the still simpler form $E = I z$,

where z = total impedance of the circuit

$$= \sqrt{R^2 + \left(\omega L - \frac{I}{\omega C}\right)^2}.$$

This expression for the impedance may be regarded as the complete one, since, by making the terms R, L, or C zero, the value of the impedance in any of the simpler cases may be obtained.

In the vector diagram $\tan\phi = \dfrac{E_{L\,max} - E_{C\,max}}{E_{R\,max}}$,

$$\therefore\ \phi = \tan^{-1}\frac{\left(\omega L - \frac{I}{\omega C}\right)}{R}.$$

This, again, is a complete expression, since the value of ϕ in any of the simpler cases may be obtained at once by making L or C zero as the case may be. If R is zero, and L and C have any values whatever, provided ωL is not equal to $\frac{I}{\omega C}$, $\phi = \frac{\pi}{2}$. On the other hand, if R is not zero, and either L and C are both zero or have values such that $\omega L = \frac{I}{\omega C}$, then ϕ is zero.

The equation $E = I z$ is an expression of Ohm's Law for the A.C. circuit, and z is often referred to as the *apparent resistance* of the circuit.

Resonance.—When the capacity reactance becomes numerically equal to the inductive reactance, i.e. $\omega L = \frac{1}{\omega C}$, the second term under the square root in the equation

$$E = I\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

becomes zero, and

$$E = IR.$$

The circuit then behaves as if inductance and capacity were both absent, and is said to be in a state of electrical resonance. This condition may give rise to disastrous effects.

As a practical example, let the resistance, capacity, and inductance in the circuit shown in fig. 13 be as follows:

$$R = 0.2 \text{ ohm}, \ C = 50 \text{ microfarads}, \ L = 0.2025 \text{ henry}.$$

Let a sine wave E.M.F. of effective value 100 volts at 50 cycles per sec. be applied to the circuit.

$$\text{Then } Z = \sqrt{R^2 + x^2},$$

$$\text{and } x = x_L - x_c = (2\pi \times 50 \times 0.2025) - \left(\frac{10^{-6}}{2\pi \times 50 \times 50}\right)$$

$$= 63.7 - 63.7 = 0,$$

$$\therefore Z = 0.2 \text{ ohm},$$

$$\therefore I = \frac{100}{0.2} = 500 \text{ amperes}.$$

$$\text{Let } V_L = \text{P.D. across the inductance}$$

$$= \omega L I = 500 \times 63.7 = 31{,}850 \text{ volts},$$

$$V_c = \text{P.D. across the capacity}$$

$$= \frac{I}{\omega C} = 500 \times 63.7 = 31{,}850 \text{ volts}.$$

It will be seen from this example that quite a low applied E.M.F. may cause a large current to flow which gives rise to very large P.D.s across the capacity and inductance.[1]

In an A.C. supply system the capacity and inductance are distributed, but behave in the same way as those in the simple circuit just referred to. Since the cores of the cable form the plates of the condenser exceedingly high P.D.s will exist between them when resonance occurs which frequently causes a break-down of the insulation of the cable.

Resonance is most likely to occur at no load, or at very light loads, and in any case is very much more serious then, since the only resistance in circuit is that of the cables, which is very low. Not only must resonance with the fundamental frequency be avoided, but also resonance with the frequency of any harmonics which may be present in the E.M.F. wave form.

[1] Practical use is sometimes made of the condition of resonance to obtain the very high pressure required for the break-down test of cables.

We will now consider the effect of changing the frequency to 25 cycles per sec. in the above example.

$$x_L = 2\pi \times 25 \times .2025 = 31.85 \text{ ohms.}$$

$$x_c = \frac{10^{-6}}{2\pi \times 25 \times 50} = 127.4 \text{ ohms.}$$

$$x = -95.55 \text{ ohms.}$$

$$\therefore z = \sqrt{.04 + 9130}$$

$$\doteqdot 95.55 \text{ ohms.}$$

$$I = \frac{E}{z} = \frac{100}{95.55} \doteqdot 1.048 \text{ amperes.}$$

$$V_L = \omega L I = 31.85 \times 1.048 \doteqdot 33.4 \text{ volts.}$$

$$V_c = \frac{I}{\omega C} = 127.4 \times 1.048 \doteqdot 133.6 \text{ volts.}$$

$$\phi = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right) = \tan^{-1}\left(\frac{-95.55}{.2}\right)$$

$$= \tan^{-1}(-477.75)$$

$$\doteqdot -89° 50'.$$

Effect of Variation of Voltage and Frequency.—The effects of the variation of the applied E.M.F. and the frequency can be readily traced by plotting for the various cases curves of impedance, reactance, phase angle, and current on a base of (*a*) applied E.M.F., (*b*) frequency.

I. *Resistance and Inductance.*—

$$x = \omega L.$$

x therefore is independent of the voltage, and varies directly as the frequency.

$$z = \sqrt{R^2 + x^2}.$$

z is also independent of the voltage, and is a function of the frequency.

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right).$$

ϕ is also independent of the voltage, and its tangent varies directly as the frequency.

$$I = \frac{E}{z}.$$

I varies directly as the voltage, and inversely as the impedance in all cases.

The curves are shown in figs. 15*a* and 15*b*.

It will be noticed that the curve of z (fig. 15*b*) cuts the axis of ordinates at a value equal to R, and is asymptotic to the curve of x, i.e. when $f = o$, $z = R$, and as f increases z becomes more and more nearly equal to x.

ϕ is zero when f is zero, and approaches $\frac{\pi}{2}$ as f approaches infinity.

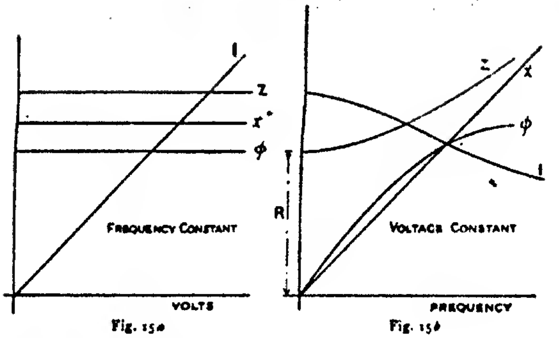


Fig. 15a Fig. 15b

II. *Resistance and Capacity.*—

$$x = \frac{1}{\omega C}.$$

x therefore is independent of the voltage, and varies *inversely* as the frequency.

$$z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2},$$

and as before is independent of the voltage, and is a function of the frequency.

$$\phi = \tan^{-1}\left(\frac{1}{\omega C R}\right),$$

and is independent of the voltage, and its tangent varies inversely as the frequency.

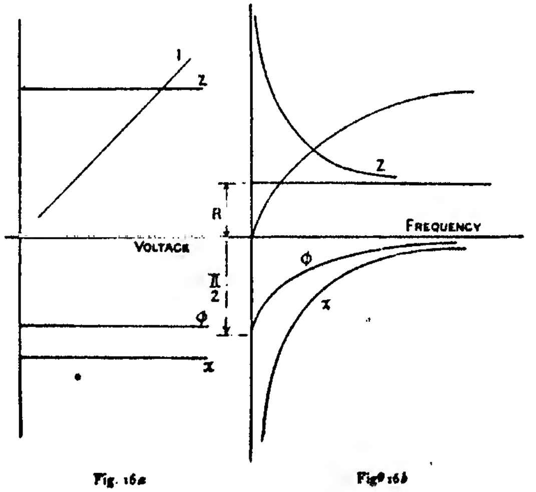


Fig. 16a Fig. 16b

From fig. 16b it will be seen that the curve of *z* is asymptotic to the

axis of ordinates, and also to the horizontal straight line representing R, i.e. z approaches infinity as f approaches zero, and approaches the value R as f approaches infinity. ϕ is negative (i.e. the current *leads* the E.M.F.), and approaches $\frac{\pi}{2}$ as f approaches zero, and approaches zero as f approaches infinity.

III. *Resistance, Inductance, and Capacity.*—

$$x = x_L - x_C = \omega L - \frac{1}{\omega C}.$$

x therefore is independent of the voltage, and varies with the frequency.

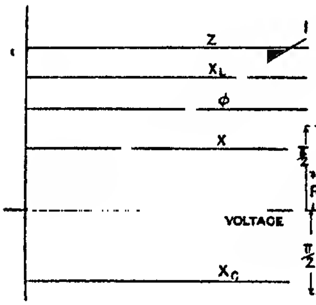


Fig. 17a

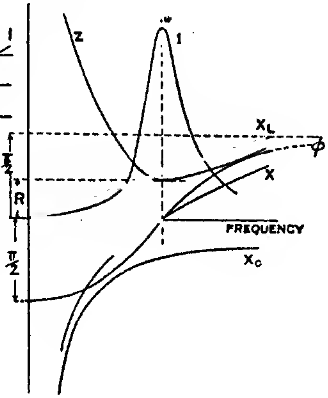


Fig. 17b

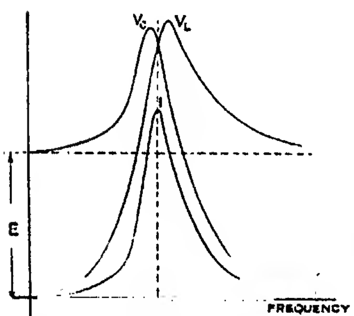


Fig. 17c

z and ϕ are also independent of the voltage, and are functions of the frequency.

As will be seen from fig. 17*b*, z decreases as f is increased, until it reaches a minimum value R. The inductive and capacity reactances are then numerically equal, and ϕ is zero (resonance). As f increases to infinity, z approaches the value x_L and ϕ the value $+\frac{\pi}{2}$. For the sake of clearness, the curves of V_c and V_L (P.D.s across condenser and inductance respectively) are shown separately in fig. 17*c*. When f is zero, V_L is obviously zero, since I is then zero. The whole applied E.M.F. therefore is devoted to producing V_c, and $V_c = E$. $V_c\left(=\frac{I}{\omega C}\right)$ will come to a maximum before I, since before its maximum I is increasing much less rapidly than ωC. Also $V_L (= \omega L I)$ will come to a maximum after I, since immediately

after its maximum I is decreasing very slowly, and ωL is increasing steadily. As f approaches infinity V_c approaches zero, and consequently V_L approaches E.

Impedances in Parallel.—So far only series arrangements of resistance, inductance, and capacity have been dealt with. Parallel arrangements will now be considered, and in order to cover all cases, a general case of two impedances connected in parallel (as in fig. 18), and each consisting of a resistance, inductance, and capacity, will be taken.

Let the resistance, inductance, and capacity in the branches 1 and 2 of the circuit be R_1, L_1, C_1, and R_2, L_2,

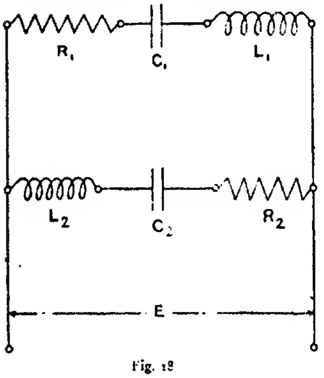


Fig. 18

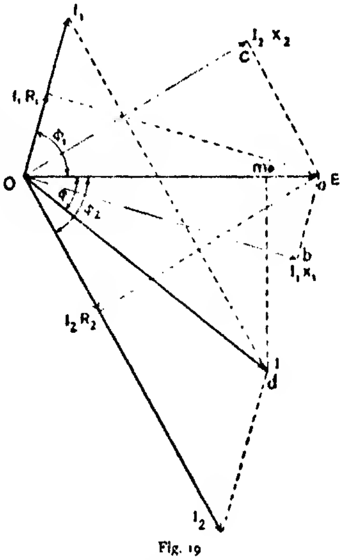


Fig. 19

C_2 respectively; and let the effective value of the applied E.M.F. be E. The reactance of branch 1 is

$$x_1 = \omega L_1 - \frac{1}{\omega C_1}.$$

$$\text{Similarly,} \quad x_2 = \omega L_2 - \frac{1}{\omega C_2}.$$

In this case we have a P.D. equal to E, which is common to both branches.

Clearly the current I in the main circuit is the resultant of the currents I_1 and I_2 in the branches 1 and 2.

This is shown in the vector diagram,[1] fig. 19. In branch 1 there are two components of E, $I_1 R_1$ and $I_1 x_1$. Similarly in branch 2 there are two components, $I_2 R_2$ and $I_1 x_1$. Bearing these facts in mind, the construction of the vector diagram will be readily followed.

Let us resolve each current into two components, one in phase with E and one at right angles to E $\left(\frac{\pi}{2}\text{ out of phase}\right)$.

[1] Since the effective values are the important ones, it is the usual practice to let the vectors represent *effective values*, as has been done in fig. 19. If instantaneous values are required, they are readily obtained by multiplying the projections by $\sqrt{2}$.

Components in phase with E (watt components of the currents).

$$I \cos \phi = I_1 \cos \phi_1 + I_2 \cos \phi_2.$$

Components $\frac{\pi}{2}$ out of phase with E (wattless components of the currents).

$$I \sin \phi = I_1 \sin \phi_1 + I_2 \sin \phi_2.$$

Considering the triangle $O a b$, the angle $O a b$ is equal to ϕ_1,

$$\therefore \cos \phi_1 = \cos O a b = \frac{a b}{O a} = \frac{I_1 R_1}{E}.$$

If z_1 is the impedance of branch 1,

$$\cos \phi_1 = \frac{I_1 R_1}{I_1 z_1} = \frac{R_1}{z_1}.$$

In an exactly similar way it may be shown that

$$\cos \phi_2 = \frac{R_2}{z_2},$$

and that

$$\cos \phi = \frac{R}{z},$$

where R_2 and z_2 are the resistance and impedance of branch 1, and R and z are the resultant resistance and resultant impedance of the main circuit.

Again, considering the triangle $O a b$,

$$\sin \phi_1 = \sin O a b = \frac{O b}{O a} = \frac{I_1 x_1}{E} = \frac{I_1 x_1}{I_1 z_1} = \frac{x_1}{z_1};$$

and similarly,

$$\sin \phi_2 = \frac{x_2}{z_2},$$

$$\sin \phi = \frac{x}{z},$$

where x_1, x_2, and x are respectively the reactance of branch 1, the reactance of branch 2, and the resultant reactance of the main circuit.

We may now rewrite the equations of the components as follows:—

$$I \frac{R}{z} = I_1 \frac{R_1}{z_1} + I_2 \frac{R_2}{z_2}, \text{ and } I \frac{x}{z} = I_1 \frac{x_1}{z_1} + I_2 \frac{x_2}{z_2}.$$

$$\text{But } I = \frac{E}{z}, \quad I_1 = \frac{E}{z_1}, \text{ and } I_2 = \frac{E}{z_2}.$$

$$\therefore E \frac{R}{z^2} = E \frac{R_1}{z_1^2} + E \frac{R_2}{z_2^2}; \text{ or } \frac{R}{z^2} = \frac{R_1}{z_1^2} + \frac{R_2}{z_2^2}.$$

$$\text{And similarly, } \frac{x}{z^2} = \frac{x_1}{z_1^2} + \frac{x_2}{z_2^2}.$$

The expression $\frac{R}{z^2}$ is called the *conductance* of the circuit, and is usually denoted by g.

We may therefore write

$$g = g_1 + g_2,$$

where g_1 and g_2 are respectively the conductances of the branches 1 and 2.

The resultant conductance of n conductances in parallel is therefore the sum of the n conductances.

The expression $\frac{x}{z^2}$ is called the *susceptance* of the circuit, and is usually denoted by b.

$$\therefore\ b = b_1 + b_2,$$

where b_1 and b_2 are the susceptances of the branches 1 and 2 respectively.

The resultant susceptance of n susceptances in parallel is therefore the sum of the n susceptances.

In the triangle O m d formed by drawing $d\,m$ perpendicular to O a, O m is the component of I in phase with E, and $m\,d$ is the component of I $\frac{\pi}{2}$ out of phase with E.

$$\begin{aligned}
\text{Now } (\mathrm{O}\,d)^2 &= (\mathrm{O}\,m)^2 + (m\,d)^2. \\
\therefore\ \mathrm{I}^2 &= (\mathrm{I}\cos\phi)^2 + (\mathrm{I}\sin\phi)^2 \\
&= (\mathrm{I}_1\cos\phi_1 + \mathrm{I}_2\cos\phi_2)^2 + (\mathrm{I}_1\sin\phi_1 + \mathrm{I}_2\sin\phi_2)^2 \\
&= \left(\mathrm{E}\frac{\mathrm{R}_1}{z_1^2} + \mathrm{E}\frac{\mathrm{R}_2}{z_2^2}\right)^2 + \left(\mathrm{E}\frac{x_1}{z_1^2} + \mathrm{E}\frac{x_2}{z_2^2}\right)^2 \\
&= \mathrm{E}^2(g_1 + g_2)^2 + \mathrm{E}^2(b_1 + b_2)^2. \\
\text{But } \mathrm{I} &= \frac{\mathrm{E}}{z}. \\
\therefore\ \frac{\mathrm{E}^2}{z^2} &= \mathrm{E}^2\{(g_1 + g_2)^2 + (b_1 + b_2)^2\}; \\
\text{or } \frac{1}{z} &= \sqrt{(g_1 + g_2)^2 + (b_1 + b_2)^2}.
\end{aligned}$$

$\frac{1}{z}$ is called the *admittance* of the circuit and is usually denoted by y.

We may therefore write a general equation for n impedances in parallel:

$$y = \frac{1}{z} = \sqrt{(g_1 + g_2 + \ldots + g_n)^2 + (b_1 + b_2 + \ldots + b_n)^2},$$

where g_1, g_2, &c., are the conductances, and b_1, b_2, &c., are the susceptances of the impedances z_1, z_2, &c.

The phase angles ϕ, ϕ_1, ϕ_2, &c., may be determined as follows:—

$$\tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{\frac{x}{z}}{\frac{\mathrm{R}}{z}} = \frac{\frac{x}{z^2}}{\frac{\mathrm{R}}{z^2}} = \frac{b}{g},$$

$$\therefore\ \phi = \tan^{-1}\left(\frac{b}{g}\right).$$

$$\begin{aligned}
\text{Similarly, } \phi_1 &= \tan^{-1}\left(\frac{b_1}{g_1}\right). \\
\phi_2 &= \tan^{-1}\left(\frac{b_2}{g_2}\right). \\
\phi_n &= \tan^{-1}\left(\frac{b_n}{g_n}\right).
\end{aligned}$$

As a practical example, let us calculate the currents and phase angles in the circuit shown in fig. 20. Let the constants have the following values:—

$R_1 = 5$ ohms, $C_1 = 50$ microfarads, $R_2 = 20$ ohms, $L_2 = 0.2$ henry, and $E = 100$ volts at a frequency of 50.

$$\text{Then } z_1^2 = R_1^2 + \left(-\frac{1}{\omega C_1}\right)^2$$
$$= 25 + \left(\frac{-10^6}{2\pi \times 50 \times 50}\right)^2$$
$$= 25 + 4058 = 4083.$$
$$z_2^2 = R_2^2 + (\omega L_2)^2$$
$$= 400 + (2\pi \times 50 \times .2)^2$$
$$= 4350.$$
$$g_1 = \frac{R_1}{z_1^2} = \frac{5}{4083} = .00122; \quad b_1 = \frac{x_1}{z_1^2} = \frac{-63.7}{4083} = -.01558;$$
$$g_2 = \frac{R_2}{z_2^2} = \frac{20}{4350} = .0046; \quad b_2 = \frac{x_2}{z_2^2} = \frac{62.8}{4350} = .01445;$$
$$y = \sqrt{g^2 + b^2}$$
$$= \sqrt{(.00122 + .0046)^2 + (-.01558 + .01445)^2}$$
$$= .00682.$$
$$y_1 = \frac{1}{z_1} = \frac{1}{64}.$$
$$y_2 = \frac{1}{z_2} = \frac{1}{66}.$$
$$\therefore I = Ey = 100 \times 0.00682 = 0.682 \text{ ampere.}$$
$$I_1 = Ey_1 = \frac{100}{64} = 1.563 \text{ amperes.}$$
$$I_2 = Ey_2 = \frac{100}{66} = 1.515 \text{ amperes.}$$
$$\phi = \tan^{-1}\left(\frac{b}{g}\right) = \tan^{-1}(-.194) \doteqdot -10^\circ\ 59'.$$
$$\phi_1 = \tan^{-1}\left(\frac{b_1}{g_1}\right) = \tan^{-1}(-12.75) \doteqdot -85^\circ\ 31'.$$
$$\phi_2 = \tan^{-1}\left(\frac{b_2}{g_2}\right) = \tan^{-1}(3.14) \doteqdot +72^\circ\ 20'.$$

A negative value of ϕ indicates, as before, that the current is leading the E.M.F. Some confusion may arise on referring this to the vector diagram, since ϕ is at the first glance apparently positive when the current is leading.

This difficulty will be removed at once if it is remembered that in all cases ϕ is measured *from* the current vector *to* the E.M.F. vector.

Resonance in Parallel Arrangements.—A condition in which the main circuit behaves as if inductance and capacity were absent occurs in parallel arrangements. This resonance differs in its effects from resonance in series circuits, since the current in the main circuit is then a *minimum* instead of a maximum. As before, when the condition of resonance occurs, ϕ is zero.

$$\therefore \tan\phi = \frac{b}{g} = 0,$$

$$\text{and therefore } b = 0.$$

In all but the simplest arrangements the equation obtained by express-

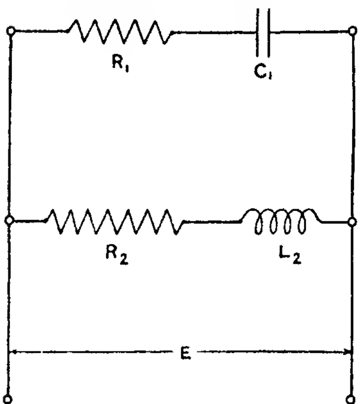


Fig. 20

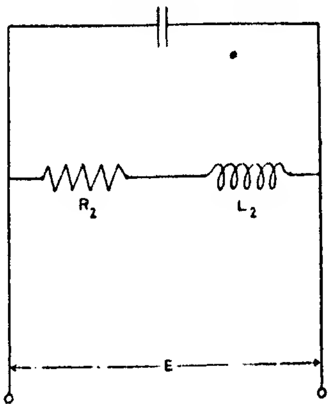


Fig. 21

ing b in terms of the constants of the circuits is very cumbrous. We will, however, consider one practical case.

It is often required to secure unity power-factor (i.e. $\cos\phi = 1$) in a circuit which is itself inductive.

This may be accomplished by placing a condenser of suitable value in parallel with the inductive circuit.

In fig. 21 let R_2 and L_2 be respectively the resistance and inductance of such a circuit. Let E be the applied E.M.F. and let C_1 be the capacity of the condenser required to bring the current and E.M.F. into phase in the main circuit. Then $\phi = 0$ and $b = 0$.

$$b = \frac{-\dfrac{1}{\omega C_1}}{\left(-\dfrac{1}{\omega C_1}\right)^2} + \frac{\omega L_2}{R^2 + \omega^2 L_2^2} = 0,$$

$$\therefore -\omega C_1 = -\frac{\omega L_2}{R^2 + \omega^2 L_2^2},$$

$$C_1 = \frac{L_2}{R_2^2 + \omega^2 L_2^2}.$$

Thus if $L_2 = 0.2$ henry; $R_2 = 5$ ohms; and $f = 50$ cycles per sec.,

$$C_1 = \frac{0.2}{25 + (100\pi \times 0.2)^2} = 50.4 \times 10^{-6} \text{ farads}$$
$$= 50.4 \text{ microfarads.}$$

This method of securing unity power-factor has hitherto been little employed, since the cost of manufacturing reliable condensers of the high capacity required in practice was excessive. Recently, however, suitable condensers at a reasonable cost have been produced.

CHAPTER V

ALTERNATING-CURRENT INSTRUMENTS

Ammeters, voltmeters, and supply meters, suitable for use with alternating currents, have already been described in Chapters V and VI of the previous article. This chapter will be devoted to wattmeters, and certain essentially alternating-current instruments, such as power-factor meters, frequency meters, and synchroscopes.

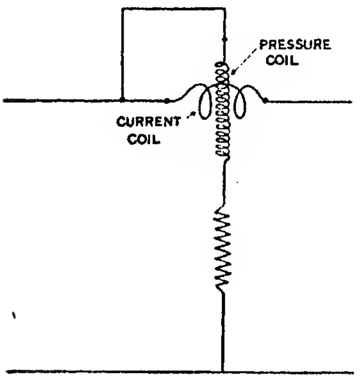


Fig. 21.

Wattmeter.—A wattmeter is an instrument for measuring the mean power in a circuit. They are rarely employed on D.C. circuits since the power is easily calculated as the product of current and pressure.

In alternating-current circuits this simple relationship does not hold, and therefore wattmeters, which, as will be seen later, record the true mean power in the circuit, have to be used.

By far the greater number of wattmeters in commercial use are of the electrodynamic type. In this type a moving coil is pivoted within a fixed coil, and carries a pointer moving over a scale. The fixed coil is constructed of heavy wire or strip, and carries the line current. The moving coil is of fine wire, and has a large non-inductive resistance connected in series with it. This coil is connected in parallel with the circuit, and therefore carries a current proportional to the P.D. between the line wires.

The fixed and moving coils are generally referred to as the current and pressure coils respectively.

For convenience in altering the range of the instrument, the current coil is frequently wound in two separate sections which may be connected in series or in parallel with one another as required. The method of connecting such a wattmeter in a single-phase circuit is shown in fig. 22.

The deflecting force is at any instant directly proportional to the product of the moving-coil current, and the field produced by the fixed coil, when the plane of the pressure coil is at right angles to the plane of the fixed coil. Since the field produced by the fixed coil is proportional to the current in the line wire, and the current in the moving coil is proportional to the P.D. between the line wires, the deflecting force at any instant is proportional to the product of e and i (where e is the instantaneous P.D. between the line wires, and i the instantaneous current in the line wire), and therefore to the instantaneous power in the circuit. The mean deflecting force will be, therefore, proportional to the true mean power in the circuit. The controlling couple is provided by light spiral springs, and is therefore proportional to the angle of deflection.

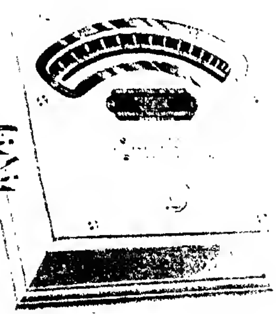


Fig. 23.—Weston Wattmeter

The deflecting couple, even over a fairly wide movement of the pressure coil, is very approximately proportional to the mean power, and the scale of the instrument is therefore open and practically evenly divided. The general appearance of a well-known wattmeter of this type is shown in fig. 23.

The fixed and moving coils of a wattmeter are very clearly shown in fig. 24. The instrument is shown connected up for measuring power in a three-phase circuit, using an artificial neutral point (see Chapter VII).

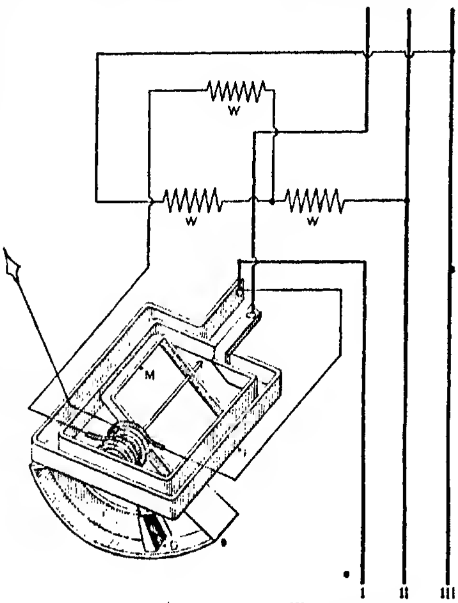


Fig. 24.—Hartmann & Braun Wattmeter

In order that instruments of this class may read correctly, the inductance, and in a lesser degree the capacity, of the coils must be made as small as possible.

The effect of the inductance of the fine-wire moving coil is minimized by a high non-inductive resistance connected in series with it. In addition,

the employment of any masses of metal in the supports or case of the instruments has to be avoided, since the eddy currents set up in them affect the readings. In any case, at very low power-factors the readings cannot be relied upon.

By careful compensation electrodynamic wattmeters, suitable for use as standards under ordinary conditions, can be constructed. Where extreme accuracy is required it is better to employ an instrument of the electrostatic type, such as the Addenbrooke wattmeter.

This instrument is similar in construction and principle to the Addenbrooke voltmeter, described in Chapter V of the previous article, except that a *single needle and quadrants instead of octants* are used. *A very complete description of an instrument of this kind, together with details* of its use at the National Physical Laboratory, will be found in vol. 51, no. 221, of the *Journal of the Institution of Electrical Engineers*, "The Use of the Electrostatic Method for the Measurement of Power", by Messrs. Paterson, Rayner, and Kinnes. The needle forms the pressure element, and it is usually arranged that the potential applied to it is from 100 to 200 volts. The quadrants have a P.D. of about 2 volts between them supplied by the potential drop in a shunt resistance through which the current passes. The quadrants thus form the current element.

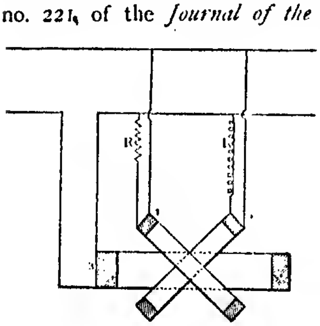


Fig. 25

The following advantages and disadvantages may be pointed out:—

Advantages.—1. High accuracy even at very low power-factors.

2. Wide range of both current and voltage for which a single instrument may be used.

3. The instrument itself is independent of changes of frequency and wave-form. The small error due to the inductance of the shunt is easily calculated and corrected for. It is seldom necessary to do this, since the error is negligible, except when the current is very large and the power-factor very low.

Disadvantages.—1. The movement of the mirror is very slow owing to the small controlling forces.

2. For large currents the shunt resistance has to be artificially cooled.

3. The instrument is not portable.

Power-factor Meters.—Although in experimental work the power-factor may generally be more accurately determined by calculation from wattmeter-readings, in power-stations it is of importance to have instruments which will give a direct indication of the power-factor.

In one type of power-factor meter, the moving part consists of two fine-wire coils rigidly fixed at right angles to one another, and mounted on a spindle to which the pointer of the instrument is attached. A single fixed coil encloses the moving coil. When used as a single-phase instrument, the two coils of the moving element are connected in parallel across

the mains (see fig. 25). The currents in the two coils are made to differ in phase by approximately 90°. This is secured by a non-inductive resistance R connected in series with one coil, while a large inductance I is connected in series with the other. The fixed coil is connected in series with one line wire. When the current in the main circuit is in phase with the applied P.D., the current is in phase with the current in pressure coil 1. The force acting on the other coil is very small, since the currents in the fixed and the moving coils 2 are nearly 90° out of phase. Under these conditions, therefore, the pair of moving coils will tend to lie in such a position that coil 1 is in the same plane as the fixed coil. If the power-factor of the circuit change and the current lag behind the applied P.D., then there will be a force acting on coil 2 of the combination, and the actual position of the pair of coils will depend on the ratio of the forces acting on the two coils.

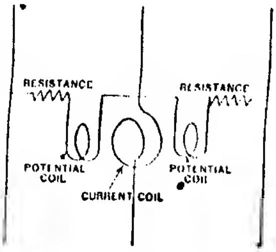


Fig. 26

If the power-factor is 0, the current lagging 90° behind the pressure, the currents in the main coil and in the coil 2 of the combination will be very nearly in phase, and therefore this coil will tend to lie in the same plane as the fixed coil. The position of this pair of coils with reference to the fixed-current coil will serve, therefore, as an indication of the power-factor of the circuit to which the instrument is connected.

When a two-phase system is used, the necessity for a highly inductive circuit to produce a current lagging nearly 90° behind the applied potential difference disappears. The second coil (2, fig. 25) in the power-factor indicator is connected either directly or through a transformer to the other phase of the system. A power-factor indicator used in this way has the advantage over the single-phase instrument of being independent of the frequency for the accuracy of its reading. The phase difference between current and potential difference in the circuit 2 is always 90°, whereas with the single-phase instrument the phase difference between current and potential difference varies with the frequency, and the reading of the meter on a circuit of frequency different from that for which it has been calibrated will be wrong.

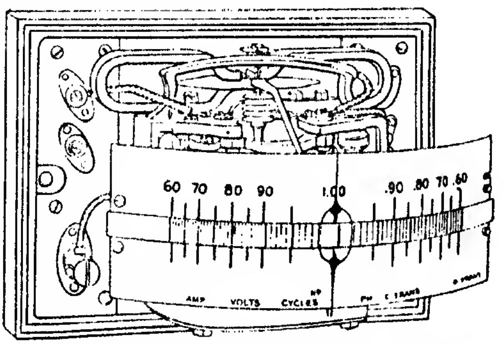


Fig. 27.—Horizontal Edgewise Power-factor Indicator (British Thomson-Houston Company, Ltd.)

The connections for a power-factor indicator on a three-phase circuit

are shown in fig. 26. When the current in the fixed coil lags behind the pressure in the phases, it comes more into phase with the current in one of the moving coils, and more nearly 90° out of phase with the current in the other, the force on one coil will be increased and on the other diminished, so that the pair (which are, of course, rigidly fixed to the same spindle) will take up a position to indicate the power-factor of the circuit. Fig. 27 shows an instrument made by the British Thomson-Houston Company for three-phase circuits. The vane seen at the top of the instrument is made of aluminium, and moves between the poles of a permanent magnet, so as to damp the motion of the revolving spindle to which it is attached.

Fig. 28.—Interior of Power-factor Meter

For higher pressures and currents the instruments are connected to the mains through transformers.

In another type of power-factor meter, all the coils are fixed, and the resultant field acts on a pair of light soft-iron vanes attached to the spindle which carries the pointer.

Fig. 28 shows the interior of a power-factor moter of this class, manufactured by the British Westinghouse Company. The current coils are so arranged as to produce a uniform rotating field. In the two-phase type of instrument two current coils placed at an angle of 90° are used, and are connected, usually through current transformers, to two line wires of different phases. The three-phase instrument has three current coils placed at 120° and star-connected.

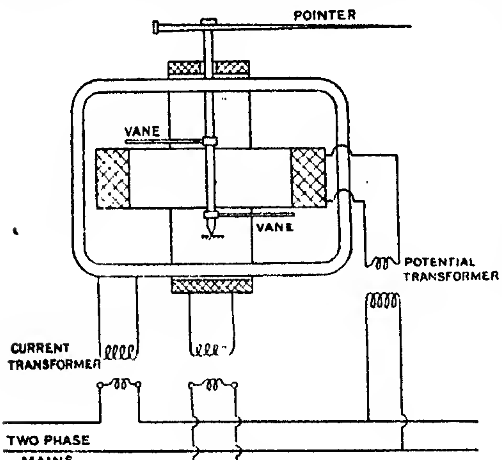


Fig. 29

A fine-wire pressure coil is connected across one pair of line wires, usually through a potential transformer, and serves to magnetize the soft-iron vanes. The arrangement of the coils and vanes in a two-phase instrument are shown in fig. 29. The iron vanes will set themselves along the line in which the rotating field vector lies at the instant when the magnetization of the vanes is a maximum. If the phase angle between the current and P.D. of the mains alters by a given amount, this line will shift by an equal amount

The position of the pointer, therefore, indicates the phase angle. For convenience, however, the scale is graduated in values of the power-factor.

Synchroscopes.—These instruments are used when it is desired to parallel an A.C. machine with others already running on the bus-bars. In principle they are similar to power-factor meters, but the windings are all of fine wire, and in nearly every case the instruments are of the single-phase type.

The general appearance of a synchroscope made by the British Westinghouse Company is shown in fig. 30.

Fig. 30.—Synchroscope for One-, Two-, or Three-phase Circuits

The rotating field is produced by a pair of coils placed at 90°. One coil has a non-inductive resistance, and the other one a highly-inductive coil in series with it. The necessary phase difference for producing the rotating field is thus obtained. The two coils are connected in parallel to the secondary of a potential transformer, whose primary is connected across the bus-bars. The single coil is connected through a potential transformer across one pair of terminals of the incoming machine.

The position of the pointer then indicates the phase angle between the E.M.F. of the bus-bars and of the incoming machine. If the machine is running too fast the pointer will rotate in the one direction, while if it is running too slow the pointer will rotate in the opposite direction. When the machine is in phase and running synchronously, the pointer comes to rest at the zero mark.

Frequency Meters.—The type of frequency meter most commonly

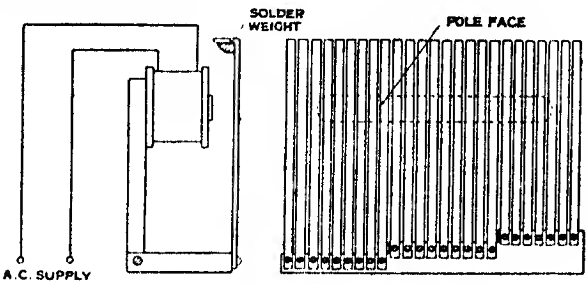


Fig. 31

used in this country consist of a system of tuned vibrating reeds acted upon by electromagnets excited from the supply of which the frequency is being measured. The reeds consist of thin steel strips of different lengths, and tuned by means of solder weights at the end to have natural frequencies varying over the range for which the instrument is designed. The general arrangement is shown diagrammatically in fig. 31.

The magnet applies alternating impulses to the reeds, and that reed whose natural frequency is equal to that of the supply will be thrown into violent vibration. The other reeds will be affected less and less as their frequency is farther from the frequency of supply.

The readings of the instrument may be doubled by superimposing a steady current upon the alternating current flowing in the magnet coil. This current polarizes the reeds, so that they get only one impulse in place of two for every cycle of the alternating current. This type of instrument may also be used as a tachometer by connecting a D.C. supply to the magnet coil through a rotating contact-maker fixed on the shaft of the machine.

In America, an induction type frequency meter is widely used. This consists essentially of two induction voltmeter elements having a common disc to which the pointer is attached.

CHAPTER VI

POLYPHASE CIRCUITS

Two-phase Circuits.—A generator provided with two windings, placed so that when one winding lies under the poles the other is midway between the poles, will generate two E.M.F.s with a phase difference of $\frac{\pi}{2}$ between them. The external circuit may consist of four wires, so that the phases are kept entirely separate, as shown in fig. 32; or the inner pair of wires may be replaced by a single wire, thus forming a three-wire circuit, as shown in fig. 33.

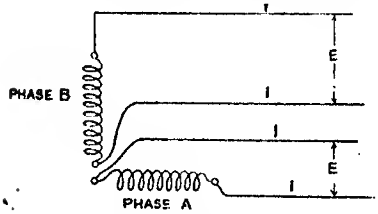


Fig. 32

In the first case it is obvious that the line P.D. E between each pair of line wires will be equal to E_p, the phase E.M.F., and the line current I will be equal to the phase current I_p. In the second case the middle wire carries a current which is the vector sum of the currents I_A and I_B carried by the outer wires. On referring to the vector diagram fig. 34, it will be evident that this resultant current is equal to $\sqrt{2}\,I$; since the P.D. between the outer wires is a *difference* of two potentials it will be equal to the vector *difference* of the two E.M.F.s E_A and E_B. From the figure its value numerically is evidently $\sqrt{2}\,E$.

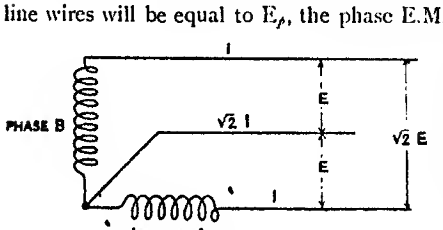


Fig. 33

Three-phase Circuits.—By employing three separate windings on a generator so arranged that each winding is displaced two-thirds of the pole-

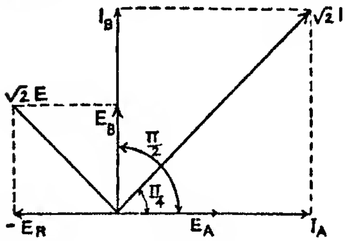


Fig. 34

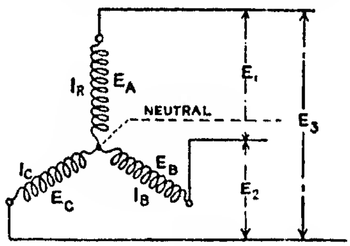


Fig. 35

pitch from the next, three E.M.F.s differing in phase from one another by $\frac{2}{3}\pi$ (120°) may be obtained. The external circuit might consist of six wires, the phases thus being kept entirely separate.

In practice this method is never adopted, and the number of wires is always reduced to three (or at most four) in the following ways:—

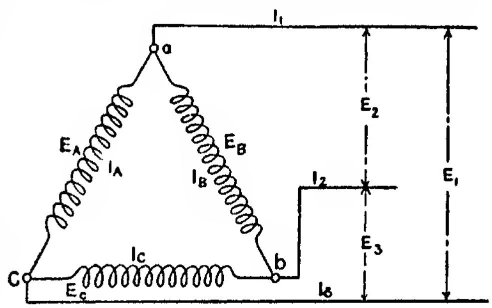


Fig. 36

I. *Star Connection.*—The ends of the three phases are connected together, and three line wires are used, one being connected to the beginning of each phase (see fig. 35). This arrangement is known as a star connection. Occasionally a fourth wire connected to the common junction of the ends of the three-phases (the neutral point) is added, giving a star connection with neutral.

II. *Mesh or Delta Connection.*—The end[1] of each phase is connected to the beginning of the next, and three line wires connected to the three junctions of the phases are used. This arrangement is shown in fig. 36, and is known as a mesh or delta connection.

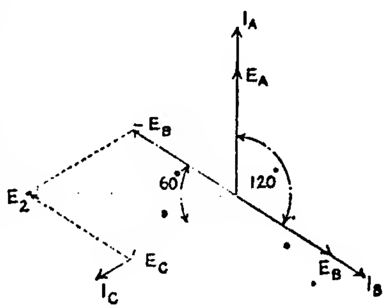


Fig. 37

Currents and P.D.s in Star System. — If the vector diagram for a star system be drawn, it will at once be evident that the sum of the

[1] Thus the end of phase B is connected to the beginning of phase A at the point *a*.

phase currents at any instant is zero (see fig. 37). Therefore if a fourth *wire (neutral wire) be used the current in it will always be zero, provided the load on the system is balanced.*

From fig. 37 it will be seen that the current carried by any line wire will be equal to that in the phase to which it is connected.

$$\text{Thus } I_1 = I_A,\ I_2 = I_B, \text{ and } I_3 = I_C.$$

On the other hand, the P.D. between each pair of line wires is equal to the vector difference of the E.M.F.s in the pair of phases to which these line wires are connected.

$$\text{Thus } E_2 = E_C - E_B.$$

Since the angle between E_C and $-E_B$ is 60°, and E_2 bisects this angle,

$$E_2 = 2\,E_C \cos 30$$
$$= \sqrt{3}\,E_C;$$

or, in general, the line P.D. E is equal to $\sqrt{3}$ times the phase E.M.F. E_p.

Currents and P.D.s in Delta System.—The line P.D. is clearly equal to the phase E.M.F.

$$\text{Thus } E_1 = E_A,\ E_2 = E_B,\ E_3 = E_C;$$
$$\text{or, in general, } E = E_p.$$

The current in any line wire is the resultant of the currents in the two phases to the junction of which the line wire is connected.

Thus I_1 is the resultant of I_A and I_B. Let us consider the point "*a*".

Then by Kirchhoff's Law,

$$i_1 + i_a + i_b = 0.$$

It must clearly be remembered that in Kirchhoff's Law currents flowing *towards* the junction are called *positive* and those flowing *away from* it *negative*.

In the vector diagram, however, currents flowing from the *beginning* towards the *end* of a phase are considered *negative* and those flowing from the *end* towards the *beginning* of a phase are considered *positive*.

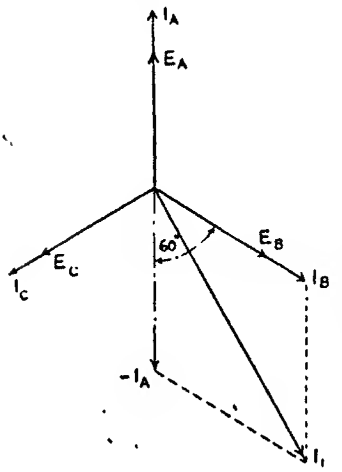


Fig. 38

Therefore in the equation $i_1 + i_a + i_b = 0$, obtained by applying Kirchhoff's Law, $+i_b$ must (since it is a current flowing from the beginning to the end of a phase) in the vector equation be replaced by $-I_B$ (see fig. 38).

$\therefore\ I_1 + I_A - I_B = 0$, is the vector equation for I_1, and $I_1 = I_B - I_A$.

Since the angle between I_a and $-I_A$ is 60°,

$$I_1 = 2\,I_a \cos 30$$
$$= \sqrt{3}\,I_a.$$

In general, I, the line current, is equal to $\sqrt{3}$ times I_p, the phase current.

Summary.—

(*a*) Two-phase 4-wire.........$E = E_p$, $I = I_p$.

(*b*) Two-phase 3-wire.........$E = E_p$, $I = I_p$, for outer wires,

$I = \sqrt{2}\,I_p$, for middle wire.

(*c*) Three-phase Star.........$E = \sqrt{3}\,E_p$, $I = I_p$,

and if a neutral wire is used the P.D. between the neutral and any line wire is equal to E_p.

(*d*) Three-phase Delta.......$E = E_p$, $I = \sqrt{3}\,I_p$.

It may be noted that in deducing the above relationships the following assumptions have been tacitly made:—

1. As far as the P.D.s are concerned, it is assumed that there is no load on the system, and that the E.M.F.s generated in the phases are equal to one another.

2. As far as the currents are concerned, it is assumed that the load on the system is balanced.

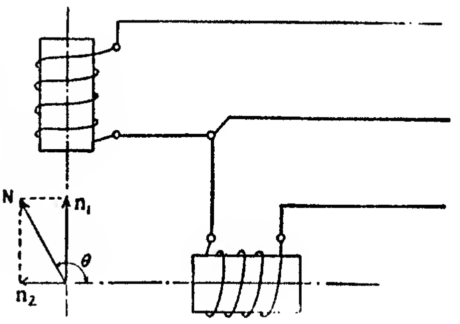


Fig 39

Other polyphase systems, notably four-phase and six-phase, are occasionally used, but by far the greater number of A.C. installations in this country are three-phase, either star or delta connected.

Production of a Rotating Field.—One great advantage of polyphase systems is their suitability for producing a rotating magnetic field, such as that required in an induction motor. As an example of the production of a rotating field, we may take the case of a pair of coils placed at right angles to one another and connected to a two-phase supply, as shown in fig. 39. Since the currents in the two coils are $\frac{\pi}{2}$ out of phase with one another, the fluxes produced by them will be $\frac{\pi}{2}$ out of phase. We may thus write

$$n_1 = N_{\max.} \sin \omega t,$$
$$n_2 = N_{\max.} \cos \omega t,$$

where n_1 and n_2 are the instantaneous values of the fluxes produced by the coils 1 and 2 respectively.

It is assumed that the coils are exactly similar, and that therefore the maximum values of the fluxes are equal. Let N be the resultant of these fluxes, and at any instant let its vector make an angle θ with the horizontal axis (see fig. 39).

Now since n_1 is confined to a vertical axis, it must always be equal to the projection of N on the vertical axis.

$$\therefore \; N \sin\theta = n_1 = N_{\text{max.}} \sin\omega t.$$

$$\text{In a similar way } N \cos\theta = n_2 = N_{\text{max.}} \cos\omega t.$$

$$\text{From which } \tan\theta = \tan\omega t. \quad \therefore \; \theta = \omega t,$$

and it follows that as the resultant flux makes an angle θ, with the horizontal axis, which varies directly with the time t, the resultant flux must rotate uniformly with an angular velocity $\omega = 2\pi f$, where f is the frequency of the supply. From the above equations it also follows that (since $\theta = \omega t$) $N = N_{\text{max.}}$

The resultant flux, therefore, has a constant value $N_{\text{max.}}$, and rotates with an angular velocity equal to 2π times the frequency of the supply in the case of a simple two-pole field.

If a number of coils are used giving a field having $2p$ poles, then the angular velocity of the rotating field will be $\frac{1}{p}$ of the velocity of the simple two-pole field.

In the simple example considered it will be evident that the direction of rotation of the field will depend on the way in which the coils are wound, and that the direction of rotation may be reversed by interchanging one pair of leads.

A rotating field can be produced with equal facility by a three-phase arrangement, and in this case also the direction of rotation of the field may be reversed by interchanging one pair of leads. The winding on the majority of induction motor stators is a three-phase one. The two-phase arrangement is always used for producing the rotating or gliding field employed in A.C. meters, ammeters, and overload relays.

CHAPTER VII

POWER IN A.C. CIRCUITS

The *instantaneous* power in an A.C. circuit is given by the product of the instantaneous E.M.F. and the instantaneous current.

Thus $w = ei$. The *mean* power, as will be seen from figs. 40, 41, depends on the phase relationship of the E.M.F. and current. In the case where E and I are in phase (fig. 40),

$$\text{if } i = I_{\text{max.}} \sin\omega t, \text{ and } e = E_{\text{max.}} \sin\omega t,$$

$$\text{then } w = I_{\text{max.}} E_{\text{max.}} \sin^2\omega t, \text{ or } w = \tfrac{1}{2} I_{\text{max.}} E_{\text{max.}} (1 - \cos 2\omega t).$$

The curve of w is, therefore, a sine wave, having as its axis the

horizontal straight line given by $\frac{1}{2} I_{max.} E_{max.}$, and having a frequency *double* that of E and I. The mean power w is, therefore, equal to $\frac{1}{2} I_{max.} E_{max.}$, or, since $I_{max.} = \sqrt{2}\, I$ and $E_{max.} = \sqrt{2}\, E$, $W = E I$.

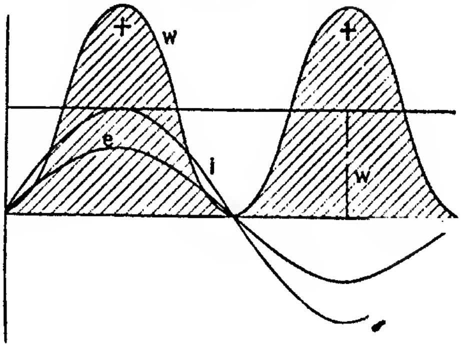


Fig. 40

In every case the curve of w is a sine curve of double frequency, but the mean power varies from $+ E I$ to $- E I$, as the phase difference between E and I varies from 0 to π, and has the value zero when E and I are $\frac{\pi}{2}$ out of phase (fig. 42). In fig. 41 the current is shown lagging behind the

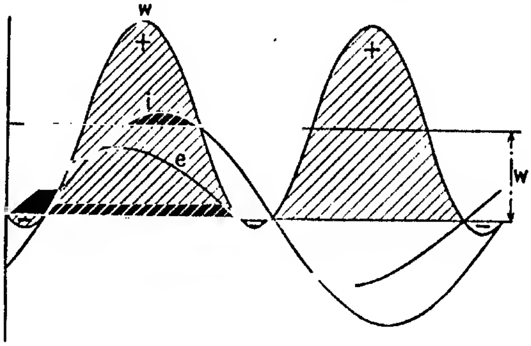


Fig. 41

E.M.F. by an angle less than $\frac{\pi}{2}$. The curve of w now dips below the axis of abscissæ, and consequently the area under the curve is not now all positive. During certain periods the area is negative, indicating that during these times energy is being given back from the circuit. The mean power W is still positive, since the positive areas are greater than the negative ones, but it is less than it was when the current was in phase

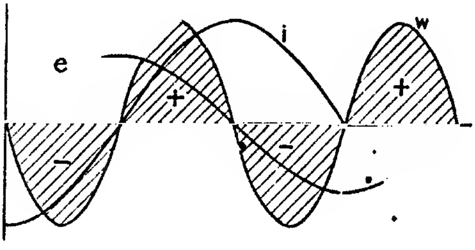


Fig. 42

with the E.M.F. In fig. 42 the current is lagging by $\frac{\pi}{2}$, and the positive and negative areas under the curve of w are equal, and in this case $W = 0$.

Taking a general case, where the current is out of phase with the E.M.F. by an angle ϕ, which may have any value between $-\pi$ and $+\pi$, we have

$$e = E_{\text{max.}} \sin \omega t,\ i = I_{\text{max.}} \sin(\omega t + \phi),$$
$$\text{and } w = E_{\text{max.}} I_{\text{max.}} \sin \omega t \sin(\omega t + \phi).$$

Now $\{\sin \omega t \sin(\omega t + \phi)\}$ may be expressed in the form

$$\{\tfrac{1}{2}\cos\phi - \tfrac{1}{2}\cos(2\omega t + \phi)\},$$
$$\therefore\ w = \tfrac{1}{2} E_{\text{max.}} I_{\text{max.}} \cos\phi - \tfrac{1}{2} E_{\text{max.}} I_{\text{max.}} \cos(2\omega t + \phi).$$

The power is thus seen to consist of a constant part, $\frac{1}{2} E_{\text{max.}} I_{\text{max.}} \cos\phi$, and a part which is a sine wave of double frequency. As we have already seen, the constant part is equal to the mean power,

$$\therefore\ W = \tfrac{1}{2} E_{\text{max.}} I_{\text{max.}} \cos\phi$$
$$= E\, I \cos\phi.$$

E I is the apparent power or volt-amperes in the circuit, and the factor $\cos\phi$ by which the *apparent power* has to be multiplied to give the *true power* is called the power factor.

$$\text{Thus } \cos\phi = \frac{\text{true power}}{\text{apparent power}}.$$

From the equation $W = E\, I \cos\phi$, it will be seen that when ϕ exceeds $\frac{\pi}{2}$ in either the positive or negative direction the mean power becomes negative, i.e. power is being taken from the circuit. This will be made clearer by an example. Suppose a single-phase alternator is connected to A.C. mains. As long as ϕ is less than $\frac{\pi}{2}$ the alternator will be giving power to the mains.

As soon as ϕ exceeds $\frac{\pi}{2}$ the alternator will take power from the mains, and will therefore be running as a synchronous motor.

Power in Polyphase Circuits.—In a two-phase circuit the total power is given by $W = W_1 + W_2$, where W_1 and W_2 are the powers in phases 1 and 2.

Since the P.D. E between either outer wire and the middle wire is equal to the corresponding phase E.M.F., and since the current I in an outer wire is equal to the current in the phase to which it is connected, the total power when the load is balanced is $W = 2\, E\, I \cos\phi$.

Similarly, in a three-phase circuit, $W = W_1 + W_2 + W_3$.

If the circuit is *star-connected*, the line P.D. is equal to $\sqrt{3}$ times the phase E.M.F., and the line current is equal to the phase current,

$$\text{i.e. } E = \sqrt{3}\, E_p \text{ and } I = I_p.$$

Therefore, for a balanced load,

$$W = 3\,E_p\,I_p \cos\phi$$
$$= \sqrt{3}\,E\,I\cos\phi.$$

If the circuit is *delta-connected*, the line P.D. is equal to the phase E.M.F., but the line current is equal to $\sqrt{3}$ times the phase current,

$$\text{i.e. } E = E_p \text{ and } I = \sqrt{3}\,I_p.$$
$$\text{As before, } W = 3\,E_p\,I_p\cos\phi$$
$$= \sqrt{3}\,E\,I\cos\phi.$$

In a two-phase system the power may be measured by a single wattmeter if the load is balanced. The current coil must be connected in series with one of the outer line wires, and the pressure coil connected between an outer and the middle line wire. The total power is then twice that indicated by the wattmeter. If the load is unbalanced, then two wattmeters connected, as shown in fig. 43, must be used, the total power being then the sum of the powers indicated by the two wattmeters.

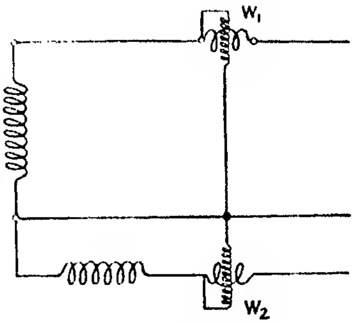


Fig. 43

In a three-phase system the total power is the sum of the powers in the three phases. The power in a three-phase star connection may be measured by a single wattmeter, if the load is balanced, by connecting the wattmeter as shown in fig. 44. If the neutral point is not available an artificial one may be made by connecting to a common point three large non-inductive resistances, the other ends of which are connected one to each line wire (see fig. 24).

The single wattmeter then measures $\frac{1}{3}$ of the total power. This method is not available if the connection is delta, since the current in any line wire is not equal to the phase current.

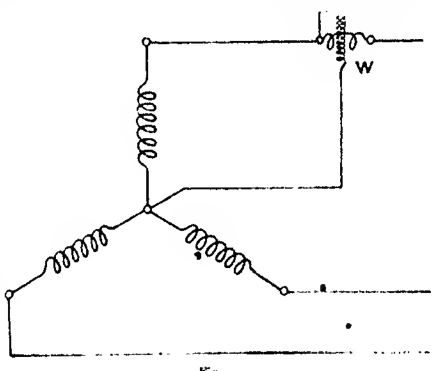


Fig. 44

Two-Wattmeter Method.—This method, which is equally applicable to star and to delta systems, and gives the true power whether the load is balanced or not, is very widely used for power measurements on three-phase circuits.

It has the additional advantage that the value of the power-factor is easily obtained from the ratio of the two wattmeter readings.

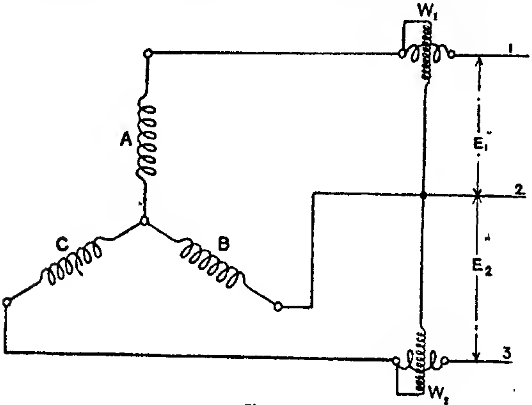


Fig. 45

The two wattmeters are connected to the circuit, as shown in fig. 45.

Let us consider a star-connected system, and let phase currents and E.M.F.s be indicated by symbols with the suffixes *a*, *b*, or *c*, while line currents and P.D.s are indicated by symbols with the suffixes 1, 2, or 3. Then considering instantaneous values, the instantaneous power read by the wattmeter W_1 is given by

$$w_1 = e_1 i_1 = (e_a - e_b) i_1.$$

Similarly, the instantaneous power read by the wattmeter W_2 is given by

$$w_2 = e_2 i_3 = (e_c - e_b) i_3,$$

$$\text{and } w_1 + w_2 = (e_a - e_b) i_1 + (e_c - e_b) i_3.$$

Since in a star system the line current is equal to the corresponding phase current,

$$i_1 = i_a \text{ and } i_3 = i_c,$$

$$\therefore w_1 + w_2 = e_a i_a + e_c i_c - e_b (i_a + i_c).$$

But the sum of the currents in the three phases is at any instant zero,

$$\therefore i_a + i_c = -i_b,$$

$$\therefore w_1 + w_2 = e_a i_a + e_b i_b + e_c i_c$$
$$= w_a + w_b + w_c$$
$$= w,$$

where w is the total power in the circuit at any instant.

The wattmeters read, of course, the mean power and not the instantaneous power, so that if W_1 and W_2 are the two wattmeter readings, then the total mean power W is given by

$$W = W_1 + W_2.$$

Since no assumption has been made as to the equality of I_A, I_B, and I_C, the above relation holds whether the load is balanced or not.

Since $w_1 = e_1 i_1$, W_1 is the *vector product* of E_1 and I_1. Similarly, W_2 is the *vector product* of E_2 and I_3.

From the vector diagram (fig. 46) it will be seen that the angle between E_1 and I_1 is $(30 + \phi)^\circ$, while that between E_2 and I_3 is $(30 - \phi)^\circ$,

$$\therefore\ \underset{\cdot}{E_1}\,\underset{\cdot}{I_1} = E_1 I_1 \cos(30 + \phi),$$

$$\text{and } \underset{\cdot}{E_2}\,\underset{\cdot}{I_2} = E_2 I_3 \cos(30 - \phi),$$

where ϕ is the angle of phase difference *in degrees* between the current and E.M.F. in any phase.

$$\text{We therefore have } W_1 = E_1 I_1 \cos(30 + \phi),$$

$$W_2 = E_2 I_3 \cos(30 - \phi),$$

and hence, if the phases are symmetrical and equally loaded $E_1 I_1 = E_2 I_3$,

$$\frac{W_2 - W_1}{W_2 + W_1} = \frac{E_1 I_1 \{\cos(30 - \phi) - \cos(30 + \phi)\}}{E_2 I_3 \{\cos(30 - \phi) + \cos(30 + \phi)\}}$$

$$= \frac{\sin 30 \sin \phi}{\cos 30 \cos(-\phi)}$$

$$= \frac{1}{\sqrt{3}} \tan \phi, \text{ since } \cos(-\phi) = \cos \phi.$$

$$\therefore\ \phi = \tan^{-1} \sqrt{3} \left(\frac{W_2 - W_1}{W_2 + W_1} \right).$$

This simple expression for the phase angle only holds when the phases are symmetrical and the load is balanced.

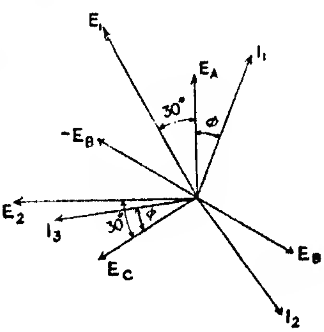


Fig. 46

When the two wattmeter readings are equal and in the same direction, i.e. $W_1 = W_2$, ϕ is clearly zero, and the power-factor is unity. This indication of unity power-factor is often very useful in experimental work, say, for example, in a synchronous motor test at unity power-factor.

When the power-factor is 0.5 (i.e. $\phi = 60$ or -60) one wattmeter reading is zero.

This serves as a useful indication of 0.5 power-factor.

The two-wattmeter method is also employed for energy measurements on three-phase systems. Either two separate supply meters are used and the algebraic sum of the records is taken, or a single meter having two elements, one connected in place of each separate meter, is used. Although the case of a star-connected system has been considered above, it can readily be shown in a similar way that the same results hold for a delta-connected system.

In experimental work it is often convenient to let one wattmeter do duty for both W_1 and W_2. This can be done by means of a switch which changes over the wattmeter from the one line wire to the other. The switch is provided with short-circuiting devices, so that the supply is not interrupted when the switch is being moved from the one position to the other.

The readings are necessarily taken successively, but usually this is no serious disadvantage. If the load is balanced the wattmeter current coil may be connected permanently in one line wire, the pressure coil being successively connected between this line wire and each of the others. As before, the algebraic sum of the wattmeter readings gives the total power, but in this case only when the phases are symmetrical and equally loaded.

CHAPTER VIII

WAVE FORM

It has been assumed throughout the foregoing chapters that the currents, P.D.s, &c., were of pure sine-wave form. In practice it very frequently happens that the wave contains harmonics, and departs considerably from the sine form. This is of some considerable practical importance, since the iron and copper losses in A.C. machines vary with the wave form, and the strain placed upon the insulation of a system depends very largely on the shape of the pressure wave. The general shape of the wave depends not only on which harmonics are present, but also on the phase positions of the harmonics relatively to one another and to the fundamental. As a simple example, a wave form consisting of a fundamental and a third harmonic is shown in figs. 47 and 48.

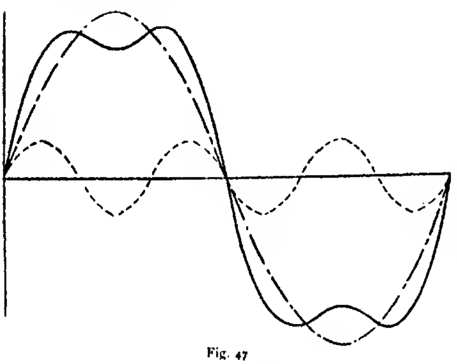


Fig. 47

In fig. 47 the phase position of the harmonic is similar to that of the fundamental, and the resultant wave is flat-topped. In fig. 48 the phase position of the harmonic has been shifted forward by $\frac{1}{6}$ of a period of the fundamental (i.e. its phase is opposite to what it was in fig. 47).

The resultant wave is now peaked.

In all practical machines the positive and negative halves of a wave are similar.

It may be shown mathematically that when this is the case only *odd* harmonics are present. This at once dismisses even harmonics, and in analysing a wave form we have only to deal with odd harmonics.

For the methods employed in wave analysis the reader is referred to a textbook of alternating-current theory.[1]

[1] For instance, La Cour and Bragstad, *Theory and Calculation of Electric Currents.*

Effective Value of a Wave containing Harmonics.—It may be shown that in all calculations involving irregular waves each harmonic contributes its effect quite independently, and that the effect of the complete irregular wave is the same as the joint effect of the component waves. Thus the heating effect of the irregular wave is equal to the sum of the heating effects of the fundamental and each harmonic. In calculating the effective value of the irregular wave we first obtain the effective value of the fundamental and of each harmonic present, and take the square root of the sum of their squares.

Form Factor.—The form factor has already been defined as the ratio $\frac{I}{I_{mean}} = \gamma$, and the mean value is determined by taking the sum of the mean values of the fundamental and the different harmonics present. The form factors corresponding to figs. 47 and 48 are 1.058 and 1.25 respectively. It will be seen that the more peaked the wave form is the greater the form factor becomes. In the limiting case of a flat-topped wave (i.e. a rectangular wave) the effective and mean values are obviously equal, and $\gamma = 1$.

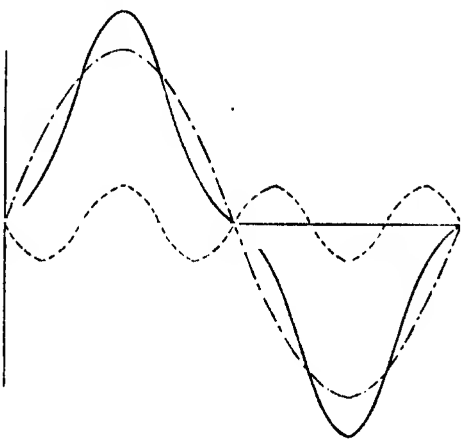


Fig. 48

Effect of Inductance and Capacity on the Wave Form.—When an irregular E.M.F. is applied to a circuit containing inductance or capacity the current is the resultant of the currents due to the fundamental and harmonics.

Let f equal the frequency of the fundamental, and suppose that the E.M.F. is applied to an inductive circuit of which the resistance is negligible.

$$\text{Then } I_1 = \frac{E_1}{2\pi f L},$$

$$I_3 = \frac{E_3}{6\pi f L},$$

$$I_n = \frac{E_n}{2n\pi f L},$$

where E_1, I_1 refer to the fundamental, E_3, I_3 to the 3rd harmonic, and so on.

Hence, if a_n be the amplitude of the nth harmonic in E.M.F. wave, its amplitude in the current wave will be proportional to $\frac{a_n}{n}$. The harmonics

of the E.M.F. wave are therefore reduced in the current wave, the reduction being greater the higher the harmonic.

An inductance therefore tends to make the current wave smooth.

A capacity has the opposite effect, for

$$I_1 = E_1\, 2\pi f C,$$
$$I_3 = E_3\, 6\pi f C,$$
$$I_n = E_n\, 2\pi n f C.$$

The amplitudes of the harmonics are therefore multiplied in proportion to their frequency, and the current wave is much more irregular than the E.M.F. wave.

Resonance.—With an irregular wave form each harmonic is capable

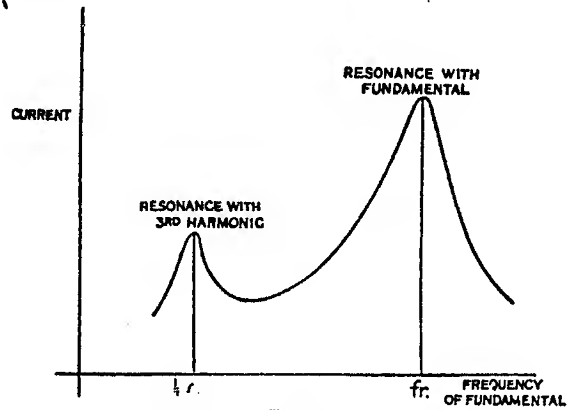


Fig. 49

of producing resonance. As an example, let f_r be the frequency at which the fundamental resonance occurs in a circuit to which an E.M.F. of irregular wave form is applied.

As long as the frequency of the fundamental exceeds f_r resonance cannot occur. If, however, the frequency of the fundamental is reduced to $\frac{f_r}{n}$, then the frequency of the nth harmonic becomes equal to f_r, and resonance with this harmonic takes place. A resonance curve for an E.M.F. wave containing a 3rd harmonic is shown in fig. 49.

It is therefore of some importance to examine the conditions for resonance *with the harmonics*, in an installation in which the E.M.F. wave form is irregular.

Effect of Wave Form on the Performance of Electrical Apparatus.—It has been found that any deviation of the wave from sine form in a circuit leads to an increase in the eddy currents both in the copper conductors of the circuit and in the iron linked with the circuit. In addition, the pressure drop for a given current is a minimum when the wave is of pure sine form. For these reasons, a sine wave gives

the highest efficiency in cables, transformers, and electrical machinery generally.

The stress put upon the insulation is less, however, with a flat-topped wave than with a sine wave, and the hysteresis losses are less with very peaked waves. In lighting circuits a flat-topped wave form is of some advantage, since the current remains for a longer period near its maximum value, and the frequency at which fluctuations of the light become visible is therefore lower.

With arc lamps some trouble has been caused by the loud humming produced with a peaked wave form.

A choking coil placed in series with the arcs suppresses the harmonics, and the difficulty may be got rid of in that way. The best wave form for an installation is the one that departs least from sine form.

At the present day, in the design of A.C. generators, every effort is made to secure as near an approach as possible to a sine-wave form. The deviation from sine form in modern machines rarely exceeds 5 per cent.

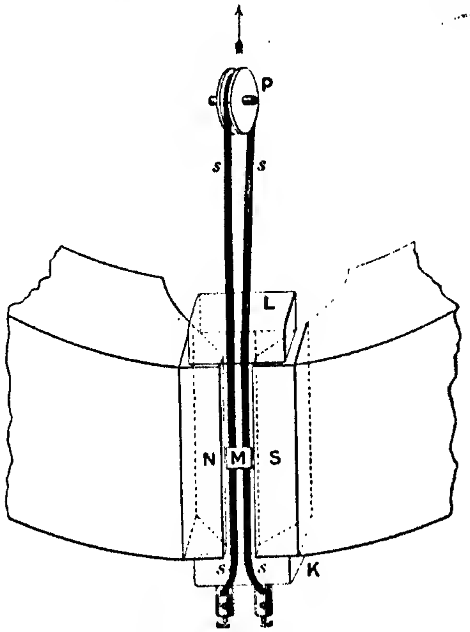


Fig 50.—Duddell Oscillograph[1]

The Delineation of Wave Form.—The wave form given by a generator may be determined by measuring the P.D. at the terminals of a condenser, which, by means of a rotating contact-maker on the shaft of the machine, is connected once every revolution across one pair of terminals of the machine. By shifting the contact-maker by small steps a set of readings may be obtained, from which a complete wave may be plotted.

A piece of apparatus which will give a continuous record of the wave form is, however, more generally useful. Such a piece of apparatus is called an oscillograph, and several types are in use. An oscillograph is often of special value from its ability to give a record of transient conditions, such as those occurring when transformers are switched in or out, &c.

Duddell Oscillograph.—This instrument, which was originally suggested by Professor Blondel, is largely used in this country. It consists

[1] Figs. 50 and 51 are taken from the *Journal of the Institute of Electrical Engineers*, by permission.

essentially of two strips of phosphor-bronze *s s* (fig. 50), which are stretched between the poles of a powerful magnet N S, by means of a spring-balance attached to a pulley P. The circuit is so arranged that the alternating current passes up one strip and down the other.

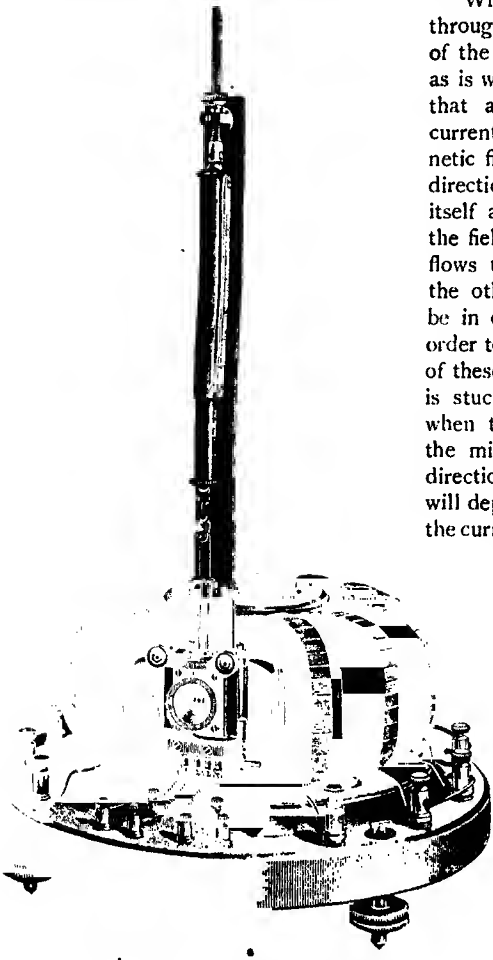


Fig. 51.—Duddell Oscillograph, general view

When any current is sent through this arrangement both of the strips will be deflected, as is well known from the fact that a conductor carrying a current, when placed in a magnetic field, tends to move in a direction perpendicular both to itself and to the direction of the field, but, since the current flows up one strip and down the other, the deflections will be in opposite directions. In order to indicate the deflection of these strips a little mirror M is stuck on to them, so that when the strips are deflected the mirror will turn, and the direction in which it will turn will depend on the direction of the current flowing in the strips.

Such, in its barest outline, is the instrument known as the Duddell Oscillograph. A general view is shown in fig. 51. The conditions that it has to fulfil, in order that it may serve as an *accurate* recorder of the current, require that a special construction of the parts should be adopted. In the first place, the moving parts must be made of such a size that they will be able to follow the rapidly changing forces that are made to act upon them. If the strips are to follow very rapidly changing forces they must be able to move with great rapidity, or, in other words, their "free periodic time" must be very small. In an actual oscillograph, such as we are considering, the strips, when deflected, will complete a back-and-forth movement

in one three-thousandth of a second, and with this instrument it is easy to record quite accurately the wave form of an alternating current with a frequency of 100.

It is not sufficient, however, that the moving parts be small and that their free periodic time be small, since, even with these conditions perfectly fulfilled, the instrument will still not record accurately unless the movement be "damped".

Supposing that we have an instrument in which these conditions are fulfilled, the difficulty that now arises is that, although we know that the instrument is being deflected by a varying amount, it is impossible to distinguish the various deflections; when the alternating current is sent through the strips all that is seen is a band of light reflected from the mirror on the screen. In order to actually see the wave some means must be devised of spreading the curve out; if the deflection be horizontal, the spot of light must be moved vertically so that it is not in the same vertical position for every deflection. This is effected in a variety of ways, the simplest of which is by the use of an oscillating mirror. The spot of light, when it has been deflected, is not viewed directly, but its reflection in a rapidly turning mirror is observed. In the case above taken, in which the motion of the spot is horizontal, the mirror is turned round a horizontal axis, so that an up or down vertical motion is given to the image of the spot, which is projected on to a screen and traces out the wave form.

The mirror is usually actuated by a cam driven by a small synchronous motor running off the supply of which the wave form is being taken. A shutter is connected to the shaft of the motor, and cuts off the light until the mirror has returned to its original position.

In recording transient phenomena a photographic method is employed, the light from the filament mirrors being projected on to a rapidly moving photographic plate or cinematograph film.

Irwin Hot-wire Oscillograph.—This instrument depends for its action on the unequal expansion of two wires which carry respectively the sum and the difference of two currents, one a steady polarizing current and the other an alternating current derived from the current or pressure of which the wave form is to be determined.

The two hot wires $C C_1 D_1 D$ and $E E_1 F_1 F$ (see fig. 52) are fixed at their lower ends, and are electrically connected at D E. They pass over an ivory pulley and cross diagonally, but are insulated from one another. Two horizontal wires (omitted in the figure for the sake of clearness) connect C_1 to D_1 and E_1 to F_1.

These wires cross each other without touching and permit of the free elongation of the vertical wires, but prevent their moving horizontally, and also practically limit the current in the vertical wires to the part below these cross wires. The wire between F_1 and the pulley is tied, by a loop of insulating thread, to the wire between D_1 and the pulley, and the other pair of wires are similarly tied.

The wires are kept taut by the tension of a spiral spring by which the pulley is suspended. A mirror is fixed across the wires near the points, where they are tied together, and when the wire $E E_1 F_1 F$ is heated more

than the other the mirror makes an angular movement in the direction shown by the arrows. The motion of the wires is damped by the immersion of the element (up to the level indicated by the dotted line) in an oil bath.

The working principle is briefly as follows:—In fig. 53, which shows

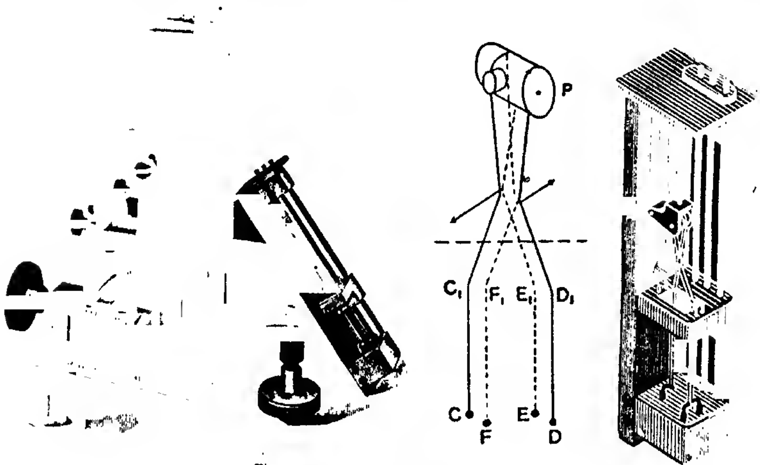


Fig. 52.—Irwin Hot-wire Oscillograph

the connections of the pressure element, CD and EF are the hot wires. If the resistance of CD is equal to that of EF, equal r say, then a steady current, I_s say, will flow in each wire.

If now an alternating current of which i is the instantaneous value be sent through both wires in series, then the rate of heating at any instant will be in the one wire $(i + I_s)^2 r$, and in the other $(i - I_s)^2 r$.

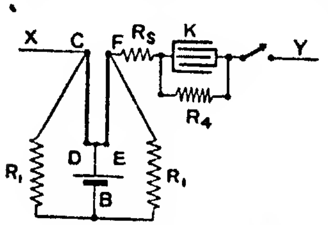


Fig. 53.—Connections of Pressure Element

The deflection of the mirror at any instant is proportional to the difference of the rate of heating in the two wires, i.e. to $4 i I_s r$. Since I_s and r are constant, the deflection is proportional to the instantaneous value of the current. Owing to thermal lag the deflection lags slightly behind the current. This is compensated for in the case of the pressure element by connecting in series with the wires a shunted condenser K R_4, which causes the current in the hot wires to lead the P.D. between X and Y by an amount equal to the lag of the deflection behind the current i. In the case of the current element the correction is made by shunting the wires by a resistance R and connecting an inductance S in series (see fig. 54). The time constant $\frac{L}{R}$ of S and R must, if exactly similar

wires are used for the pressure and current elements, be equal to C R_4, the product of the capacity of K in farads and the resistance of R_4 in ohms.

The viewing arrangements are very similar to those employed with the Duddell Oscillograph. The synchronous motor is of special design, and can be made to run steadily at sub-synchronous speeds. This is of advantage when slow periodic variations have to be detected, since a number of complete waves instead of a single one are seen on the screen. For the tracing of transient phenomena the motor is stopped, and the record is obtained photographically upon a rapidly moving plate or cinematograph film.

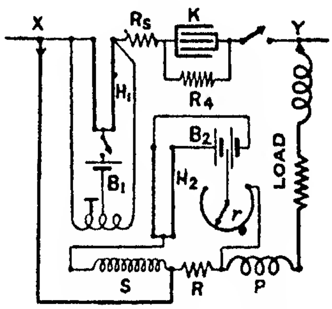


Fig. 54.—Connections of Current and Pressure Elements

Cathode Ray Oscillograph.—This is a type of oscillograph occasionally met with, although in this country its use is not so general as that of the types just described. It depends for its action on the deflection of a pencil of cathode rays by a magnetic field proportional to the current or P.D. of which the wave form is being determined.

CHAPTER IX

HIGH-FREQUENCY MEASUREMENTS

The low-frequency currents which have been dealt with in the preceding chapters have in practice frequencies varying from, say, 25 to 100 cycles per second. When the frequency is of the order of 1000 or over, the currents are said to be high-frequency currents or oscillatory currents.

The latter name is, however, generally reserved for currents whose frequency is very high, say, a million or over. It must be understood that these terms are relative only, and that there is no hard-and-fast distinction between them.

It is found that the constants of an electric circuit are not the same for low and for high frequencies. High-frequency measurements form a class of their own, and the methods employed differ from those suitable for low-frequency measurements. Up till the present time high-frequency currents have only been practically employed in radio telegraphy and telephony, although recently their use for power transmission has been suggested. The rapid advance of radio telegraphy and telephony during recent years has brought the subject of high-frequency measurements into great prominence, and it is proposed in this chapter to describe briefly a few of the methods commonly employed.

Measurement of High-frequency Current.—The excessive reactance

of A.C. ammeters having *coils* through which the current passes precludes their use for the measurement of high-frequency currents, since their introduction entirely alters the conditions existing in the circuit. For this reason the greater number of current-measuring instruments employed in high-frequency measurements are of the hot-wire type. For fairly large currents, such as the current in a transmitting aerial, an ammeter of the ordinary stretched-wire form already described[1] may be employed, but large errors are possible. For small currents the electrical thermometer, invented by Snow Harris and shown in fig. 55, is more suitable. The bulbs A and B are connected by a U-tube, the lower part of which is filled with liquid. The bulb A contains a fine wire (or a number of fine wires) through which the current to be measured is passed. The heat generated in the fine wires causes the air in the bulb A to expand, thus forcing the column of liquid up the limb of the tube connected to B.

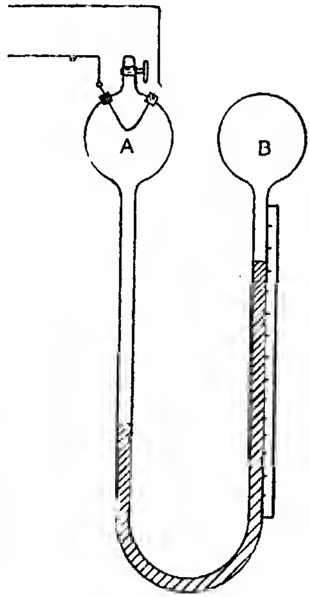


Fig. 55

The bulb B is added to prevent errors due to change in the temperature of the surrounding air.

In another class of hot-wire instruments a thermo-junction is placed near or in contact with the fine wire, the temperature of which is recorded by a sensitive galvanometer or milliammeter connected to the thermo-junction. The recording instrument may be calibrated to read currents directly by passing a series of known steady currents through the hot wire. It has recently been found by Messrs. Campbell and Dye[2] that air-core and iron-core transformers suitably designed give accurate results for currents of from 1 to 50 amperes at frequencies from 50,000 to 2,000,000. The transformers are used in conjunction with a thermo-couple and milliammeter.

The Einthoven String Galvanometer shown in fig. 56 is an instrument of entirely different principle, and is exceedingly sensitive.

A fine metallic filament or "string", as shown in fig. 57, is placed in an intense magnetic field.

The filament carries the current to be measured, and is deflected to and fro as the current alternates. The natural period of the string is adjustable by altering the tension, and may be made as small as $\frac{1}{1000}$ second. The material of the string varies according to the purpose for which the instrument is to be used, and is either a fine copper or silver filament, or in certain cases a silvered-glass filament. The deflection of the filament is

[1] See Chapter V, D.C. Measurements.
[2] Paper read before the Royal Society, March, 1915.

observed either by a microscope or from an image of the filament projected on to a screen.

A record of the movement of the filament may also be taken photographically. The filament is incapable of following the alternations of

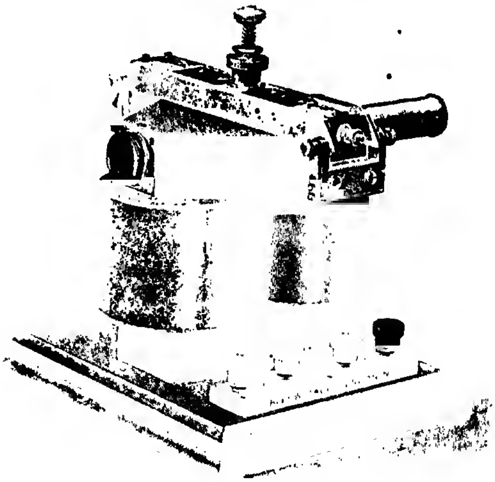


Fig. 56.—Einthoven Galvanometer

currents of the frequencies of those employed in radio-telegraphy, but it has been largely used for measuring the strength of received signals through the medium of the uni-directional currents in the detector circuit.

From its great sensitiveness it is admirably suited for such a purpose.

Measurement of Inductance and Capacity.—When a steady current is passed through a homogeneous conductor, the current is uniformly distributed over the cross-section of the conductor. If an alternating current be employed, it may be shown that the current distribution is no longer uniform, but that the current density in successive shells increases as the shells are nearer the outer surface and farther away from the axis of the conductor. Although at low frequencies this effect is not strongly marked, at very high frequencies the current is almost entirely confined to a very thin shell at the outer surface of the conductor.

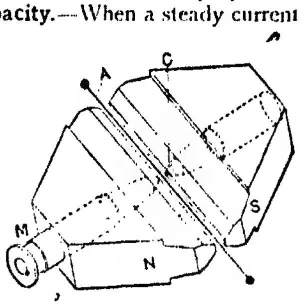


Fig. 57.—Magnets and Filament of Einthoven Galvanometer

This phenomenon is known as the *skin effect*. As a result of this effect the inductance of a circuit is *less at high frequencies* than at low ones. For this reason it is generally advisable to make inductance measurements at approximately the normal frequency employed in the circuit.

This remark also applies to capacity measurements. At the high

potentials and frequencies employed in radio-telegraphy a glow discharge is liable to occur, which *increases the capacity*.

In many cases inductance and capacity measurements may most conveniently be made at relatively low frequencies of, say, a hundred or so cycles per second. Very many methods of this kind have been employed, although a number of them are chiefly of academic interest.

Others, such as the Anderson Method for the measurement of inductances, the De Sauty Method for the comparison of capacities, and the Fleming Method for the absolute measurement of capacity, are in practical use, and will be found described in many textbooks and electrical laboratory handbooks.[1] Our attention will, however, here be confined to the making of measurements at frequencies of the order of those employed in radio-telegraphy.

The Fleming Cymometer.—This well-known instrument provides a simple and accurate means of measuring, not only high-frequency inductance and capacity, but also wave-length and frequency. The general arrangement is shown diagrammatically in fig. 58.

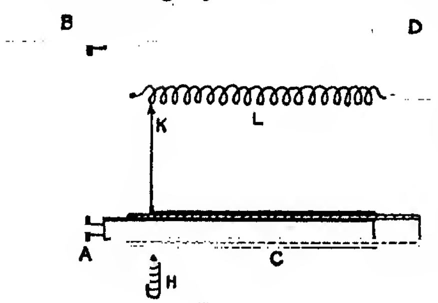


Fig. 58

The instrument in itself consists of a closed circuit containing a standard inductance L and a standard capacity C, both continuously variable. The capacity consists of several tubular condensers connected in parallel. Each condenser consists of an outer tube sliding over an ebonite tube, which forms the dielectric and separates the movable outer tube from the fixed inner tube. The handle H which moves the outer tubes also moves a sliding contact K over the standard inductance (which consists of a helix of stout copper wire mounted on an insulating tube), so that the inductance and capacity are simultaneously varied. The connection between the capacity and inductance consists of a rectangular copper bar, the section A B of which is detachable. The instrument has a natural electrical frequency which is given by

$$f = \frac{1}{2\pi\sqrt{CL}},$$

where C = capacity in *farads*,

L = inductance in *henrys*.

By varying C and L the natural frequency may be varied in the usual type of instrument over a range in which the highest frequency is about 12 times the lowest. If, now, a straight conductor, forming part of an oscillatory circuit in which a high-frequency current is flowing, be placed

[1] See Dr. Fleming's *Handbook of the Electrical Laboratory and Testing-room.*

parallel and fairly near to the straight part B D of the copper connecting bar of the cymometer, a high-frequency current will be produced in the instrument, which will have a maximum value when the natural frequency of the cymometer has been made equal to the frequency of the current in the oscillatory circuit. The indicator ordinarily used is a neon or carbon dioxide vacuum tube connected across the terminals of the tubular condenser. The tube glows with maximum brilliancy when the natural frequency of the cymometer has been adjusted to equal the frequency of the oscillatory circuit. When quantitative results are required the bar A B is replaced by a connector containing a hot wire with a thermo-couple connected to a sensitive galvanometer, from the reading of which the effective value of the current in the cymometer is obtained.

Measurement of Capacity.—In order to measure a capacity C_x by means of the cymometer this capacity is connected in series with a standard inductance L_s and a spark-gap G (see fig. 59). This forms an oscillatory circuit in which high-frequency currents are produced by means of an induction coil J which causes oscillatory discharges to take place. The cymometer handle is moved until the glow in the vacuum tube is a maximum. The value of the *oscillation constant* is then read off on the scale over which a pointer attached to the sliding contact K moves. The oscillation constant is $\sqrt{C\,L}$, and for convenience the scale gives the value of this when C is expressed in microfarads and L in centimetres (*absolute* electromagnetic units).

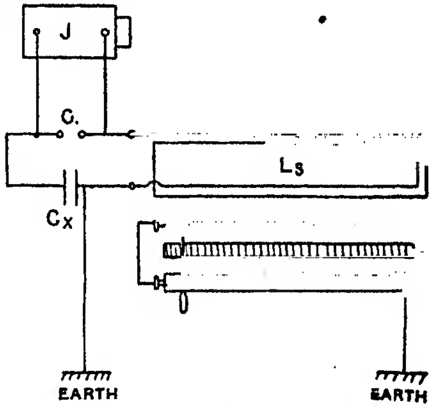


Fig. 59

When the cymometer and oscillatory circuit are in resonance their oscillation constants are equal.

$$\therefore \sqrt{C\,L} = \sqrt{C_x L_s}$$

where C_x = unknown capacity in *microfarads*,

L_s = standard inductance in *centimetres*.

Calling the value of $\sqrt{C\,L}$, as read off from the cymometer scale, O_c,

$$C_x = \frac{O_c^2}{L_s} \text{ microfarads.}$$

A suitable variable standard inductance, of which the values are given in centimetres, is supplied with the cymometer.

Measurement of Inductance.—The measurement of an unknown

inductance is carried out in a similar way. A capacity is first carefully measured as described above. The standard inductance is then replaced by the inductance to be measured. The value of the oscillation constant at resonance is then determined, and as before—

$$\sqrt{CL} = \sqrt{C_s L_x},$$

where L_x is the unknown inductance, and C_s the capacity just measured and used as a standard.

$$\therefore L_x = \frac{O_c^2}{C_s} \text{ centimetres.}$$

A very small inductance is best measured as the difference of two inductances. In this case the inductance to be measured is connected in series with the standard inductance.

If O_c is then the oscillation constant for resonance,

$$L_s + L_r = \frac{O_c^2}{C_s},$$

$$\text{and } L_x = \frac{O_c^2}{C_s} - L_r.$$

Measurement of Frequency.—The frequency of an oscillatory current flowing in a circuit may be directly determined by the use of the cymometer. The straight copper bar of the cymometer is placed parallel and fairly near to a straight conductor of the circuit in which the oscillatory current is flowing. It may here be remarked that it is very important, in making *any* measurement with the cymometer, to place it as far from the oscillatory circuit as is consistent with a clear indication.

A close coupling between the cymometer and the oscillatory circuit allows the circuits to react upon one another in such a way that currents of two different frequencies are superimposed upon one another in *both circuits*, neither frequency being equal to the natural frequency of either circuit. In order to avoid this the coupling must be made as loose as possible. With this precaution the cymometer handle is moved until resonance occurs, and the value of the frequency is read off directly on the frequency scale of the cymometer.

Another scale, which gives directly the wave lengths corresponding to the various positions of the cymometer handle, is also provided. This allows of the direct measurement of the wave length of the electromagnetic waves radiated by an aerial.

Several other instruments based on the same principle are in common use.

They differ in the use of a detector and telephone, or else a hot-wire ammeter, as an indicator of resonance. The Marconi Wavemeter and the Dönitz Wavemeter are well-known examples of this class, and will be found described in most textbooks of Wireless Telegraphy.

Measurement of High-frequency Resistance.—Owing to the skin effect already mentioned, the resistance of a conductor is *greater* for high-

frequency currents than for low-frequency ones. In effect, the available cross-section of the conductor is diminished owing to the current being practically confined to a thin surface shell when the frequency is high. For a given straight circular-section wire the high-frequency resistance may be calculated by the use of the following formulæ, which are due to Lord Rayleigh:—

If $\frac{R'}{R}$ = ratio $\frac{\text{high-frequency resistance}}{\text{steady-current resistance}}$,

c = circumference of the wire in *centimetres*,

f = frequency of the current,

ρ = steady current resistivity of the material of the wire expressed in *absolute C.G.S. units*,

$k = \frac{fc^2}{\rho}$ = a constant.

Then, if k is less than unity,

$$\frac{R'}{R} = 1 + \frac{k^2}{48} - \frac{k^4}{2880},$$

and if k is very much greater than unity, say, at least equal to 6,

$$\frac{R'}{R} = \tfrac{1}{2}\sqrt{k} + \tfrac{1}{4}.$$

It will be seen from these formulæ that if the circumference c of the wire is very small, the high-frequency resistance differs very little from the steady-current resistance.

Hence, if the conductors employed in high-frequency circuits be made of stranded cables, consisting of *very fine insulated* wires, the increase of resistance may be made negligible.

The measurement of a high-frequency resistance by a direct method is seldom practicable, since this involves the enclosing of the resistance in the air bulb of an electric thermometer. Several indirect methods for the determination of the high-frequency resistance of an aerial are employed, and depend upon the measurement of the decrement of the aerial. For a description of the apparatus and methods employed, the reader is referred to a textbook of Wireless Telegraphy.

3. Continuous-current Generators

CHAPTER I

GENERAL PRINCIPLES

In the following article an attempt has been made to present the fundamental principles of the continuous-current generator, and also to demonstrate, in an elementary manner, the chief guiding rules in the design of the various types of these machines. While attempting to preserve the excellences of the old article, its sphere of usefulness, it is hoped, is extended by the introduction of the more elementary concepts, and also by the introduction of matter other than design. The writer realizes to the full the great difficulty of treating such a subject in the short space at his disposal, and also in a manner to appeal to the class of readers for whom it is intended.

Use has been made of the standard treatises on the subject, and also of the I.E.E. proceedings, &c. The writer is indebted to these sources for his information. Much progress has been made in the design of direct-current generators within the last decade, and the exacting requirements of modern specifications have rendered radical changes necessary and imperative.

The Function of the Dynamo.—The dynamo is essentially a generator of electric pressure, and its function is to maintain this pressure when current flows.

The principle of electromagnetic induction, discovered by Faraday, states that, when a conductor is moved across a magnetic field so as to cut the lines, an electromotive force is induced in the conductor. If the conductor forms part of a closed circuit, current will flow from the higher potential to the lower. The magnitude of this electromotive force will depend on the rate of cutting of the lines of force. The absolute unit of E.M.F. is that produced in a conductor when one line of force is cut per second. The practical unit, called the volt, is 10^8 times the absolute unit.

The direction of the E.M.F. induced in the conductor depends on the direction of motion across the field of force. It may be found by the following rule: Place the forefinger of the right hand along the direction of the lines of magnetic force. Extend the thumb at right angles to the fingers, and place the hand so that the thumb points in the direction of the motion. The middle finger, bent so as to be at right angles to the palm of the hand, then points in the direction of the induced E.M.F.

A rectangular loop of wire revolving in a magnetic field about an axis at right angles to the lines of force is shown in fig. 1.

The ends of the loop are connected to two metal rings, and a circuit R is connected to the two rings. When the armature is rotated uniformly in a counter-clockwise direction the sides of the loop have an E.M.F. induced in the directions shown by the arrows. The two inductors are moving in the field in opposite directions, and hence the directions of the E.M.F. will be opposite. The conductors are connected at the ends, so that these E.M.F.s are additive, and therefore the total E.M.F. will be twice that which is induced in a single inductor. It will be seen that when the plane of the loop is in the same direction as the lines of the field, the rate of cutting is a maximum, and also when the loop is at right angles to the field the E.M.F. is zero. When the conductors have rotated through 180° the direction of the E.M.F. in the conductors is reversed, and also the direction of the current in the external circuit. The E.M.F. is an alternating one, and the current it produces is likewise alternating in value and direction. For many applications a current constant in value and uniform in direction is required, and we must now enquire how this result is to be accomplished. In the first place, we shall consider how constancy of direction can be attained. This is effected by connecting the ends of the loop to a split metal ring, the two halves being insulated from each other. By placing the brushes, which are of sufficient width to span the insulation between the segments, in such a position that they pass from one segment to the other when the induced E.M.F. is zero, the current in the external circuit is uniform in direction, while still alternating in the loop itself. In effect, we have simply altered the connection of the loop to the external circuit at the instant when the induced E.M.F. changed in direction. That this is so will be seen from fig. 2.

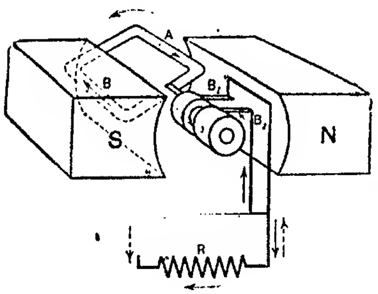


Fig. 1

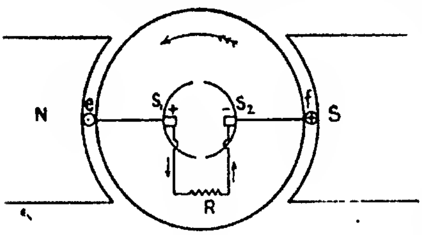


Fig. 2a

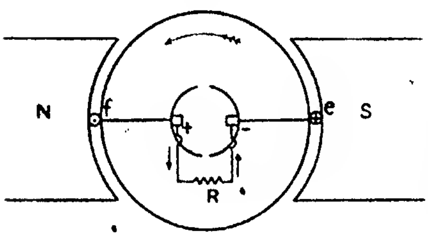


Fig. 2b

In fig. 2*a* the inductors *e* and *f* are under N. and S. poles respectively, and the current flows from segment S_1, through the positive brush and external circuit to segment S_2 to the inductor *f*. When the loop has rotated through 180° (fig. 2*b*) the current flows through *e* and *f* in the opposite direction, but the segments S_1 and S_2 have also moved through 180°, and the current will flow from S_2 to S_1 in the same direction around the external circuit as before. We have therefore rectified the current in the external circuit, but it still fluctuates in value from zero to a maximum as the loop moves from between the poles to a point midway under the pole. We shall now consider the means whereby constancy of E.M.F. and constancy of current in the external circuit can be obtained. Consider first a bipolar ring armature with a number of coils wound evenly around the periphery of the hollow cylinder, and let the end of each coil be joined to the beginning of the next through a commutator segment; then, provided the number of conductors in a single circuit are sufficiently great, the E.M.F. generated will be sensibly constant. This division of the commutator into as many segments as there are coils was invented by Pacinotti in 1860, but was little used till Gramme took it up. The brushes are placed on the neutral line of zero field, so that the short-circuiting of a coil or coils has little or no effect on the E.M.F. at the armature terminals. The instantaneous value of the total E.M.F. in a circuit consisting of several coils in series is equal to the sum of the instantaneous E.M.F.s induced in each coil. That this addition of E.M.F.s produces constancy will be readily seen by considering a bipolar ring armature with, first, one coil only per circuit.

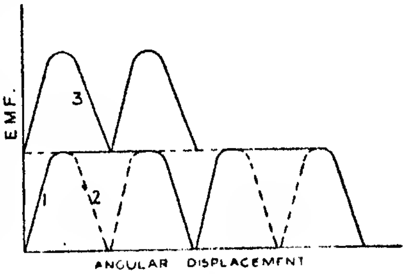


Fig. 3

The full-line curve, marked 1 in fig. 3, shows the variation of E.M.F. with angular displacement for one coil only. Now let another coil be wound 90° away from coil 1, and let it be joined in series with it. The instantaneous E.M.F. of coil 2 is shown by the dotted lines in fig. 3. The total E.M.F. at the brushes is obtained by adding the ordinates of the two curves, and this gives curve 3. It will be seen that the fluctuation in E.M.F. is reduced from 100 per cent from the mean value to about 30 per cent. It is quite clear that by adding more coils and connecting them in series, the end of one coil being connected to the beginning of the next through a commutator segment, the fluctuations can be readily reduced, and with about 30 coils per circuit an E.M.F. is generated the fluctuations of which can only be detected by the oscillograph or telephone. The commutator segments perform two functions. In the first place, they serve as the junction-points between successive coils, and, secondly, their true function is called out when the coil passes from under one pole to the next of opposite sign. The E.M.F. is then reversed, and the current likewise must

change sign. It is thus necessary to short-circuit the coil midway between the poles in order to cause the current to die away and reverse its direction. This is done by the action of the brush bridging over two or more segments.

There are two distinct types of windings, "Ring" and "Drum". In the "Ring" type, which is now practically obsolete, the winding consisted of a number of spirals wound on a hollow cylinder, the beginning and end of each coil terminating in a commutator segment. Its great advantage was that the voltage between adjacent wires, being but a small fraction of the terminal voltage, the insulation of the wires presented little difficulty. It had many disadvantages, however, chief among which was the large amount of inactive winding. The armature resistance and the weight of wire are therefore relatively large. In practice, drum windings are used, and we will now proceed to explain the rules for the connection of the ends. In the first place we will consider the ordinary bipolar drum type having ten coils. The periphery is divided into 20 equal parts, and from segment 1 we proceed to the front end of wire 1, and from the far end of wire 1 to the far end of wire 10, and then from the near end of wire 10 to segment 2, and from segment 2 to wire 3, and so on. The forward pitch in terms of the elements or winding spaces must be an odd number, and the back pitch must also be odd and must differ by two from the forward pitch. In our case, if Z = number of elements and p = pairs of poles, then the back pitch

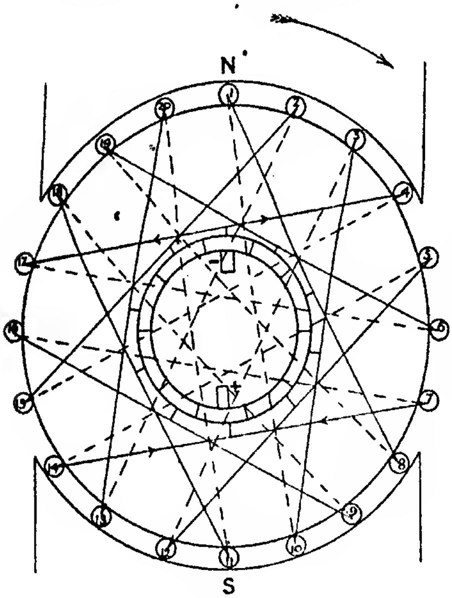


Fig. 4

$$y_b = \frac{Z \pm 2}{2p},$$

$$y_f = y_b - 2.$$

$$\text{In our case } y_b = \frac{20-2}{2} = 9,$$

$$\text{and } y_f = 7.$$

In formulating rules for the guidance of the armature winder the order of numbering of the elements is in the direction of the backward pitch at

the back end of the armature. In the case of drum windings each coil side, whether consisting of only one wire or of several wires, is regarded as a winding element. The "hand" of the armature winding is determined by the order in which successive connections to the commutator are made. Starting from a commutator sector and following the coil side nearest the iron core along till we reach the next junction to the commutator, then if this falls to the right of our starting-point the winding is right-handed, and if to the left then left-handed. The order of progression round the commutator fixes the hand of the winding.

It will be noticed in our previous example that with clockwise rotation the currents in the front connector of elements 14 and 7 are in opposite directions, and so the brush must be placed at the segment to which 14 and 7 are connected. The current enters at the negative brush and flows through two parallel paths to the positive brush. At the moment shown, the paths through the armature are

$$-\left|\begin{matrix} 17 & 6 & 19 & 8 & 1 & 10 & 3 & 12 & 5 & 14 \\ 4 & 15 & 2 & 13 & 20 & 11 & 18 & 9 & 16 & 7 \end{matrix}\right| +$$

Windings may be divided into two divisions, viz. "Lap" and "Wave". Wave windings are employed in machines of low output and are also well adapted for high-speed machines of intermediate output. The important characteristic of the simplex wave winding is that there are only two circuits through the armature, the number of circuits being entirely independent of the number of poles. Their use has been greatly extended with the introduction of interpoles, since a larger current per circuit can then be allowed. The absence of circulating currents also in such windings renders their adoption desirable.

Multiple-circuit or "lap" windings are used for large outputs, and in the simpler form have as many circuits as there are poles. The pitch of a winding is the difference between the numbers of the elements which are joined together. The rear and forward pitches must be odd numbers, and must be such that the winding closes on itself after taking in every element as we proceed from the starting-point till the original conductor is reached.

We will consider first Multiple Circuit Lap Windings. These may be either long pitch or short pitch. Here the front and rear pitches must be odd, and differ by two. If the rear pitch is less than the pole pitch, then we get the chord winding of Swinburne, and the conductors between the coil sides which are short-circuited at adjacent brushes carry currents in opposite directions, so that their demagnetizing effect is neutralized. A moderate degree of chord winding is beneficial, since in the case of toothed armatures the short-circuited coils fall in different slots. Their inductance is thereby decreased, and their commutating properties are improved. They are used largely on crane motors and on machines without interpoles.

The mean pitch which equals $\frac{y_b + y_f}{2}$ is an even number, and is made

equal to $\frac{Z}{2p}$, where Z = total number of winding elements. The number o inductors must be so chosen that this condition is fulfilled. Fig. 5 represents an ordinary lap winding. The radial lines represent inductors in the slots, and the connecting lines inside represent the end connections to the commutator, and those outside, the end connections at the back of the armature. The thick black line traces out one coil or section of the armature between two commutator segments. The diagram shows forty-eight inductors and twenty-four commutator bars. An inductor at the front end is connected to the 9th ahead of it, and at the back to the 7th behind it. The conductors will generally be arranged in two layers in a slot, so that the conductors per slot must be a multiple of two. It is convenient to number elements in the top layer odd and those in the bottom even.

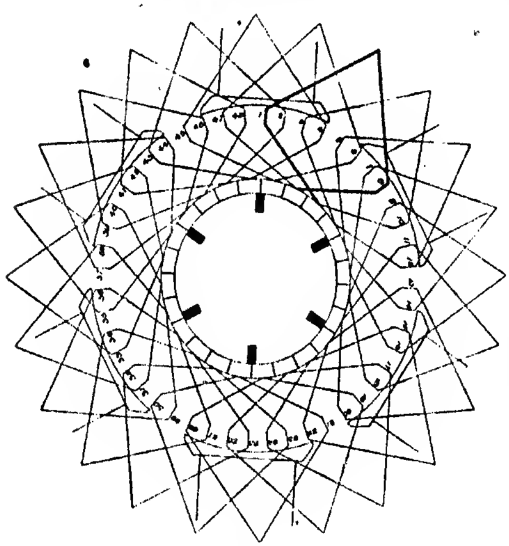


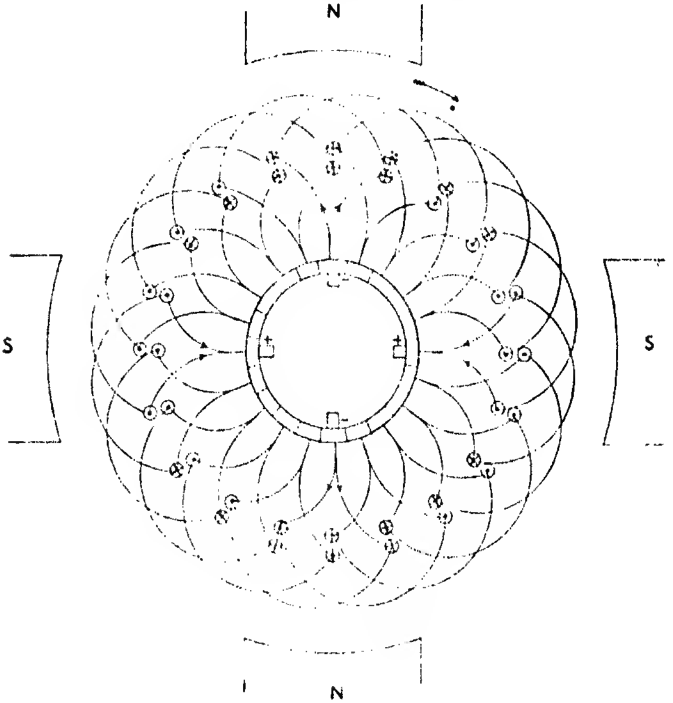
Fig. 5

Assuming any direction of rotation and polarity for the poles, it is readily seen that there are as many circuits in the armature as there are poles and also as many collecting-points or brushes. Brushes of positive polarity are joined to form one common terminal, and similarly with those of negative polarity. Generally as many brushes as there are poles are used, in order to shorten the length of the commutator, but by connecting segments which are $\frac{360°}{p}$ apart, only two sets of brushes are necessary, with, of course, a longer commutator.

Fig. 6 shows a short chord winding with twenty slots and one coil per slot. The slot pitch taken is three forward and two back. The direction of the currents in the bars is indicated by crosses (away from observer) and dots (towards the observer). This exaggerated case shows how the bars in the interpolar zone carry currents in opposite directions. With this arrangement, however, the E.M.F. is reduced, due to the differential action which is thus set up. Starting from the negative brush to trace through the winding, it is clear that there are four paths to the positive brush. It is clear from the diagram that in tracing out any two consecutive winding

units the second unit overlaps the first. Hence the name lap winding. Its characteristic feature is that in passing from one winding unit to the next we come to an element within the embrace of the two elements forming the first winding element.

Owing to the fact that circulating currents are nearly always present in lap windings, they are fitted with equalizing connections. In order to obtain points at the same potential for the addition of equalizing rings



it is essential that the number of slots be divisible by the pairs of poles. This is seldom the case in practice, for the number of slots are so chosen as to suit the use of wave windings, which for an even number of pairs of poles must have an odd number of slots. In this case "stepped windings" are used.

Multiplex Lap Windings.—Armature windings may be simplex, duplex, triplex, &c., i.e. they may be wound with one, two, three, &c., independent windings. In the case of the duplex winding there are two independent windings, each with its segments interleaved between those of the other. Two distinct commutators might be provided, one at either end of the armature, to which each winding might be connected to its

separate commutator. The use of such windings is somewhat restricted, and is only called for when very large currents have to be collected.

Let a = pairs of circuits,
and p = pairs of poles.
Z = number of coil sides or elements;

$$\text{then the rear pitch } y_r = \frac{Z \mp b}{2p},$$

$$\text{and the forward pitch } y_f = -\left(\frac{Z \mp b}{2p} - \frac{2a}{p}\right),$$

$$\text{and the resultant pitch } y = y_r + y_f = \frac{2a}{p}.$$

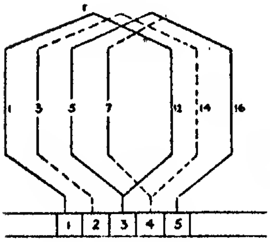


Fig. 7

We will illustrate this with a duplex lap winding having 6 poles and 60 inductors. The rear pitch in slots, assuming full pitch winding and one coil per slot, is 5, and in elements is 11. The back pitch $y_f = y_r - 4 = 7$, so that we go from element 1 to element 12, and then back to element 5. From element 12 we go to the third segment from our starting-point.

Fig. 7 shows the development of a few coils. The general march of the winding is sufficiently evident from the diagram, and we have

1—12	24	36	48	60	1st winding.
5—16	17—28	29—40	41—52	53—4	
9—20	21—32	33—44	45—56	57—8	
13—24	25—36	37—48	49—60	1	
3—14	26	38	50	2	2nd winding.
7—18	19—30	31—42	43—54	55—6	
11—22	23—34	35—46	47—58	59—10	
15—26	27—38	39—50	51—2	3	

The brushes must overlap at least 2.5 commutator segments in order to collect current from the two windings simultaneously.

In the winding shown above, there are 12 circuits in parallel from the negative to the positive brushes. In a multipolar dynamo $\frac{Z}{2p}$ is not necessarily a whole number, and thus the addition or subtraction of b is necessary in the formulæ for the rear and forward pitches. It may be necessary to use $+b$, so that the rear pitch in slots may be a whole number. The use of $-b$ gives a degree of chord winding depending on its value. It will be noticed that there is a greater difference in the

two component pitches than in the simplex lap winding. Again, the winding may form one single closed helix, and yet there may be as many parallel paths through the armature as in the case of the two independent windings.

The condition for a duplex winding singly re-entrant is that the average pitch $\left(y_a = \frac{y_r + y_f}{2}\right)$ and the number of segments or coils $\left(\frac{Z}{2}\right)$ shall have no common factor but unity. If the common factor is 2, then we have a duplex winding doubly re-entrant. If 3, a triplex winding triply re-entrant, and so on. In every case the brush width must exceed

$$m - 1 \text{ sectors where } m = \frac{\text{circuits}}{2p}.$$

To secure the effective brush width the brushes may be staggered.

Multiplex windings singly re-entrant are preferable to multiplex windings multiply re-entrant, since each coil in the former passes successively through every parallel path during a revolution.

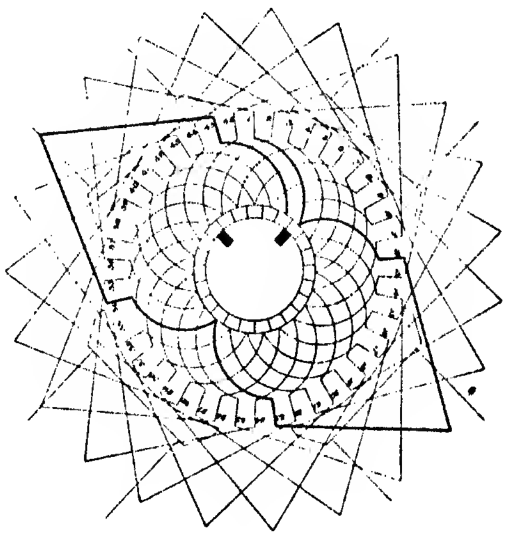


Fig. 8

We will now turn to Wave Windings. The characteristic of the simplex wave is that there are only two paths through the winding, no matter how many poles there may be. The average pitch here is $\frac{Z \pm 2}{2p}$. Both pitches must be *odd* numbers, and if the average pitch be odd, both the front and rear pitches may be equal to the average pitch. Should the average pitch be even, then the front pitch requires to be less or greater than the mean pitch by 1, according as the rear pitch is greater or less than the mean pitch by 1,

$$\text{i.e. } y_f = y_a \pm 1,$$
$$\text{and } y_r = y_a \mp 1.$$

Fig. 8 shows an ordinary simplex wave singly re-entrant winding. The thick-black line shows a section or coil between two commutator segments. A reference to the figure shows that the pitch at each end is continuously

forward, and not alternately forward and back. In the figure there are 46 elements.

$$\text{The mean pitch} = \frac{46 \pm 2}{4} = 11,$$

and the front and back pitches are equal to the mean pitch,

$$\text{i.e. } y_R = 11, \text{ and } y_f = 11.$$

Fig. 9 shows a 4-pole wave winding with 22 elements.

$$\text{The mean pitch} = \frac{22 \pm 2}{4} = 5,$$

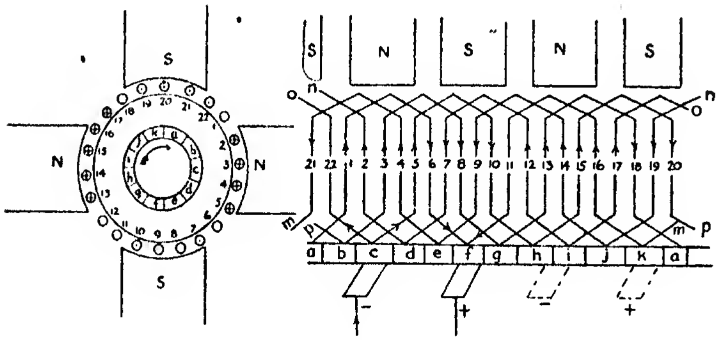


Fig. 9

and front and rear pitches are each 5. The two circuits from the negative to the positive brushes are—

	5	10	15	20	3	8	13	18	1	6	
−	22	17	12	7	2	19	14	9	4	21	+
							16	11			

It will be noticed that the winding is continuously forward, and is zigzag in character. Hence its name. The positions for the brushes can be ascertained by tracing the currents. A positive brush should be placed where two currents meet at a segment, and a negative brush where two arrowheads go in opposite directions through the winding. It is clear that only two sets of brushes are required so far as the collection of current is concerned. In order to reduce the length of commutator, however, it is usual to use as many brush sets as there are poles. This frequently leads to sparking troubles, due to the unequal division of currents at the brushes, owing to the difference of contact resistance, &c., and gives rise to what is called "selective commutation". The addition of extra brush sets in no way alters the distribution of current in the windings. With coil windings in two layers, for complete similarity of all coils, it is necessary that

the rear pitch in elements = rear pitch in slots × coil sides per slot + 1,

$$\text{i.e. } y_{rl} = \frac{y_r - 1}{e_s},$$

where y_n = rear pitch in slots,
y_r = rear pitch in elements,
e_s = elements per slot,

and the slot pitch should not greatly exceed $\frac{\text{slots}}{\text{poles}}$.

If all the slots in a toothed wave armature are to be filled, then Z must be a multiple of the number of slots,

$$\text{or } Z = e_s \times n_s,$$
$$\text{where } n_s = \text{number of slots},$$
$$\text{and } y \text{ average} = \frac{e_s \times n_s \pm 2}{2p}$$

must be a whole number. e_s is a multiple of 2, and it is clear that if y average is to be a whole number, the number of slots *must not* be divisible by the pairs of poles. With more than 2-coil sides per slot, in multipolar machines, only certain numbers of coil sides per slot will give a wave winding and at the same time fill all the slots. The following table shows the possible number of coil sides per slot to fill all the slots:—

Pairs of circuits 1.

Poles.			Coil sides per slot.					
4	...	...	1	2		6	—	10
6	...	...	1	2	4	—	8	10
8	...	...	1	2		6	—	10
10	...	...	1	2	4	6	8	—
12	...	...	1	2		—	—	10
14	...	...	1	2	4	6	8	10

In using wave windings on the smaller types of machines and on motors it will often be found convenient to use a number of slots which is not an even multiple of the commutator bars; as, for example, in an armature wound with 198 inductors.

99 commutator bars.
50 slots.

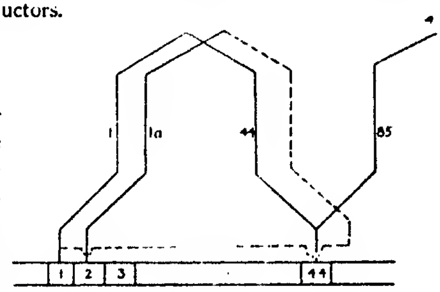


Fig. 10

In this case a dead section, or two dead inductors, may be used to fill up. This dead section is called a "dummy" coil. Sometimes, also, instead of a dummy coil, a "supplementary" coil is used. Thus, consider the case of a winding for a 4-pole machine having 43 slots and 86 coils with 2 turns per coil. We have y average $= \frac{172 \pm 2}{4} = 42\frac{1}{2}$, which is not a whole number. To make the winding suitable for a wave we must use 85 coils, giving $\frac{170 \pm 2}{4} = 42$ as the average pitch;

our rear and front pitches are now 43 and 41. The supplementary coil is connected as shown in fig. 10.

Again, duplex wave windings and triplex wave windings are used when a case arises in which a pattern already exists in which the number of poles is larger than the current per path calls for, and in which the bars and sectors become too numerous and small for cheap production with lap windings. It is possible to vary the number of circuits in such a case, the number always being a multiple of 2. For large current, low voltage, and high speed, multiplex wave windings are frequently used. The consideration which governs the choice of an armature winding is the value of the total current. In order to limit the reactance voltage the current per armature path without special commutating poles should not exceed 150–200 amperes. With interpoles the value is 300–400 amperes per path. The use of wave windings is limited in the case of high-speed, low-voltage machines by the number of sectors per pair of poles falling too low, the limit usually being 30 for constant E.M.F.

Equalizing Connections. — With a lap winding we have as many parallel circuits as there are poles. It is thus practically impossible to avoid pressure differences in the various circuits, due to unequal strength of the magnetic circuits, to eccentric boring of the poles, unequal quality of iron in each pole, brackets, and projections on the yoke, &c. Such pressure differences cause local currents of considerable magnitude to circulate through the winding and brushes. This may cause serious sparking troubles at the commutator, due to unequal loads on individual lines of brushes. In order to counteract this it is necessary with lap windings and also multiplex wave windings to use equalizing rings or cross connectors. These will be used to couple up points in the armature circuits which should be at the same potential. By their use the local currents are prevented from flowing through the brushes.

The following table represents standard practice in the use of equalizing rings:—

	Lap Windings.		Wave Windings. $a = 2$.	
No. of Poles.	No. of Rings.	No. of Connectors.	No. of Rings.	No. of Connectors.
4	3	—	—	—
6	5–8	15–24	8–12	16–24
8	8–12	32–48	16–20	32–40
10	12	60	24	48
12	12	72	30	60
14	12	84	30	60
16	12	96	30	60
18	12	108	30	60
20	12	120	30	60

Section of rings used, 20 millimetres × 4 millimetres for lap, and 20 millimetres × 2 millimetres for wave.

Equalizing Connections for Multiplex Wave Windings. — These tend to reduce the tendency towards selective commutation. The condition for symmetrical equalizing connections is that

$$\frac{\text{segments}}{\text{pairs of circuits}} = \text{a whole number.}$$

If $\frac{\text{pairs of poles}}{\text{pairs of circuits}}$ is a fraction, and $\frac{\text{segments}}{\text{pairs of circuits}}$ is not a whole number, the parallel branches do not contain equal numbers of wires, and the E.M.F.s are unequal, and points of equal potential are absent. Perfectly symmetrical points for connection by equalizers are obtained with

(*a*) $\frac{\text{poles}}{\text{circuits}}$ = a whole number when $\frac{\text{slots}}{\text{pairs of circuits}}$ = whole number.

(*b*) $\frac{\text{circuits}}{\text{poles}}$ = a whole number when $\frac{\text{slots}}{\text{pairs of poles}}$ = whole number.

If $\frac{\text{slots}}{\text{pairs of circuits}}$ is not a whole number, the dissymmetry is due only to the position of the bars in the slots, and can be removed by adding "2 *a*" dead bars (a = pairs of circuits). As an illustration of the use of equalizing rings, we will take an armature winding for a 6-pole machine with 141 slots, and 282 coils, each having a single turn. The slots should be divisible by the pairs of poles for perfectly symmetrical equalizing connections. There are 16 equalizing rings of section 20 millimetres × 1 millimetre. The machine has an effective output of 575 kilowatts at 600 volts and 735 r.p.m. (variable voltage generator). The winding goes from coil 1 to 96, 3 to 98, &c.

Ring.	I.	II.	III.	IV.	V.	VI.	VII.	VIII.
Coil.	1	7	13	19	25	31	37	43
Coil.	95	101	107	113	119	125	131	137
Coil.	189	195	201	207	213	219	225	231
Ring.	IX.	X.	XI.	XII.	XIII.	XIV.	XV.	XVI.
Coil.	48	54	60	66	72	78	84	90
Coil.	142	148	154	160	166	172	178	184
Coil.	236	242	248	254	260	266	272	278

As another example, we will take a double wave for a 12-pole variable-voltage machine, 710 b.h.p., 600 volts, 72 r.p.m. *Winding* 596 coils (single turn), stepped winding singly re-entrant. 298 slots; 20 equalizing rings 20 millimetres × 2 millimetres. The winding is as follows: Calling one vertical coil-side coil 1, and the next coil 2, &c. Segment 1 to coil 1 and 50′, and coil 50′ to segment 100. Segment 100 to coil 100, and then to coil 149′. Coil 149′ to segment 199, and then to coil 199, the pitch of the coils being 49 and 50 alternately. 50′ denotes the bottom element in position of coil 50.

Ring.	I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.
Coil.	1	294	587	284	577	274	567	264	557	254
Coil.	299	592	289	582	279	572	269	562	259	552
Ring.	XI.	XII.	XIII.	XIV.	XV.	XVI.	XVII.	XVIII.	XIX.	XX.
Coil.	448	145	438	135	428	125	418	115	408	105
Coil.	150	443	140	433	130	423	120	413	110	403

The E.M.F. Equation.—

Let Z = total number of conductors on the armature,
N = flux per pole,
n = revolutions per second,
a = pairs of circuits,
p = pairs of poles.

Then in one revolution $2\ N p$ lines are cut, and in one second $2\ N n p$ lines. The voltage induced in 1 conductor is therefore $\frac{2\ N n p}{10^8}$.

The conductors per circuit are $\frac{Z}{2\,a}$.

$\therefore$ the E.M.F. between the brushes $= Z\,N\,n\,.\frac{p}{a} \times 10^{-8}$ volts.

With a lap-wound machine (simplex) $p = a$, and we have the E.M.F. $= N\,Z\,n \times 10^{-8}$ volts.

The inductors and end connections are of high conductivity annealed copper which is prepared electrolytically. Where the current is small and the sectional area of the inductor does not exceed 0.15 square centimetre, circular wires are used. The space-factor of the slot with such winding may sink as low as 0.25. It is much more usual, however, to use rectangular copper strip. In a low-voltage machine for, say, 110 volts with rectangular bars, the space-factor rises from 0.35 for small outputs at low speeds to 0.55 for large outputs. At 250 volts for normal speeds and outputs the ratio rises from 0.4 to 0.52, from 40 to 200 kilowatts.

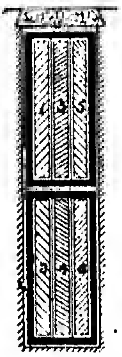


Fig. 11

The arrangement of the elements in the slot and the slot insulation usual for armatures is given below. The slot insulation usually consists of 1½ turns of presspahn and mica 0.4 millimetre thick, and one layer of 0.15 millimetre tape with ½-inch lap. This is placed on both top and bottom layer. The micanite usually extends about ½ inch beyond the armature core at each end. The space thus taken up in the depth is 3.2 millimetres and in the width 1.2 millimetres. To retain the conductors against the action of centrifugal force wooden wedges are used in large machines, the slot being overhung, as shown in fig. 11. This prevents the radiation of heat from the coils. With smaller machines bands of special steel wire, non-magnetic, of No. 18 to 22 S.W.G., are wound in widths of 2 to 3 centimetres.

The Commutator and Brush Gear.—The successful operation of modern multipolar generators depends to a large extent on the correct design of commutator and brush gear. The essential points with regard to the commutator to consider are as follows:—

1. The bars must be held in place absolutely rigidly. It should not be possible to drive down a bar by striking the commutator surface with a mallet or hammer.

2. The bars must not be liable to be loosened by unequal expansion of the commutator due to frequent heating and cooling.

3. Sufficient radiating surface must be provided to dissipate the watts lost, due to brush contact resistance and friction, without excessive rise of temperature.

4. It must be possible to reduce the diameter of the commutator by a reasonable amount in the course of successive turnings and grindings without in any way weakening or loosening the bars.

5. The copper bars and the mica insulation between them must wear evenly, to avoid the formation of ridges of mica between bars, which will cause chattering of the brushes and sparking.

As regards the first point, this depends on the goodness of the end clamping arrangements for holding bars.

Fig. 12 shows different types of end-clamps in general use. Fig. 12*a* shows an arrangement which has given every satisfaction. The top part of the clamping ring is slightly coned in order to give a good true seating for the bars, whilst the clamping down is done by the under side of the ring. This section of clamp-ring was adopted after very careful experiments and with excellent results. The commutator sleeve is of cast iron, while the end ring is of cast steel. Fig. 12*b* is a design by Mr. Parshall used on a 1600-kilowatt generator, and is arranged so that a segment of the clamp can be removed and repairs effected on a portion of the commutator without disturbing the rest. From a study of the method of computing reactance voltage per segment, given in Chapter III, it will be seen that in order to keep the value of this within the prescribed limits it is necessary to have a large number of commutator segments. In fact, in most modern machines above, say, 200 kilowatts, and wound for voltages of 220 or 550 volts, it will be found that it is impossible to keep the value of reactance voltage per segment within limits unless the maximum number of commutator-bars is employed; that is, unless the number of commutator-bars is equal to half the number of armature inductors. This, at ordinary speeds, and with the proportion of magnetic surface generally employed, will necessitate a commutator diameter equal to something like 0.7 to 0.8 of the armature diameter, as the thickness of the commutator-bars at the top must, for mechanical reasons, be not less than 0.2 of an inch, and should be more if possible. The thickness of mica generally employed between the segments is about 0.03 of an inch. As a rule, pure mica will be found too hard for this purpose, except for small commutators. If it is employed it will generally be found, after some months' working, that ridges of mica have formed between the segments, which, if not trued down with an emery wheel, will rapidly get worse and cause sparking to appear at the brushes. Several insulation-makers manufacture a special quality of mica plate for use between commutator segments. Care must be exercised in selecting these materials, however, as, although a few types give excellent results, others are quite worthless.

The insulating end-rings for insulating the commutator end-clamps from the bars are built up of mica strips stuck together by an adhesive material, such as shellac. They are then moulded under hydraulic pres-

sure, the moulds being heated to drive off the shellac used as binding-material.

It is a great mistake to make these end-rings too thick, a thickness of

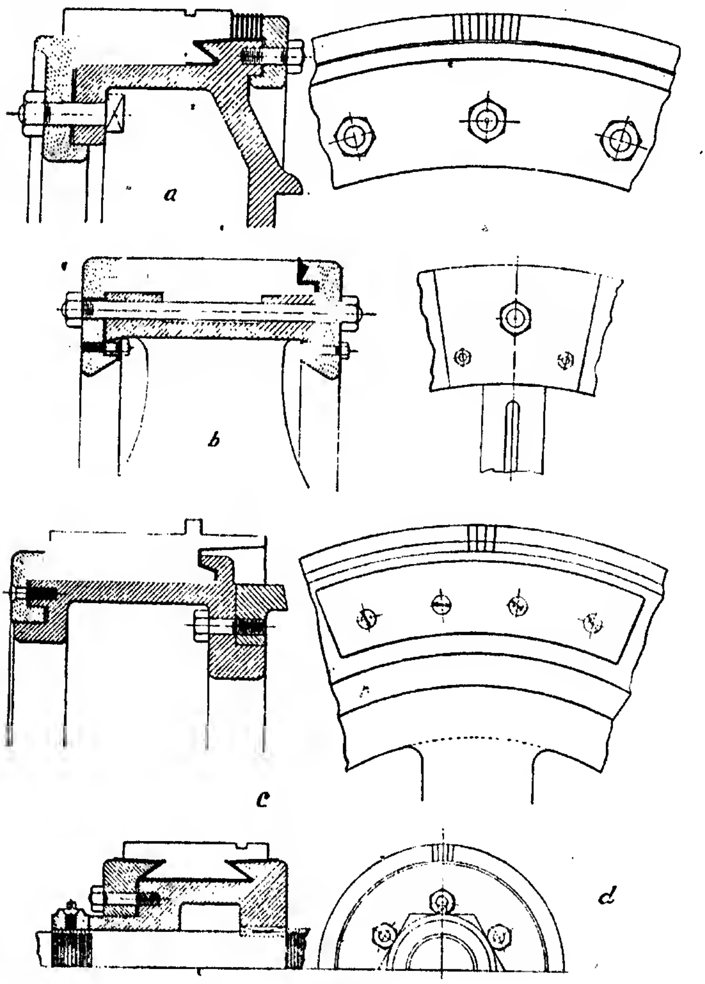


Fig. 12

one-tenth to one-eighth of an inch being ample for generators wound for 500 to 600 volts. If made much thicker than this the rings are liable to have soft places and ridges on the surface, which will cause trouble due to dropped bars, as the commutator expands and contracts under the changes of temperature it will meet with in practice.

The temperature rise at the surface of the commutator will depend on the losses. The brush-friction loss is given in watts by the formula:—

9.81 × peripheral speed of the commutator in metres per second
× total brush area in square centimetres
× brush pressure in kilogrammes per square centimetre
× coefficient of friction.

The brush pressure is usually 0.25 kilogramme per square centimetre and the coefficient of friction is 0.3. The watts lost due to contact resistance are equal to the volts drop across the brushes × the total current.

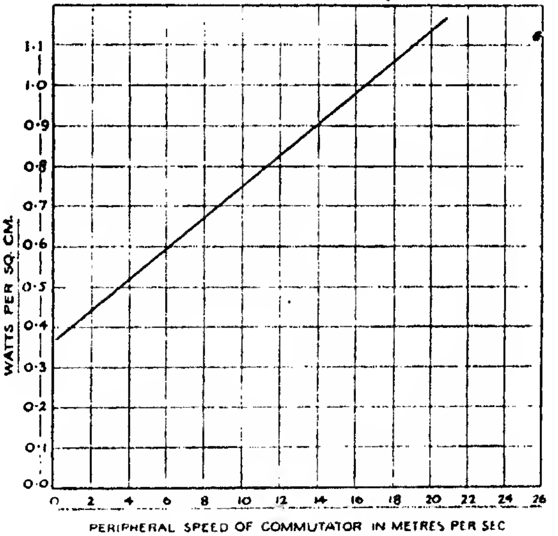


Fig. 13

As a general rule, for soft brushes the volts drop across the brushes is about 1.5 and for hard carbon about 2. The watts per square centimetre can then be found by the use of the graph in fig. 13.

The number and size of brushes will depend on the current per brush-arm. The soft varieties of carbon are used for low voltages up to about 220 volts, and a permissible current of 8.5 amperes per square centimetre is generally taken. For higher voltages the harder varieties are used, and density of 7 amperes per square centimetre is used. The use of carbon brushes is now general, and it would be quite impossible to run modern traction generators, or, indeed, any generator on variable load, with any other type of brush without excessive cost of upkeep of both commutator and brushes. The part which the contact resistance plays in commutation will be emphasized in the chapter on commutation. There are many types of brushes made which combine the good features of both carbon and

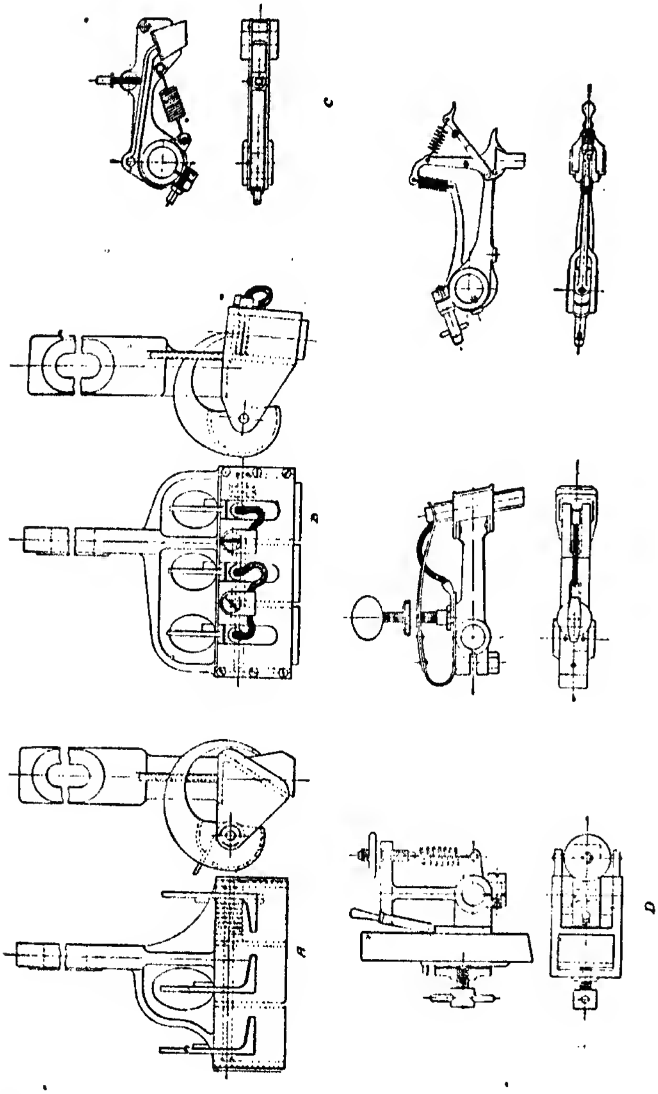


Fig. 14

copper. Among these may be mentioned the Bronskol and the "Endruweit" brushes. These are used for low voltages and large currents, and have met with a varying degree of success. The large frictional losses with carbon brushes may be greatly reduced by impregnating them with paraffin wax.

Types of Brush-holders.—The successful operation of dynamos

depends in no small measure on the design of the brush-holders. These should be light in structure and with a natural period of vibration very different from the period of the disturbing forces. Besides being light in construction they should nevertheless be substantial and strong. Fig. 14 illustrates various types of brush-holders.

The reaction type shown in fig. 14, A gives very good results when used on a commutator running quite true, and having no movement due to vibration. In large generators coupled direct to the crank-shaft of slow-speed engines there is always a slight eccentric movement, due to the working of the crank-shaft in the bearings, which causes the brushes to pump up and down slightly. In the reaction-type brush this contact movement has in time a bad effect on the contact face of the holder,

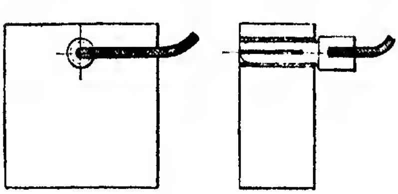


Fig. 15

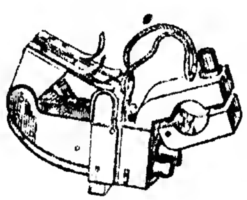


Fig. 16

and causes heating trouble. This is remedied in fig. 14, B by the addition of a flexible connection between the carbon block and holder.

The carbon blocks are of the shape shown in fig. 15. A small brass tube is let into the block, and into this a split-pin slides. From this split-pin a flexible connection is carried to the holder. When a new carbon block has to be inserted the split-pin is pulled out of the old block, which is then removed from the holder and a new block is slipped in place, and the split-pin re-inserted into the brass tube. Fig. 14, C shows the girder or finger type. Very good results can be obtained from this type of brush, but it has many disadvantages. The distance between the carbon block and brush-spindle brings the end of the brush-spindle opposite and close to commutator lugs which are at a considerably different potential from it. If care is not taken in arranging the brush-gear there is a liability to flashing over between these when the machine is subjected to the heavy and sudden overloads to which traction and power generators are subjected. Fig. 16 shows the brush-holder used on Messrs. Siemens Brothers' machines. It is light in structure and accommodates two brushes, so that it is possible to remove one without interfering with the working of the machine. In small and moderate-size machines it is usual to support the brush-gear from a ring mounted on the bearing, but in larger machines the brush-holder ring carrying the collecting gear is supported from the magnet yoke by means of brackets bolted to facings on the yoke. Fig. 17 illustrates this construction. It has the merit that it leaves the commutator free for inspection.

The Magnetic Circuit.—The different types of field systems in use are illustrated in fig. 18. The bipolar type shown in fig. 18, A is very

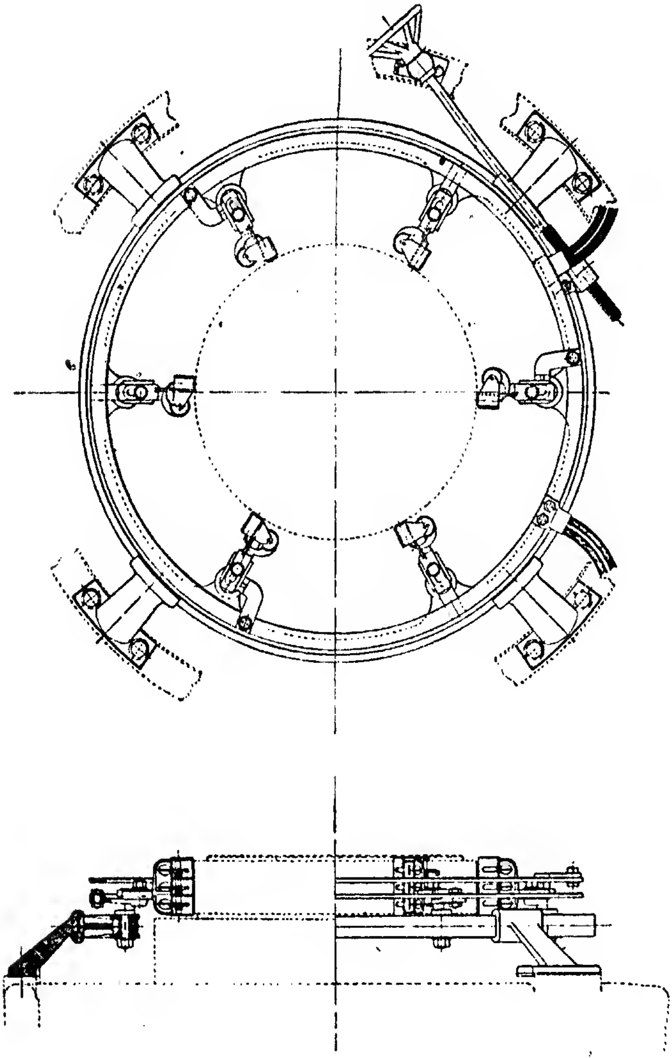


Fig. 17

little used except for very small machines. The general practice now is to employ yoke-rings of cast iron or cast steel with steel poles bolted to them, as in fig. 18, G. For machines having an armature not exceeding 1 metre in diameter the best results are usually obtained by making the yoke-rings of cast steel. The cast-steel or laminated-steel poles are usually bolted on inside the yoke. In some cases solid poles and laminated pole-

shoes are used, but it is perhaps cheaper to laminate the whole pole. Above 1 metre diameter of armature, cast iron is preferable for the yoke, being stiffer and stronger than cast steel, and not so liable to alter shape after machining. Wherever possible a pole of circular section should be used, since it is the most economical section, giving the largest area for

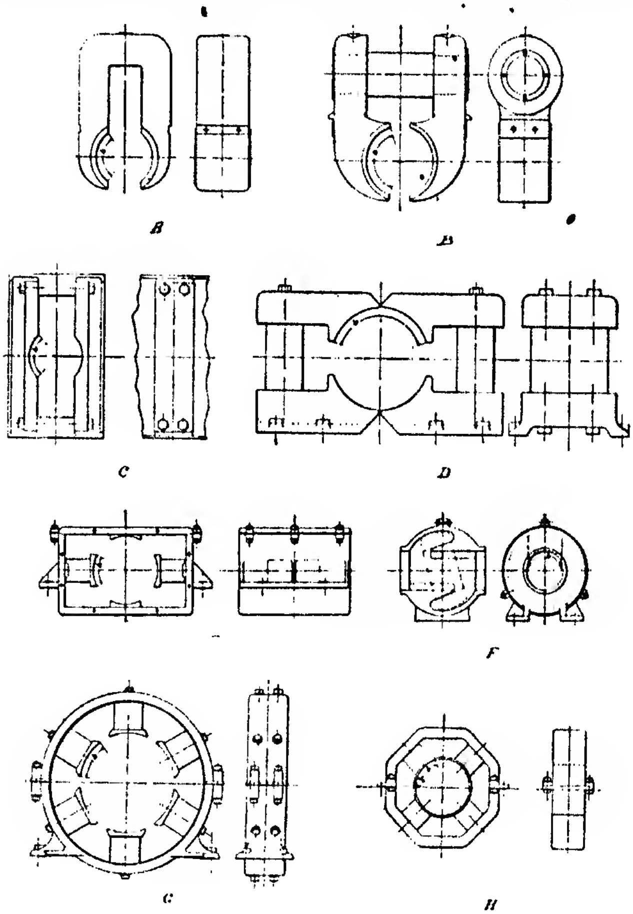


Fig. 18.—Types of Field Magnets

the least perimeter. With laminated poles it is, of course, necessary to use a rectangular section, and the most economical rectangular section is the square. Observance of these points leads to considerable saving in the amount of field-copper required.

The pole span is usually from 0.7 to 0.75 in machines without interpoles, and in interpolar machines this is reduced to 0.65 to 0.7. The edges

of the pole-shoes should either be chamfered or rounded off, or should be made of an elliptical shape to prevent too sudden variations of flux in the teeth when entering or leaving the pole-face. Bad cases of sparking are frequently cured by this artifice.

The magnetic flux consists of two parts, one which crosses the air-gap, which we will call N_a, and is cut by the conductors, and the other, N_l, which does not cross the air-gap, and is called the leakage flux. The total flux, which passes through the yoke and enters the pole $N_p = N_a + N_l$,

and the ratio $\frac{N_p}{N_a}$ is called the leakage factor.

The magnetizing coil has to provide such a magnetic potential difference as will carry the flux from C to B, via core, teeth, gap, pole, and yoke, and,

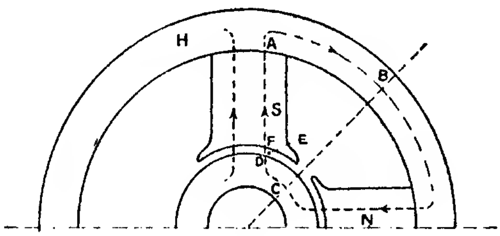


Fig. 19

since it maintains this magnetic P.D. on one side, it will also maintain it on the other side towards H, since the two paths are in parallel.

Let A_y = sectional area of the yoke perpendicular to the lines of force,
A_p = sectional area of the pole perpendicular to the lines of force,
A_g = sectional area of the gap per pole perpendicular to the lines of force,
A_t = sectional area of the teeth per pole,
A_c = sectional area of the core.

In order to determine the ampere turns necessary to force the flux across each part of the path a curve is required for each of the different materials used in the circuit, connecting induction density and ampere turns per centimetre length of path. Such curves for cast iron and sheet steel, &c., will be given later.

The density in the yoke $= \frac{N_y}{2 A_y}$.

The density in the pole $= \frac{N_p}{A_p}$.

The apparent density in the gap $= \frac{N_a}{A_g}$.

$$\text{The actual density in the gap} = C\frac{N_a}{A_g},$$

where C = Carter's coefficient.

$$\text{The density in the teeth} = \frac{N_a}{A_t}.$$

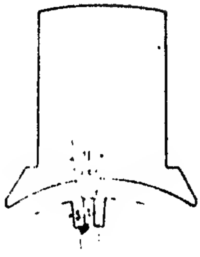


Fig. 20

The different lengths can be scaled off from the diagram, and the total ampere turns calculated.

We will consider the effect of the slots and ventilating ducts in reducing the effective value of the air-gap. Fig. 20 shows how the flux bunches on the teeth. If it were not for the slots and ventilating ducts the air-gap area per pole would be $\lambda\tau L_c$.

$$\text{Where } \lambda = \frac{\text{pole arc}}{\text{pole pitch}},$$

τ = pole pitch,

L_c = gross length of core.

The air-gap area is reduced, due to the slots in the ratio $\frac{x}{y}$, and is equal to $\frac{x}{y}\lambda\tau L_c$, where $\frac{y}{x}$ is the Carter coefficient.

Let t = width of a tooth,

and s = width of a slot,

y = pitch of slots at the armature circumference,

then $x = t + Bs$.

B depends on the slot width and on the air-gap length, and its value is obtained from fig. 21.

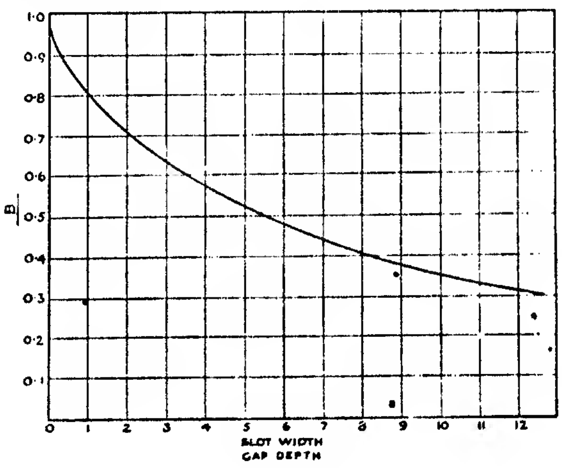


Fig. 21

The Carter coefficient for the ventilating ducts can be found in exactly the same way as that for the slots. Its value is nearly unity, however, and the calculation is seldom made. There is a small amount of fringing at the pole-tips, which tends to increase the air-gap area, but its effect is counterbalanced by the fact that the air-gap length is increased at the tips. Its effect may be considered by taking the effective pole arc

as the pole arc + gap length × constant.

The constant depends chiefly upon the ratio

$$\frac{\text{distance between pole-shoes}}{\text{length of air-gap}},$$

and values for it are given by Mr. F. W. Carter in fig. 22.

In finding the ampere turns for the teeth, the density at the top,

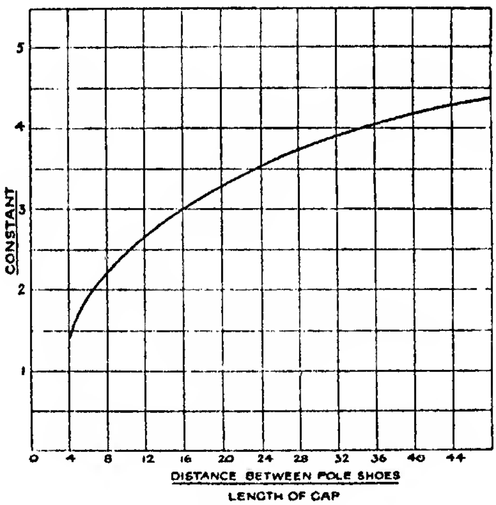


Fig. 22.—Fringing Constant

middle, and root can be found, and Simpson's rule applied to the ampere turns per centimetre for each. It is sufficiently accurate to take the density ⅔rds down the tooth. The tooth section per pole

$$= \frac{\text{tooth circumference } \tfrac{2}{3}\text{rds down tooth} \times \lambda \times \text{nett length of core}}{\text{poles}}.$$

The teeth are usually worked at a high degree of saturation for commutation reasons, and densities of 21,000 and 22,000 lines per square centimetre are common at the roots. At these densities the permeability of the iron is low, and a certain percentage of lines pass through the air in the slots.

The actual density in the teeth can be found as follows: Assume the lines to pass through the air of the slots parallel to the sides of the teeth.

Then total flux = teeth flux + slot air flux.

The apparent density in the teeth $Bt_a = \dfrac{\text{total flux}}{\text{teeth cross-section per pole}}$,

$$\frac{\text{total flux}}{\text{teeth cross-section}} = \frac{\text{teeth flux}}{\text{teeth cross-section}} + \frac{\text{slot air flux}}{\text{slot air cross-section}} \times \frac{\text{slot air cross-section}}{\text{teeth cross-section}},$$

$$\text{or } Bt_a = Bt_r + H_r\left(\frac{1 - vir}{vir}\right),$$

where Bt_a = apparent density in the teeth,

Bt_r = real density in the teeth,

H_r = density in the air corresponding to Br,

v = ventilating factor = $\dfrac{\text{core over all length} - \text{length of ducts}}{\text{core over all length}}$,

i = iron factor = 0.9 for cores,

r = fundamental ratio = $\dfrac{\text{tooth width } \tfrac{2}{3}\text{rds down tooth}}{\text{tooth pitch } \tfrac{2}{3}\text{rds down}}$.

$$\text{The density in the core} = \frac{N_a}{2A_c}.$$

Calculation of the No-load Saturation Curve.—We require to find a relation between the ampere turns required on the field to produce a flux N_a in the air-gap of the machine at no-load. This is a most useful relation, and it enables us to predict with considerable certainty the behaviour of the machines under various conditions of load. The M.M.F. between B and C in fig. 19 = ampere turns on each field coil. This must equal the sum of ampere turns for yoke, pole, gap, teeth, and core.

The flux in the yoke is $\dfrac{N_y}{2}$, and its length of path is known. The density is thus known for a given frame, and the ampere turns per centimetre of path is found from the curve for the material of which it is built. Similarly for the pole, the density is found taking the full-load leakage factor, and the ampere turns calculated. For the air-gap the ampere turns $= 0.8\, B_g\, l_g$,

where B_g = real density in the gap,

l_g = length of air-gap.

It is useful in plotting the curve to plot the ampere turns for the gap alone, and the teeth gap and core alone, and the total ampere turns against the value of the flux.

Leakage Factor.—Since there are no magnetic insulators it follows that a flux must be established wherever a magnetic difference of potential exists.

Magnetic leakage takes place between:

1. Inner faces of pole-shoes.
2. The flanks of pole-shoes.
3. Inner faces of the poles.
4. Flanks of the poles.

It is difficult to estimate the mean areas and mean lengths of path of the leakage flux, but it can be estimated by making certain approximate assumptions.

The most useful general formulæ, evolved from the usual machine proportions, are given by Mr. C. C. Hawkins as follows:—

(*a*) For 4-pole machines total leakage flux per pole
$= \text{ampere turns per pole } (7.35\,d + \mathrm{D} + 26)$;

(*b*) 6-pole leakage flux per pole
$= \text{ampere turns per pole } (8.24\,d + 7\,\mathrm{D} + 26)$;

(*c*) 8-pole leakage flux per pole
$= \text{ampere turns per pole } (8.37\,d + 5.3\,\mathrm{D} + 26)$.

D = diameter of armature in inches,
d = diameter of pole, or side of pole if square in inches.

In general the leakage factor is approximately 1.25 for machines without interpoles, and a ratio of pole arc to pole pitch of 0.75, and 1.3 for machines with interpoles.

Estimation of Leakage Factor.—
Leakage between sides of pole shoes.

Let l_s = length of shoe,
and h_s = height of shoe.

The M.M.F. acting across the tips

$$= 2(\bar{A}_g + \bar{A}_t + \bar{A}_c),$$

where $\bar{A}_g$ = ampere turns per pole for gap,
$\bar{A}_t$ = ampere turns per pole for teeth,
$\bar{A}_c$ = ampere turns per pole for core.

The permeance of the leakage path $= \dfrac{h_s \,.\, l_s}{l_1}$,

where l_1 = distance between the tips,

$\therefore$ total leakage between sides of pole shoes

$$= \frac{2 \times 0.4\pi \times 2(\bar{A}_g + \bar{A}_t) \times l_s \times h_s}{l_1},$$

neglecting $\bar{A}_c$ as negligibly small.

Leakage between Flanks of Shoes.—Assume the flux to consist of straight lines and quadrants of circles with centres at the pole tips.

Consider a small strip of flux of width dy, and whose circular part has a radius y.

The permeance $= \dfrac{h_s dy}{l_1 + \pi y}$, and the leakage flux across one path

$$= 0.4\pi \times 2(\bar{A}_g + \bar{A}_t)\int_0^{\frac{w_s}{2}} \frac{h_s dy}{l_1 + \pi y},$$

$\therefore$ total flank shoe leakage

$$= 3.2 h_s(\bar{A}_g + \bar{A}_t) \log_e\left(\frac{l_1 + \frac{\pi w_s}{2}}{l_1}\right).$$

Leakage between Inner Pole Faces.—Let θ be the angle which the pole sides make with each other and assume the lines of flux to pass across in straight parallel lines. Then flux across strip dx

$$= 0.4\pi \bar{A}\frac{x}{h} \cdot \frac{l\,dx}{W - 2x\tan\frac{\theta}{2}}$$

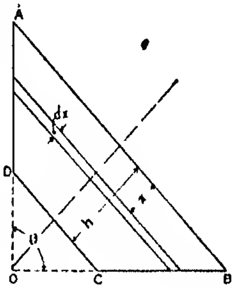


Fig. 23

where $AB = W$, $\bar{A}$ = total ampere turns per pair of poles, and l = axial length of pole.

Total side leakage per pole

$$= 0.8\pi\bar{A}\frac{l}{h}\int_0^h \frac{x\,dx}{W - 2x\tan\frac{\theta}{2}}$$

$$= 0.8\pi\bar{A}\frac{l}{h}\left(\frac{2.3\,W}{4\tan^2\frac{\theta}{2}}\log_{10}\frac{W}{W - 2h\tan\frac{\theta}{2}} - \frac{h}{2\tan\frac{\theta}{2}}\right).$$

Similarly, total flank leakage per pole

$$= 1.6\,\bar{A}_p h_p \log_e\left(1 + \frac{\pi w_p}{b_1 + b_2}\right),$$

w_s = width of shoe, w_p = width of pole core,

b_1 = distance between poles at the bottom of pole shoe,

b_2 = distance between poles at the yoke ring,

h_p = height of core.

CHAPTER II

FIELD COILS

The maximum temperature limits the amount of current a field coil can take without injury. This maximum temperature should not exceed 90° C. The mean rise of temperature can be measured by the increase in resistance of the coil from the formula

$$R_t = R_o(1 + 0.004t),$$

where R_o is resistance at 0° C. and t is the temperature above 0° C.

The ratio $\dfrac{\text{maximum temperature}}{\text{mean temperature}}$ seldom exceeds 1.2

and $\dfrac{\text{mean temperature}}{\text{surface temperature}}$ varies from 1.4 to 3.

The latter ratio depends entirely on the outside wrappings which produce local heating. The ratio 3 given above applies to the earlier machines, the field coils being covered with tape and rope, and of fair depth. When the coil is insulated by being impregnated with compound, and the external surface left bare, with a coil about 2 inches thick, the latter ratio = 1.5 approximately. The impregnating compound is a good heat conductor. Should the coil be wrapped over with tape the ratio increases. The fanning action of the armature will also affect the ratio considerably. The heating constants are figured out in different ways, and the radiating surfaces are not all equally effective. In practice it is usual to take the external cylindrical surface only in estimating the temperature rise. For impregnated coils without any insulating material on the internal surface, the watts per square centimetre of external surface for 40° C. rise of temperature of external surface vary in the manner shown in fig. 24 with the peripheral velocity of the armature.

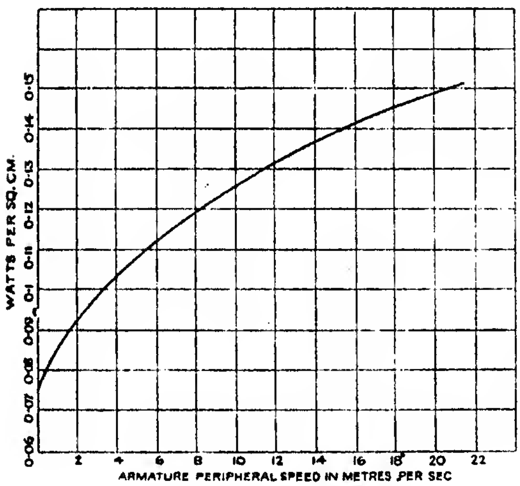


Fig. 24

The radiating surface of a coil is often increased by putting in ventilating openings. The sides of the ventilating openings are not so effective as

either the inner or outer surface. With coils 1 inch thick and spaced ½ inch apart the watts per square centimetre of external surface can be increased about 50 per cent above the values given above in fig. 24.

The Size of Wire for the Field Coils.—

Let E_s be the volts per coil.
I_s the current in amperes in coil.
S_s the number of turns per coil.
l = length of mean turn in metres.
R_s = resistance of coil in ohms (hot).

$$\text{Then } I_s = \frac{E_s}{R_s}.$$

$$R_s = l \times S_s \times \text{resistance per metre of the wire hot.}$$

Allowing 20 per cent increase in resistance due to heating,

then resistance per metre hot

$$= 1.2 \text{ resistance per metre cold } (15^\circ \text{ C.}).$$

$$\therefore I_s = \frac{E_s}{l \times S_s \times \text{resistance per metre at } 15^\circ \text{ C.} \times 1.2}.$$

$$\therefore \text{resistance per metre of the wire at } 15^\circ \text{ C.} = \frac{E_s}{\bar{A} \times l \times 1.2}.$$

$$\therefore \text{resistance per kilometre at } 15^\circ = \frac{833 \times E_s}{\bar{A} \times l}.$$

By consulting a wire table the necessary diameter of wire is found. In generators it is usual to allow 85 per cent of the terminal voltage of the machine across the coil in estimating the size of wire.

The Radial Length of Field Coil.—The watts radiated from the coil = coil external surface in square millimetres × permissible watts per square millimetre.

Let d_f = depth of field coil in millimetres.
l_f = radial length of field coil in millimetres.
s_f = space factor of the wire.
S_s = turns per coil.

$$\text{The section of the wire in coil} = \frac{d_f \times l_f \times s_f}{S_s} = A \text{ (say)}.$$

$$\text{Let } R_f = \text{resistance of coil hot}$$

$$= \frac{0.000017 \times l(\text{mm.}) \times 1.2 \times S_s}{\text{section in sq. mm.}}$$

$$= \frac{0.0000204 \times l(\text{mm.})\, S_s}{\text{section in sq. mm.}}.$$

$$\text{Watts lost per coil} = \frac{I_s^2 \times 0.0000204 \times l(\text{mm.}) \times S_s}{A}$$

$$= \frac{I_s^2 S_s^2 \times 0.0000204 \times l(\text{mm.})}{d_f \times l_f \times s_f}$$

Watts lost per coil = watts radiated, when the temperature is steady external periphery of coil × l_f × watts per square millimetre.

$$\therefore l_f \text{ (in mm.)} = I, S, \sqrt{\frac{l \times 0.0000204}{\text{external periphery} \times \text{watts per sq. mm.} \times d_f \times s_f}}.$$

f we assume average values for above quantities

$$s_f = 0.6,$$
$$d_f = 51 \text{ mm.};$$
$$\text{watts per sq. mm.} = 0.0009,$$
$$\text{and external periphery} = 1.2 \times l,$$
$$\text{then } l_f \text{ (mm.)} = \tfrac{26}{1000}\bar{A} = 0.026\,\bar{A}.$$

Weight and Depth of Coils.—The weight of field copper in coil in lbs.

$$= 0.0197 \times l \text{ in metres} \times S_t \times \text{section in sq. mm.}$$

$$= 0.0197 \times l \text{ (metres)} \times \frac{d_f \times l_f \times s_f}{S_t} \times S_t.$$

A copper wire 1 metre long and 1 square millimetre in cross-section weighs 0.0197 lb.

Substituting the value for l_f above, we have weight of field coil

$$= k \times \sqrt{d_f}. \quad (k \text{ is a constant.})$$

The larger d_f, the shorter l_f, and the smaller the radiating surface, and also the permissible loss. The section of the wire is fixed, since it depends only on the volts per coil and the ampere turns, and a lower permissible loss can only be obtained by a smaller current and a larger number of turns, and therefore a more expensive coil. The thinner the field coil the cheaper therefore the machine, but l_f increases, and also the cost of poles and yoke. An average value for d_f is 2 inches, and this value does not alter greatly on machines of widely different outputs.

In the design of the magnetic circuit the following are average values for the induction density in the various parts:—

		Density.
Pole (Wrought iron)	......	14700 lines per square centimetre.
Yoke (Cast steel)	......	11600 " " "
Yoke (Cast iron)	......	6200 " " "
Teeth (Sheet steel)	......	10000–20000 lines per square centimetre.
Core (Sheet steel)	......	12000–15000 " " "

In coils having series turns as well as shunt the current density used is about 2 amperes per square millimetre. In determining the requisite ampere turns at full load we find the drops of voltage due to

1. Armature resistance.
2. Brush contact resistance.
3. Series coils (if any).
4. Interpole coils (if any).

These drops of voltage are added to the terminal P.D. to give the internal E.M.F. The flux per pole is then obtained from the equation

$$\text{Flux per pole in megalines} = \frac{3000 \times \text{E.M.F.} \times \text{circuits}}{\text{coils} \times \text{turns per coil} \times \text{revs. per min.} \times \text{poles}}$$

From the no-load saturation curve the ampere turns required to produce the flux in the gap are found, and to this must be added the compensating ampere turns for armature reaction.

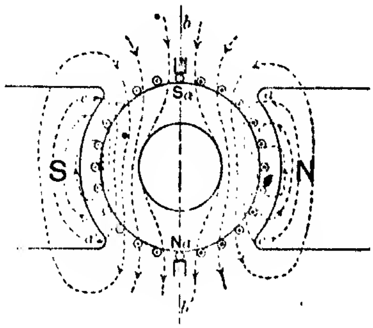


Fig. 25

Armature Reaction. — When the machine is on load the armature conductors are surrounded by a magnetic field. This armature field is shown in fig. 25. It will be seen that the armature is converted into a magnetic solenoid with S. and N. poles at the top and bottom respectively, with the direction of currents shown. It will also be seen that the general direction of the armature flux in the iron is at right angles to the main field, and hence it is called the "cross" flux. The magnitude of this flux will depend on the length of air-gap and on the degree of saturation of the teeth. It is also clear that the direction of the cross flux is the same as that of the main flux at the point *a*, i.e. the trailing tip, and in the opposite direction at the point "*a'*", the leading tip. This is clearly seen from fig. 26, which shows the development of the flux distribution.

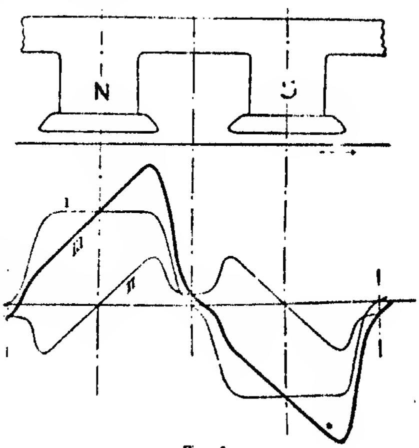


Fig. 26

If the armature teeth are not saturated, as many lines are added at one tip as are subtracted at the other, but in practice the teeth are highly saturated, and any increase of magnetomotive force due to the armature produces very little increase in magnetic flux. On the other hand, a decrease of M.M.F. produces a much larger reduction of flux, and hence the net result is that the total flux per pole is reduced.

The cross magnetizing M.M.F. between pole tips (fig. 25) $= \frac{1}{2} \lambda Z I_c$, where Z = total number of conductors and I_c = current in each con-

ductor and $\lambda = \frac{\text{pole arc}}{\text{pole pitch}}$. The cross magnetizing effect at each pole tip $= \frac{1}{2}\lambda \frac{Z}{2p} I_c$ for any number of poles, where p = pairs of poles.

A method for estimating this reduction of flux due to the armature is given in Hawkins and Wallis. It is rather long and tedious, and the following method is sufficiently accurate for practical purposes: Let ab (fig. 27) be the no-load flux, then Oa is the ampere turns per pole for this flux. On load the ampere turns at the trailing tip are Oh, which equals Oa + $\frac{1}{2}\lambda \frac{Z}{2p} I_c$. The flux density in the gap at the trailing tip is increased from cd to hi, and at the leading tip is decreased in the ratio ye to cd.

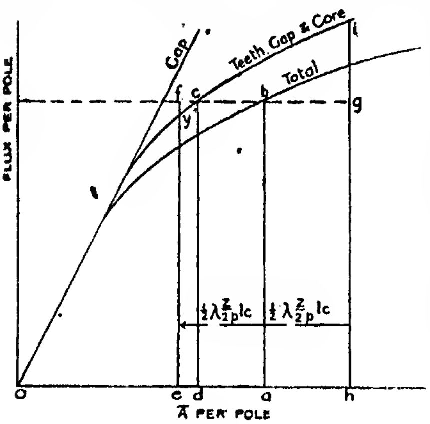


Fig. 27

The total flux per pole is reduced below its value at no load in the ratio $\frac{eyihe}{efghe}$.

It will be seen that the action of the cross M.M.F. of the armature is to twist the flux around in the direction of rotation, so that the brushes must be moved forward in order that the coil undergoing short-circuit shall be moving in a reversing field. On shifting the brushes forward, however, a state of things exists as shown in fig. 28. The magnetizing effect is now divisible into two distinct belts of conductors. The belt between A and D is obviously demagnetizing in its effect and the belt between D and C cross magnetizing. The demagnetizing ampere turns per pole

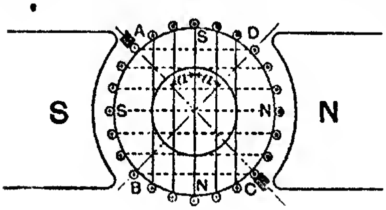


Fig. 28

$$= \frac{1}{2}\frac{Z}{2p} \times I_c \times \frac{2a}{180}.$$

The angle "a" is called the "angle of lead". Its amount will vary with the design of the machine, and is seldom more than 2 or 3 segments. With machines having interpoles, and the brushes in the neutral position, it is generally found that about 25 per cent of the armature ampere turns per pole are needed to be added to the field ampere turns per pole to compensate for armature reaction effects.

We have seen that the flux at the leading tip is reduced in value, and in order to secure a sufficiently strong reversing field it is essential that the ampere turns of the main field per pole for the gap, teeth, and core

should be greater than the cross ampere turns of the armature per pole. The greater the ratio the better the commutation. This ratio is seldom less than 1.2. To reduce the cross flux the tips of the poles and the teeth are highly saturated, and in many designs an air-gap is interposed in the path of the cross flux in the pole itself.

CHAPTER III

COMMUTATION

The subject of commutation is exceedingly difficult, and no attempt will be made to deal with its analysis. The subject is excellently treated by Messrs. Hawkins and Wallis, and its discussion will be found in several recent copies of the *Electrician*. It will suffice here to touch the salient points, and to show what precautions are taken to ensure the sparkless operation of machines. We have seen that when a coil passes under a brush its current is reversed from a value $+ I_c$ to a value $- I_c$ in a very short interval of time, depending on the width of the brush and on the peripheral velocity of the commutator. This reversal may be effected by the action of the contact resistance of the brush, or by the combined action of brush contact resistance and the presence of a reversing field. Fig. 29 represents a coil undergoing short-circuit. Let the direction of the current in the coil before entrance under the brush be considered positive.

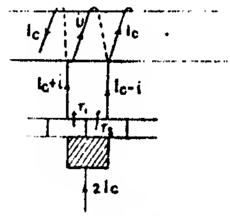


Fig. 29

Let R = resistance of total brush contact in ohms,
T = time of commutation,
L = coefficient of self-induction of the coil in henrys,
M = coefficient of mutual induction of the coil and other adjacent coils, short-circuited at adjacent brushes.

Then, if the coil is cutting no external field, we have by Kerchoff's second law

$$(I_c + i) r_1 - (I_c - i) r_2 + (L + M) \frac{di}{dt} = 0,$$

where r_1 = contact resistance of the part of the brush in contact with the leading segment,
and r_2 = contact resistance of that part of brush in contact with the trailing segment, and let the brush cover one segment.

Let the time be reckoned from the commencement of the short circuit,

$$\text{then } r_1 = R\left(\frac{T}{T - t}\right),$$

$$\text{and } r_2 = \frac{RT}{t},$$

$$\text{and} \therefore \left(I_c + i\right)\frac{RT}{T - t} + \left(L + M\right)\frac{di}{dt} - \left(I_c - i\right)R\left(\frac{T}{t}\right) = 0,$$

$$\text{and } \frac{di}{dt} = -\left(\frac{RT}{L + M}\right)\left(\frac{I_c + i}{T - t} - \frac{I_c - i}{t}\right).$$

By giving various values to $\frac{RT}{L + M}$, the value of the current in the coil (i.e. i) can be plotted in relation to the time of commutation. With a value of $\frac{RT}{L + M}$ equal to infinity, we get straight-line commutation, and the current density under the brush remains uniform. This is the ideal state, and is always aimed at. As this ratio gets smaller and smaller, the short-circuit curve of current in relation to time becomes bowed, and the brush-tip density becomes very high, and may be infinite.

$$\frac{RT}{L + M} \text{ must always exceed } 1,$$

$$\text{and } \therefore R > \frac{L + M}{T},$$

$$\text{and } \therefore 2I_cR > \frac{L + M}{T} \times 2I_c.$$

This is the average reactance volts on the assumption that the short-circuit current follows a linear law. *The average reactance volts must therefore be less than the volts drop across one brush contact when the coil is not moving in an external field.* The brush contact resistance is not uniform, but varies with current density, brush pressure, peripheral velocity, and temperature of contact, vibration, &c. The contact resistance decreases with current density, and, at values of about 5.4 amperes per square centimetre, almost inversely as the current density, the curve becoming almost hyperbolic. This variation is in the wrong direction, for, when the current density increases abnormally, it is then that the corrective action of the brush is needed most. This variation is shown in fig. 30.

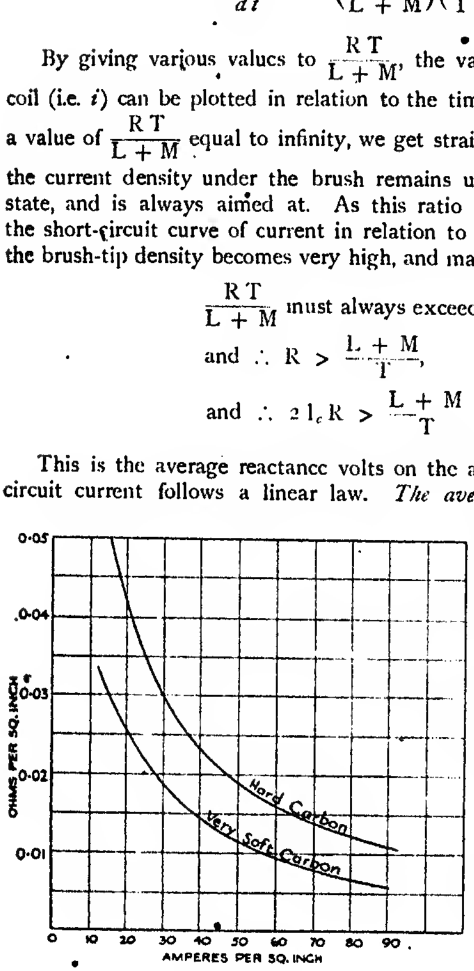


Fig. 30

The machine is supposed to be running long enough for the temperature corresponding to each current density to be reached, and the

values are the averages of the positive and negative contact resistance. At normal densities of 40 to 45 amperes per square inch the specific contact resistance for hard carbon brushes is about 0.021, and the volts drop across one contact is 0.94 volt. The decrease of contact resistance with increasing current density is closely associated with the fact that carbon has a negative temperature coefficient, and also with the fact that a blackening of the commutator results, due to particles of carbon which are worn off the brushes, especially with a high current density. The temperature of the contact is a great determining factor. After a temperature of 35° C. is reached the fall of contact resistance is more or less rapid, which is more pronounced where the current flows from carbon to metal, that is, at the negative brush. This brings out the importance of good ventilation of the commutator in suppressing sparking. It is a well-known fact that sparking troubles occur most when the machine gets hot after a long run. At low temperatures the contact resistance of the negative brushes is higher than that at the positive, and vice versa with high temperatures. Thus a machine usually sparks first at the negative brushes when it gets hot. An increase of brush pressure decreases the contact resistance. As the speed increases, the contact resistance increases in the case of a commutator, due to vibration, which is set up by the mica ridges. With a smooth slip-ring the contact resistance is almost independent of the speed.

As the load on a machine increases, the reactance voltage increases, and we have seen that, for sparkless commutation, there is a limit to its value, depending on the contact resistance of the brush, when the coil is not cutting any external reversing field. The value of the reactance voltage may be increased, however, by causing the short-circuited coil to move in a reversing field whose value increases as the current increases. This reversing field may be obtained by moving the brushes forward in the direction of rotation, or by providing a separate reversing field which is independent of the main field. Modern machines are expected to work sparklessly with fixed brush position at all loads between no load and 25 per cent overload.

In machines without special commutating poles this is effected by moving the brushes forward until the machine is about to spark at no load. There is then no reactance voltage, and the generated voltage is a little less than the sparking voltage. When the machine is carrying the required overload, the reactance voltage will be greater than the generated voltage from the reversing field by a little less than the sparking voltage. Half-way between no load and the overload, the reactance voltage will be equal and opposite to the generated voltage, and commutation will be perfect. In shunt machines, the external reversing field decreases with increase of load, due to the cross-magnetizing effect of the armature, and hence it is very necessary that the field ampere turns per pole for the gap and teeth and core should be greater than 1.2 times the armature ampere turns per pole, plus the demagnetizing ampere turns per pole. The over-compounded machine is greatly superior as far as commutation is concerned.

Slots per Pole.—In order to ensure that the coils in a slot shall be moving in a correct reversing field, it is essential that the pitch of the slots

shall not be too great. In general it may be said that there should not be less than 3.5 slots in the interpolar space, or not less than 12 slots per pole, and for large machines the number is seldom less than 14. This obviously reduces the distance through which a slot moves while the conductors it carries are commutating.

Brush Width.—The brush width affects the reactance volts very little, for, although the number of coils which are simultaneously undergoing commutation is increased, the time of commutation is proportionately increased. It is necessary, nevertheless, to limit the width of the brush, for, if it be too wide at the commencement of commutation, commutation will be retarded, and at the end there will be overcommutation. To limit this the brush should not cover more than 28 per cent of the space between the poles, or one-twelfth of the pole pitch. Measured on the commutator circumference the brush arc

$$= \frac{\text{pole pitch}}{12} \times \frac{\text{diameter of commutator}}{\text{diameter of armature}}.$$

Again, it is necessary to limit the brush width, in order to limit the current circulating through the brush, since with a wide brush the E.M.F. of several coils acts on a low resistance path through the coil and two brush contacts. In general, the brush should not cover more than three segments.

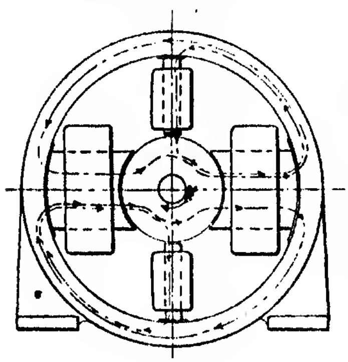


Fig. 31

Interpoles.—With the introduction of interpoles, machines which formerly were limited in output, due to sparking, can have their output considerably increased. Radical changes in design are rendered necessary, but these changes are such as to justify the use of interpoles. We have seen that for perfect commutation a reversing field must be present of such magnitude that the reactance voltage is neutralized. This reversing field is obtained by placing polar projections between the main poles, which are surrounded by coils carrying the armature current. The reactance voltage increasing directly with the armature current, it follows that, if the reversing E.M.F. is to be equal to it, the interpole field must increase proportionately with the armature current. This is accomplished by working the interpole on the straight part of the magnetization curve. The air-gap density in the interpole should not exceed 7000 lines per square centimetre at full load, and the density at the root of the interpole should not exceed 10,000 lines per square centimetre. Fig. 31 shows the distribution of flux in an interpole machine. Fig. 32 shows the development of the field distribution.

The interpole must have a number of ampere turns equal to the sum of the cross ampere turns of the armature, and the ampere turns required to

produce a flux in the interpole air-gap large enough to generate a voltage equal and opposite to the E.M.F. of self and mutual induction, and to that generated due to the cutting of the magnetic field by the end connections.

The reversing field required by the interpole can be found as follows:—

Let L_{1p} = interpole length in centimetres,
B_i = average interpole gap density in lines per square centimetre,
S_1 = turns per coil,
v = peripheral velocity of armature in centimetres per second.

Then the E.M.F. due to the interpole

$$= 2\,S_1\,B_i\,L_{1p}\,v \times 10^{-8},$$

and this must equal $(L + M)\dfrac{2I}{T}$,

from this B_i can be determined.

The length of the interpole is sometimes made the same as the length of main pole, but in some cases is made considerably shorter. In the interpole machines of the Phœnix Dynamo Company the length is considerably shorter, and a circular core with a trapezoidal shoe is used. The width of the interpole should be such that the slot in which a conductor lies is under the interpole during the whole period of commutation. The distance through which a slot moves from the instant the first coil in a slot terminates its short-circuit to the instant the last coil in the slot terminates its short-circuit is equal to the slot pitch minus the width of a commutator segment referred to the armature circumference. The flux fringes out on either side of the interpole by a distance approximately equal to the air-gap clearance, so that the effective interpole arc = actual arc + $2\delta_1$, where δ_1 = interpole air-gap length. The effective interpole arc must therefore be equal to the slot pitch + (brush arc − width of segment) $\times \dfrac{\text{diameter of armature}}{\text{diameter of commutator}}$. The arc must also be a multiple of the slot pitch in order to keep the interpole flux from pulsating in value, and to preserve a constant reluctance under the interpole.

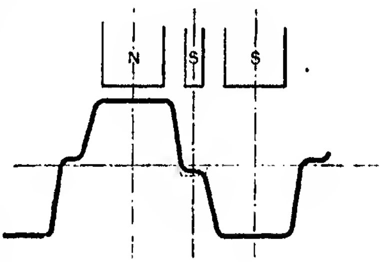


Fig. 32

The interpole excitation is generally fixed empirically in practice, and the following value for the reversing ratio $\left(\dfrac{\text{interpole ampere turns per pole}}{\text{armature ampere turns per pole}}\right)$

$$\text{namely, } 1 + \frac{k \times \delta_1 \times 2p}{D}, \text{ is used,}$$

where k = 3.5, a constant,
δ_1 = length of interpole gap in millimetres,
$2p$ = number of poles,
D = diameter of armature in millimetres.

The exact adjustment is made on test by separately exciting the interpoles and adjusting their current by a diverter till sparking disappears.

Compensating Windings.—A serious trouble on machines having sudden variations of load, such as traction generators, rolling-mill motors, and winding motors, is often caused by flashing over at the commutator. This occurs when the maximum volts between adjacent commutator segments exceeds certain limits. We have seen that the effect of armature reaction is to cause a piling up of the flux at the trailing tip in generators, and it is clear that the coils moving in that part of the field will have much larger E.M.F.s than the average generated in them. The usual limit is 40 volts per segment at 50 per cent overload. Sudden changes of armature current also tend to increase the volts per segment and to produce flashing over. In rolling-mill motors the current changes from less than full-load current to about three times full load at the instant of reversing the rolls. Hence the average volts per segment should not exceed 15 without special commutating windings.

The maximum volts per segment

$$\doteq \frac{\text{E.M.F.}}{\text{segments}} \times 2p \times \frac{1}{\lambda} \times \frac{\text{N maximum}}{\text{N minimum}} + \frac{\text{voltage drop}}{\text{segments}} \times \text{poles}.$$

N maximum = maximum flux = flux corresponding to sum of field ampere turns per pole effective for teeth, gap, and core, and the effective cross ampere turns of the armature. The flux must be read off the curve for teeth, gap, and core only. Compensating windings consist of windings which are connected in series with the armature, and which are embedded in slots in the laminated poles. These windings carry currents in opposite direction to the armature currents under the pole, and the number of ampere turns per pole in the compensating windings is made equal to the cross ampere turns per pole of the armature. This being so it is only necessary to put on the interpole a number of ampere turns sufficient to produce the reversing field.

The compensating winding completely neutralizes the armature cross field, keeps down the maximum volts per segment, and so prevents flashing over. The use of such windings makes the machine rather expensive, and in most cases the weight of copper in them is about 50 per cent greater than the weight of armature copper. The tooth density is usually about 16,500 lines per square centimetre at a point one-third the depth of the teeth from the pole face. The current density in the compensating windings is generally not greater than 230 amperes per square centimetre in the slots and 260 amperes per square centimetre on the hang out.

Reactance Volts for Full and Short-pitch Windings.—Consider first a full-pitch lap winding.

Let L_c = length of iron core,
L_e = length of one end connection,
N_s = the flux that links 1 cm. length of the slot part of the coil for each ampere conductor in the group of conductors simultaneously commutated,

N_e = the flux that links 1 cm. length of the end connections for each ampere conductor in the group of end connections simultaneously short-circuited.

S_c = turns per coil,

and let the brush cover one segment only. Since the brush covers one segment only there will be $2\,S_c$ conductors short-circuited simultaneously in the slots. The flux due to these ampere conductors for one slot

$$= N_s \times L_c \times 2\,S_c \times I \text{ lines.}$$

In the group of end connections there are S_c conductors, each carrying a current I. The flux due to these $= N_e \times L_e \times S_c \times I$. The total flux encircling the short-circuited coil

$$= 2\,S_c \times I\,(2\,N_s L_c + N_e L_e),$$

$$\text{and } (L + M) = 2\,S_c^2 \times (2\,N_s L_c + N_e L_e).$$

The time of commutation in seconds

$$= \frac{\text{segments covered by the brush}}{\text{total segments} \times \text{revs. per sec.}}$$

$$\therefore \text{ the average reactance volts (R.V.)} = (L + M)\,\frac{2\,I_c}{T}$$

$$= \frac{2\,I_c \times 2\,S_c^2\,(2\,N_s L_c + N_e L_e) \times \text{revs. per sec.} \times \text{total segments} \times 10^{-8}}{\text{segments covered by the brush}}.$$

It can be shown that an increase in the brush width has no effect on the reactance voltage so long as the group of conductors simultaneously commutated is not greater than the number of conductors per slot. If the group of conductors short-circuited at one time is greater than the conductors per slot, the reactance volts is decreased. Mr. C. C. Hawkins has investigated the subject of inductance of armature coils at some length, and the reader is referred to his book on *The Dynamo* for this. A simple and generally used system is to use 4 lines per ampere conductor per centimetre in the slots and 0.8 line per ampere conductor per centimetre on the end connections. This latter method is due to Mr. Hobart. Substituting these values for N_s and N_e in the above formula, and remembering the above statement about the width of the brush, we have for a full-pitch lap winding in two layers:—

$$\text{(R.V.)} = 4\,I_c S_c^2\,(8\,L_c + 0.8\,L_e) \times \text{R.P.S.} \times \text{segments} \times 10^{-8} \text{ volts}$$

$$= 32\,I_c S_c^2\,(L_c + 0.1\,L_e) \times \text{R.P.S.} \times \text{segments} \times 10^{-8} \text{ volts.}$$

R.P.S. = revolutions per second.

Short-pitch Lap Winding.—Here the coils simultaneously short-circuited are not in the same slot. The end-connection flux is the same, while the slot flux is reduced to one-half its value if the conductors are in adjacent slots.

$$\therefore \text{ the (R.V.)} = 32\,I_c S_c^2\left(\frac{L_c}{2} + 0.1\,L_e\right) \times \text{R.P.S.} \times \text{segments} \times 10^{-8} \text{ volts.}$$

Reactance Volts for Wave Windings (Simplex).—When one set only of brushes are used it is quite clear from the discussion on two-circuit windings that each brush short-circuits "p" coils in series. In this case the (R.V.)

$$= 32\, I_c S_c^2 (L_c + 0.1\, L_e) \times \text{segments} \times \text{R.P.S.} \times p \times 10^{-8} \text{ volts.}$$

If as many sets of brushes be used as there are poles, then there is a short-circuit path around one coil short-circuited by two positive brushes or two negative brushes, in addition to the long path already mentioned. The reactance voltage generally used is that corresponding to the longer path, since selective commutation has to be taken into account. This value is pessimistic. It is usually taken, however, that the (R.V.) is 20 per cent lower than that obtained for the longer path.

Heating and Rating of Generators.—Stefan's law states that the rate of losing heat varies as the fourth power of the difference of absolute temperatures of the hot body and surrounding air. Newton's law is taken to hold, however, for small ranges of temperature, such as are met with in dynamo practice. In the dynamo, losses of energy take place, due to armature resistance, field resistance, hysteresis, and eddy currents. These losses are converted into heat. The temperature rises till the rate of generation of heat equals the rate of dissipation, and then the temperature is steady. There are thus two periods during heating: (1) the heating-up period and (2) the stable condition.

Let θ = rise of temperature in time t seconds and

θ_0 = final temperature rise.

Then $\theta = \theta_0 (1 - e^{-at})$.

θ_0 is dependent on the loss of energy. a is independent of the loss of energy, and depends on the specific heat and efficiency of the cooling surfaces.

$$\text{When } t = \frac{1}{a}, \quad \theta = 0.632\, \theta_0.$$

This time depends on "a", and is constant for a given body and method of cooling, and is known as the "heating-time constant". The heating-time constant is the time it would take the body to reach its final temperature if the heating continued at the initial rate and if the body lost no heat during the process. That this is so can be seen by differentiating the above equation; for we have

$$\frac{d\theta}{dt} = + a\theta_0 e^{-at},$$

when $t = 0$, at the commencement of the heating period

$$\frac{d\theta}{dt} = a\theta_0,$$

$$\therefore \frac{1}{a} = \frac{\theta_0}{\frac{d\theta}{dt}}.$$

The time constants for field and armature can be calculated as follows:—

For armatures it is in minutes

$$\frac{230 \times (\text{weight of copper} + \text{weight of iron in the teeth and core}) \text{ lbs.}}{\text{permissible watts on core surface for } 40^\circ \text{ C. rise at given peripheral speed}}$$

For field coils it equals

$$\frac{230 \times \text{copper weight in lbs.}}{\text{permissible watts on coil surface for } 40^\circ \text{ C. rise}} \text{ minutes.}$$

From these equations the rise of temperature at any time of the run can be found approximately, and the overload the machine will take without exceeding a given temperature rise. The highest temperature at which a machine can be operated, with the usual insulating materials, presspahn, paper, tape, &c., is 85° C. These materials become brittle if heated for any length of time at this temperature, and pulverize, due to vibration. For continuous rating it is generally specified that the temperature rise shall not exceed 40° C. on the surface with an initial air temperature of 25° C. The usual time taken by large machines to reach this temperature rise is six hours, and machines are expected to give their full output at the specified speed and voltage for six hours without exceeding this temperature rise. Smaller machines reach their steady temperature state in less than six hours.

CHAPTER IV

DESIGN

The principal dimensions of the machine are determined from the output equation in the following manner:—

The E.M.F. generated $= N Z n \frac{p}{a} \times 10^{-8}$ volts,

where N = flux per pole in the air-gap,
Z = total number of surface conductors,
n = revolution per second,
p = pairs of poles,
a = pairs of circuits.

Let B_g = density in air-gap,
L_c = length of core,
$\lambda = \frac{\text{pole arc}}{\text{pole pitch}}$,
τ = pole pitch,
q = ampere conductors per centimetre of armature periphery,
I_a = total armature current;

$$\text{Then E.M.F.} = Z(B_g \lambda \tau L_c) n \frac{p}{a} \times 10^{-8},$$

$$\text{and } q = \frac{Z I_a}{2 a \pi D_a}, \qquad (D_a = \text{diameter of armature}).$$

$$\therefore E I_a = Z(B_g \lambda \tau L_c) n \times \frac{p}{a} \times \frac{q a \pi D_a}{Z} \times 10^{-8} \times 2$$

$$\therefore = Z B_g \lambda \frac{\pi D_a}{2p} L_c \times n \times \frac{p}{a} \times \frac{q a \pi D_a}{Z} \times 2$$

$$= B_g L_c n \lambda \pi^2 D_a^2 \times q \times 10^{-8}.$$

Now, if V = terminal volts,

E = 1.05 V on an average,

and if I = external current,

then I_a = 1.03 I on an average,

$$\therefore D_a^2 L_c = \frac{\text{watts output} \times 1.08 \times 10^8}{B_g \times n \times \lambda \times \pi^2 \times q}.$$

From the above equation we can find the value of $D_a^2 L_c$, provided we know the values of B_g and q. The value of B_g, the apparent gap density

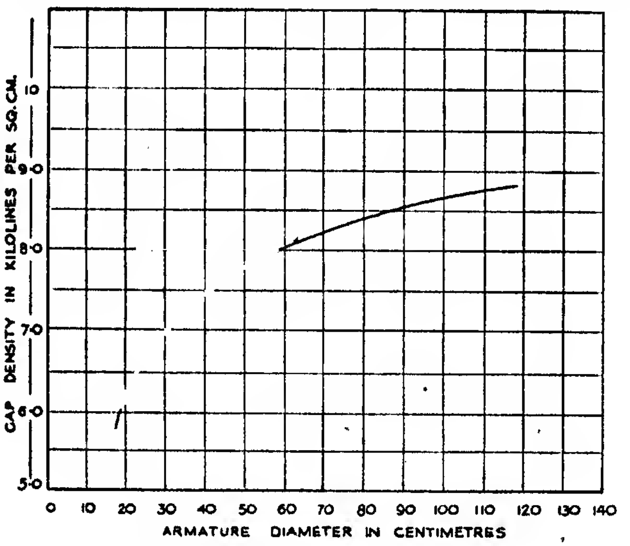


Fig. 33

is limited by the maximum tooth density. The latter should not exceed 21,000 lines per square centimetre for frequencies up to 30 cycles. It will be seen that the smaller the machine the greater the tooth taper, and therefore the lower the gap density for a given maximum tooth density. Fig. 33 shows the relation between B_g and D_a for average machines for frequencies up to 30 cycles per second. The value of q is limited by considerations of heating and sparking. It increases in large machines, for

with larger diameters it becomes possible to increase the depth of the slot without increasing the density at the roots of the teeth too much. It thus becomes possible to increase the volume of copper more than in proportion to the increase in armature diameter. Again, it is necessary to limit the value of q from considerations of sparking, for q can be increased by increasing the number of conductors or by decreasing the diameter of armature for a given rating. The reactance voltage is increased in either case, as is evident from an inspection of the formula for it. Thus q depends largely on the diameter of the machine, and hence

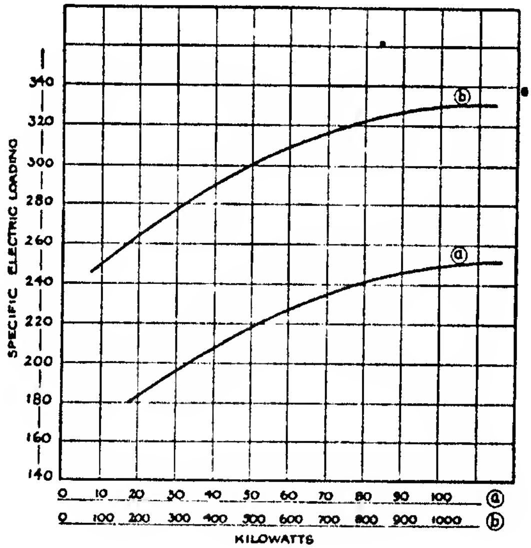


Fig. 34

on the kilowatt output. The relation for q and kilowatt output is shown in fig. 34. For interpole machines increase these values by about 20 per cent.

The separation of $D_a^2 L_c$ into its component factors is a matter of some difficulty, and is frequently settled by the relation between the electric and magnetic loading. For every value of $\frac{\text{KW.}}{\text{R.P.M.}}$ there is a value of electric loading which gives the most economical machine. Fig. 35 shows the value of electric loading, or total ampere conductors and $\frac{\text{KW.}}{\text{R.P.M.}}$ Proportions are generally used in which the pole face is approximately square. In this case we have $\frac{\pi D_a}{2p} \lambda = L_c$, and from this D_a and L_c can be deduced from the original equation for $D_a^2 L_c$. The maximum armature reaction per pole permissible affects the number of poles, and, speaking generally, the armature ampere turns per pole should not greatly exceed 7500. Good

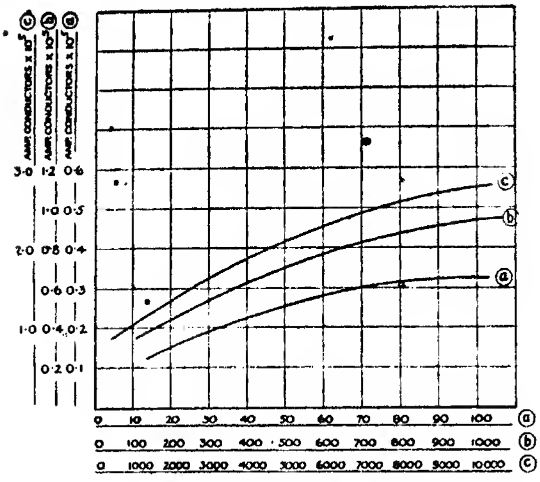


Fig. 35

results are obtained if machines are built with the number of poles in the table given below:—

No. of Poles.	Kilowatt Outputs.	
	220 to 250 volts.	500 to 550 volts.
2	10	10
4	80	80
6	200	250
8	300	400
10	350	500
12	400	600
14	500	700
16	550	800
18	700	900
20	750	1000
22	800	1100
24	900	1200

Having obtained the diameter and length of armature, the number of poles, and assuming an average value for $\frac{\text{pole arc}}{\text{pole pitch}}$, the proportions of the magnetic circuit are settled as already pointed out in earlier sections.

The flux per pole can then be settled, and the number of conductors. The type of winding will be settled by the total armature current as already mentioned, and the number of slots must be chosen to suit the

winding. The depth of slot will vary with the diameter of armature, and this relationship is shown in fig. 36. The width of the slot will be settled from the permissible magnetic density in the teeth at full load. The length of air-gap should be sufficient to ensure good mechanical clearance, and is also settled largely in non-interpole machines by the fact that the field ampere turns per pole for teeth and gap must be at least equal to 1.2 times the armature ampere turns per pole. The values given in fig. 37 are those taken from practice, and they show in a most remarkable manner how the gap can be reduced in interpole machines, and a great saving in copper effected. The reason for this

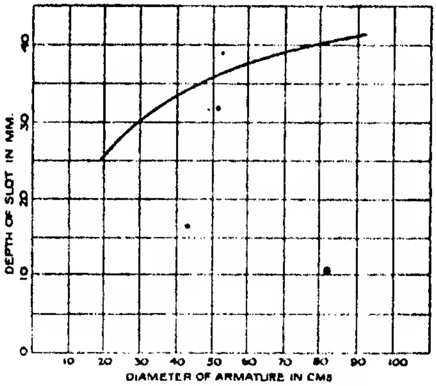


Fig. 36

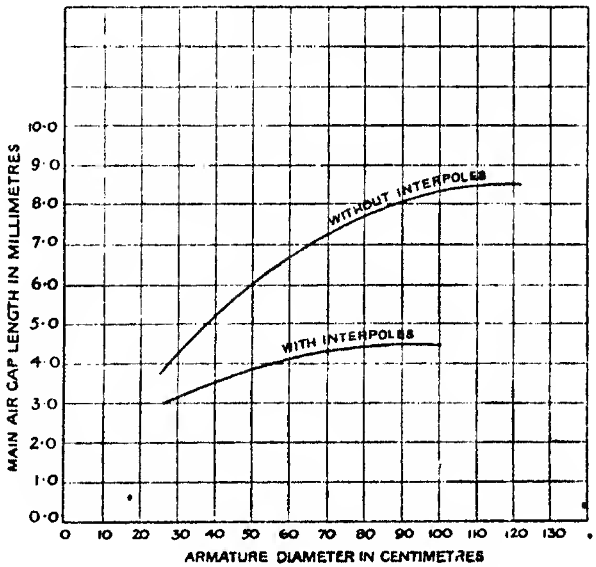


Fig. 37

reduction is that it is no longer necessary to provide a long air-gap with interpoles, since the cross-magnetizing effect is wiped out by the interpole.

Losses in D.C. Machines.—The losses in the armature are those due to hysteresis, eddy currents, and the copper loss.

The hysteresis loss is $= K_1 B^{1.6} f V$ watts,
where K_1 = the hysteresis constant, which varies with the grade of iron,
B = maximum induction density in lines per square centimetre,
f = frequency,
V = volume of the iron in cubic centimetres.

The eddy current loss $= K_2 B^2 f^2 t^2 V$ watts,
where K_2 = constant which is inversely proportional to the electrical resistance of the iron,
t = thickness of the core plates in centimetres.

To reduce the eddy-current losses, high-resistance iron can be employed. The alloys possessing this property, however, have a lower permeability than lower-resistance iron, and are much more costly and more brittle. The core is built up of thin plates, which are insulated from each other by layers of varnish or paper, and assembled directly on the shaft or on a cast-iron spider. The thickness of the laminations is usually about 0.035 centimetre. Besides the ordinary hysteresis and eddy-current losses there are additional losses due to filing of the slots, to leakage flux in the spider and end heads, and to non-uniform distribution of flux in the core. When the machine is on load, also, there is increased iron loss due to the piling up of the flux in the teeth under the trailing tip of the pole.

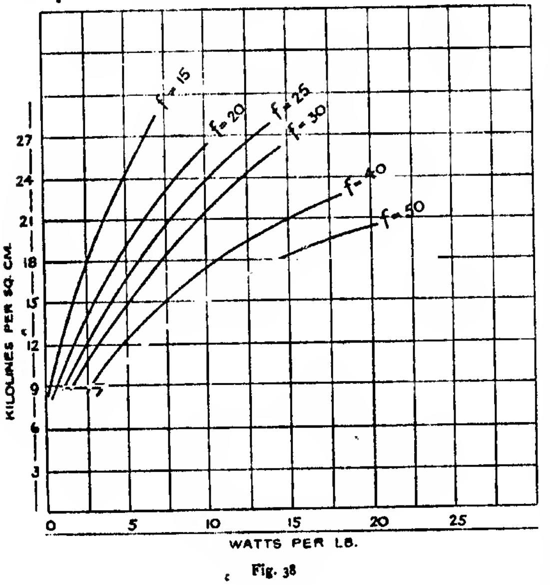


Fig. 38

The estimation of the iron losses is a difficult matter, and is usually performed with the help of results obtained from tests on similar designs. The curves in fig. 38, taken from Gray's *Electrical Machine Design*,

give test results for machines built with ordinary iron having a thickness of 0.035 centimetre. It is necessary of course to calculate the losses for the teeth and core separately. The armature copper loss is easily estimated from the resistance of the armature and the known armature current. The resistance of the armature at 15° C.

$$= \frac{0.017 \times \text{total length of the wire in the armature in metres}}{(\text{circuits})^2 \times \text{section of conductor in square millimetres}}.$$

For 40° C. surface rise an increase of resistance of 15 per cent gives the resistance hot. In estimating the temperature rise it is usual to calculate

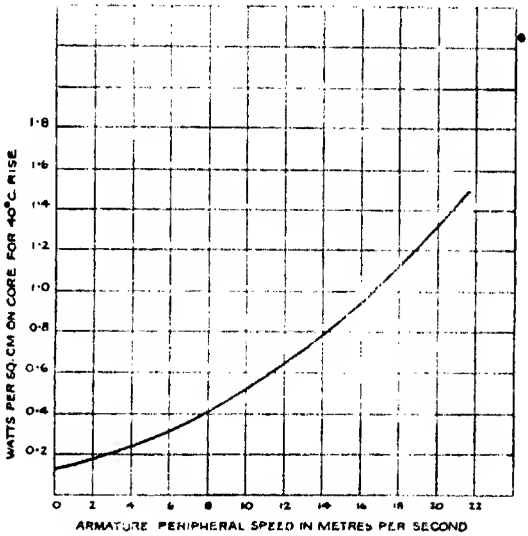


Fig. 39

the copper loss in the slots only and add this loss to the iron losses. The cooling surface is then taken as the external *core* surface only. The permissible watts per square centimetre of core surface varies with peripheral speed in the following manner, as shown by fig. 39. The losses in the field-coils are easily calculated from the resistances and currents.

The bearing loss $= 0.19 \times D_j \times l_j \times$ (peripheral speed of journal in metres per second)$^{1.5}$ watts,

where $D_j =$ diameter of journal in centimetres,

$l_j =$ length of bearing in centimetres.

The windage loss $= 0.17 \times 10^{-3} \times$ armature cylindrical surface in square centimetres $\times$ (peripheral speed of the armature in metres per second)^2 watts.

All the data for design have been given, and we will now proceed to

design a variable-voltage generator for the Ward Leonard equipment which has to work to the output diagram in Fig. 40. The root mean square current is calculated in the section on the motor, and it is there found that the kilowatts output of the generator is 65. The output of the generator

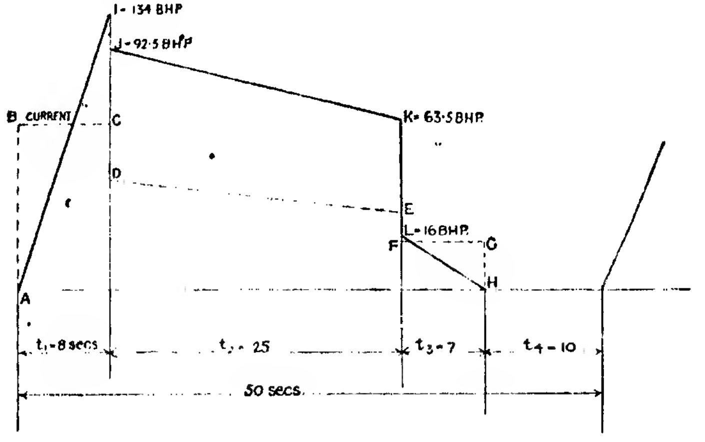


Fig. 40

is therefore 65 kilowatts, 220 volts, 1175 R.P.M. We will proceed to find the dimensions of the machine.

$$D_a^2 L_c \text{ in cubic centimetres} = \frac{\text{watts output} \times 1.08 \times 10^8}{B_g \times n \times \pi^2 \times q \times \lambda}.$$

Assuming a value for $B_g = 8000$ and $q = 300$ for interpoles (from curve),

$$\text{then } D_a^2 L_c = \frac{65000 \times 1.08 \times 10^8 \times 60}{8000 \times 1175 \times \pi^2 \times 300 \times 0.7}$$
$$= 21600 \text{ cubic centimetres.}$$

Four poles will be most suitable for this output, and, using square pole-shoes, we have

$$\frac{\pi D_a \lambda}{2p} = L_c \quad \therefore \frac{\pi D_a^3 \lambda}{2p} = 21600.$$

$$\therefore D_a^3 = \frac{4 \times 21600}{\pi \times 0.7} = 39300.$$

$$\therefore D_a = 34 \text{ centimetres and } L_c = 19 \text{ centimetres.}$$

We will take 34 centimetres and 20 centimetres.

$$\text{The area of the pole face} = \frac{\pi \times 34}{4} \times 0.7 \times 20 \text{ square centimetres}$$
$$= 376 \text{ square centimetres.}$$

$$\therefore \text{flux per pole} = 376 \times 8000 = 3 \text{ megalines.}$$

The leakage factor for the poles we will assume as 1.3.

$$\therefore \text{ flux in the poles } = 3.0 \times 10^6 \times 1.3$$
$$= 3.9 \times 10^6 \text{ lines.}$$

Assuming a density in the pole at full load of 15300 lines per square centimetre, the area of the pole = 255 square centimetres.

$$\text{The area of yoke } = \frac{3.9 \times 10^6}{2 \times 6350} = 305 \text{ square centimetres.}$$

$$\text{The area of core } = \frac{3.0 \times 10^6}{2 \times 15200} = 99 \text{ square centimetres.}$$

$$\text{The area of teeth per pole } = \frac{3.0 \times 10^6}{20200} = 148.5 \text{ square centimetres.}$$

It will be necessary to place a ventilating duct about every 7 centimetres, therefore there will be three ventilating ducts 1 centimetre wide. The net length of core iron will be therefore $(20 - 3) \times 0.91 = 15.5$ centimetres. The radial length of coil is 14.5 centimetres and the internal diameter of the yoke 69.3 centimetres. The width of pole = 13 centimetres and the diameter of poles = 34.6 centimetres. The internal diameter of core is 15.2 centimetres and the overhang of the armature coil at each end 13.0 centimetres. The length of air-gap for the main poles is 3.0 millimetres. The R.M.S. current is 294 amperes, therefore a two-circuit winding will be used.

$$\text{The coils} \times \text{turns per coil} = \frac{3000 \times 230 \times 2}{3.0 \times 10^6 \times 1175 \times 4} = 98.$$

We will use 99 coils, each having a single turn in 33 slots connected wave. The commutator diameter is 28 centimetres and the pitch of segments

$$= \frac{\pi \times 28}{99} \text{ centimetres} = 0.89 \text{ centimetre.}$$

The slot depth is 30 millimetres and width 12.3 millimetres. The depth of slot insulation + slack is 4.2 millimetres, the width is 1.8 millimetres, therefore space available for copper and tape in the depth = 25.8 millimetres and width 10.5 millimetres.

Net copper space depth = 25 millimetres,
" width = 9.3 "

(i.e. allowing tape one-eighth lap on conductors), therefore strip is 12.5 deep by 3 millimetres wide. The actual strip used was 12.4 millimetres × 2.36 millimetres, allowing extra slack in the slot.

$$\text{The resistance of the armature at } 15^\circ \text{ C.} = \frac{0.017 \times 99 \times 1.13}{4 \times 12.4 \times 2.36}$$
$$= 0.01625 \text{ ohm.}$$

1.13 metres = length of mean turn on armature. The iron losses at the given flux = 2065 watts and the I^2R loss in slots = 630 watts, therefore the total loss = 2695 watts, and the permissible loss at 20.9 metres per

second peripheral speed is $1.4 \times 2140 = 3000$ watts, so that the heating is all right. We require to determine the interpole dimensions and winding. The width of the interpole = 2.5 centimetres, i.e. twice the slot pitch, and its length = 20 centimetres. The reversal ratio is taken as 1.2.

$$\therefore \text{ ampere turns per interpole} = 1.2 \times \frac{99}{4} \times \frac{294}{2}$$

$$\text{and the turns per pole} = 1.2 \times \frac{99}{8}$$

$$= 15.$$

The section of conductor used is 17 millimetres $\times$ 1.0 millimetre, 9 strips in parallel, taped half lap. The axial length of interpole between bobbin cheeks is 127 millimetres and the depth of coil 26 millimetres. The length of copper per coil is 8.9 metres and the resistance cold is 0.00099. The resistance at 40° C. is 0.001188 and watts lost per coil = 112. The permissible loss = 128 for 40° C.

To determine the field winding the drop in the armature is 5.8 volts, the brush contact drop is 1.5 volts, and the interpole drop is 1.48 volts, therefore the total drop at full load is 8.78 volts and the full-load flux is 2.945 megalines. The ampere turns required to drive this flux through the magnetic circuit = 3800 + 25 per cent of the armature ampere turns per pole = 4755 total. At the peak load the current is 515 amperes and the total voltage drop is 13.59 volts. The corresponding flux is 3.01 megalines and the ampere turns per pole required = 5645. The axial length of the coil is 145 millimetres and the depth 27 millimetres. The length of mean turn is 0.805 metre.

$$\text{Resistance of shunt wire per kilometre at } 15° \text{ C.} = \frac{710 \times 55}{0.805 \times 5645}.$$

$$= 8.6 \text{ ohms}$$

The nearest shunt wire is 1.6 millimetres diameter bare and double cotton covered to 1.9. The space factor is 0.98.

$$\therefore \text{ turns per coil} = \frac{27 \times 145 \times 0.98}{1.9 \times 1.9}$$

$$\doteqdot 1062.$$

The length of copper in the coil = 856 metres
and resistance cold = 7.25 ohms,
resistance hot = 8.7 „
shunt current at normal load = 4.48 amperes,
„ „ peak load = 5.3 „
watts lost at normal load = 175,
permissible watts loss = 193.

The current per brush arm, using as many sets of brushes as there are poles, at R.M.S. current is $\frac{294}{2} = 147$ amperes. The current density at the peak load must not exceed 11.5 amperes per square centimetre,

and the current at the peak is 515 amperes, with the corresponding current of 257 amperes per brush arm.

$$\therefore \text{ the area of brushes per arm} = \frac{257}{11.5} \text{ square centimetres}$$
$$= 22.3 \text{ square centimetres.}$$

We will use 1.8 centimetres × 1.5 centimetres soft carbon brushes.

$$\therefore \text{ the number of brushes per arm required} = \frac{22.3}{1.8 \times 1.5}$$
$$= 8.$$

The brush friction loss in watts

$$= 9.81 \times 17.2 \times 32 \times 1.8 \times 1.5 \times 0.25 \times 0.3 = 1090 \text{ watts,}$$
$$\text{brush contact loss} = 1.5 \times 294 = 442 \quad ,,$$
$$\text{total commutator loss} = 1532 \quad ,,$$

permissible loss on commutator at 17.2 metres per second = 0.94 watt per square centimetre.

$$\therefore \text{ length of commutator} = \frac{1532}{\pi \times 28 \times 0.94} = 18.5 \text{ centimetres.}$$

Important Factors in Design.—

$$\text{The reactance voltage} = B\,m \times \text{R.P.S.} \times I_c \times L_c \times S^2 \times \frac{\text{poles}}{\text{paths}} \times 10^{-8},$$

where m = number of segments,
I_c = current in armature conductors,
L_c = length of core in centimetres,
τ = pole pitch,
S = turns per coil,
B = 38.4 for series and full-pitch lap windings
= 22.4 for short-pitch lap windings.
λ = pole arc pole pitch.

The average volts per segment

$$= 2\,S\,m\,(B_g\,\lambda\,\tau\,L_c) \times \text{R.P.S.} \times \frac{\text{poles}}{\text{paths}} \times \frac{\text{poles}}{m} \times 10^{-8}.$$

$$\therefore \frac{\text{average volts per segment}}{\text{reactance volts}} = \frac{2\,B_g\,\lambda\,\tau \times \text{poles}}{B\,S\,I_c}$$
$$= \frac{4\,B_g\,\lambda}{B\,q}.$$

This is a constant quantity, and for the same reactance voltage in each case the average voltage per segment is a fixed quantity. Therefore the number of segments must increase in the same proportion as the voltage increases. A limit is soon set by the extra expense entailed, and the mechanical limit to the width for the maximum number of segments. Thus a definite limit is set to the voltage for which a machine can be wound with a given frame. There is also a decided limitation due to large currents, for the commutator must be greatly extended, and this demands special design. We will now proceed to show that there is a

maximum limit to the output as fixed by commutation for a machine of given diameter and running at a given maximum peripheral speed. For such a case we will use the best winding for commutation, viz. the single-turn short-pitch multiple winding.

The reactance voltage for such a winding

$$= 22\,m \times \text{R.P.S.} \times I_c L_c \times 10^{-8} \text{ (all dimensions in cms.).}$$

Now $I_c = \dfrac{I_a}{\text{paths}}$ where I_a = total armature current,

$$\therefore\ I_a = \frac{\text{Reactance voltage} \times \text{paths}}{22 \times m \times \text{R.P.S.} \times L_c \times 10^{-8}}.$$

$$\text{E.M.F.} = Z\,N_a\,\text{R.P.S.} \times \frac{\text{poles}}{\text{paths}} \times 10^{-8}$$

$$= 2\,m\,B_g \lambda \tau L_c \times \text{R.P.S.} \times 10^{-8},$$

$$\therefore\ E\,I_a = 2\,m\,B_g \lambda \tau L_c \times \text{R.P.S.} \times 10^{-8} \times \frac{\text{R.V.} \times \text{paths}}{22 \times m \times \text{R.P.S.} \times L_c \times 10^{-8}}$$

$$= \frac{\text{R.V.} \times B_g \times \lambda \times D_a \times \pi}{11},$$

$$\therefore\ D_a = \frac{\text{watts} \times 3.5}{\text{R.V.} \times B_g \times \lambda}.$$

For non-interpole machines, R.V. should not exceed 2 volts,

$$\therefore\ D_a \text{ cm.} = \frac{\text{watts} \times 1.75}{B_g \times \lambda} = \frac{\text{watts} \times 2.5}{B_g}.$$

This gives the minimum diameter for the given output. From a curve connecting diameter and B_g, and assuming the maximum peripheral speed, it is possible to plot a curve showing the maximum output obtainable with a given diameter. The peripheral velocity of direct-current armatures is seldom greater than 30 metres per second, since the cost for special design increases rapidly above this speed. Again, it may be shown that, with interpole machines, the reactance voltage should not exceed 15, and there is also a definite maximum output for each diameter as far as commutation is concerned.

CHAPTER V

CHARACTERISTIC CURVES

The no-load saturation curve is obtained experimentally by running the machine on open circuit at normal speed and separately exciting the field coils. It is necessary to connect a rheostat in the field of the machine to vary the exciting current. Corresponding values of field current and terminal voltage are read simultaneously, and the curve connecting flux per pole, and ampere turns per pole on the field coil plotted. It is essential to work well over the knee of the magnetization curve in order to secure

stability. Any increase in speed of a shunt dynamo increases its terminal P.D. and also the shunt current. Above the knee of the curve the flux is increased very little, and so tends to better regulation. Further, the difference in ampere turns when hot and cold produces relatively small changes in the flux. If the open-circuit curve be plotted for a shunt dynamo, with E.M.F.s as ordinates and exciting currents as abscissæ, and any point be taken upon the curve and joined to the origin, then the resistance of the shunt circuit is given by the tangent of the angle this line makes with the abscissa axis. When this line coincides with the straight part of the curve the value of the tangent of the angle represents the maximum resistance which the shunt circuit can have to magnetize the field-magnets. This value of the resistance is known as the "critical" resistance.

External Characteristics.—An "external" characteristic is the curve connecting terminal P.D. and load current, the values of P.D. being plotted as ordinates and currents as abscissæ. The curve is obtained experimentally by running the machine at constant speed and varying the external resistance, and observing corresponding values of current and terminal volts.

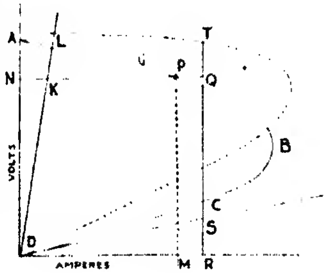


FIG. 41

In the case of a series dynamo, since the load current is also the exciting current, the terminal volts increase rapidly at first, reach a maximum, and then fall gradually. This fall of P.D. is due to the voltage drop in the armature and series coils, and to the demagnetizing effect of the armature. When the magnets are approaching saturation, any increase in external current is not followed by a corresponding increase in flux, and, further, the demagnetizing effect of the armature is proportional to the current, so that a fall of terminal P.D. results. Increase in the leakage coefficient with increasing current also contributes its share to the drop of voltage.

External Characteristic of Shunt Dynamo.—In a shunt dynamo the field coils are connected across the armature terminals, and hence the exciting current varies with the P.D. across the terminals. The external characteristic has the form shown in fig. 41. The fall of P.D. is due to the drop of pressure in the armature and brush contacts, to the demagnetizing effect of the armature. The decrease of P.D. due to these causes lowers the exciting current with a corresponding fall of P.D. This fall of P.D. goes on till the bend of the curve is reached. At this point the demagnetizing ampere turns of the armature preponderate and the voltage decreases to zero. The curve thus bends back on itself, and cuts the current axis a little to the right of the origin. Thus there is a maximum current beyond which it is impossible to go. This maximum current, however, is far beyond the thermal limits of the machine, and is about twice the full-load current. For every value of the current there are two

values of terminal P.D., and whichever is obtained will depend upon the resistance of the external circuit. The slope of the bottom portion of the curve is defined as the "critical" resistance of the external circuit within which the magnetic flux is unstable and the machine fails to excite. A shunt dynamo, short-circuited, therefore, is fairly safe from burning out, for, when the maximum current is reached, the terminal P.D. falls rapidly and the current falls to zero.

Compounded Dynamos.—Machines may be compounded (*a*) level, (*b*) over, and (*c*) differentially. The compounding effect is obtained by using series coils excited by the armature current, and providing a number of ampere turns to compensate for resistance voltage drop and armature reaction. In "level" compounded machines the series coils are so proportioned to maintain the voltage constant at the terminals. In over-compounded machines the series coils provide a M.M.F. greater than that required to compensate for voltage drops and armature reaction, and the voltage increases with the load. When the load is situated at the end of a long transmission line, constant voltage there is often required at all loads. In such a case the generator is over-compounded to compensate for the drop in the transmission line. Traction generators are generally over-compounded, giving 500 volts at no load, and 550 volts at full load. Differentially compounded generators give decreasing voltage with increase of load, and have the series coils opposing the shunt. They are used for cinematograph work, &c.

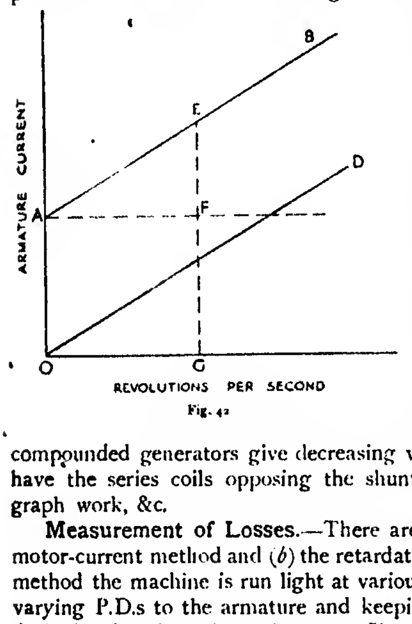


Fig. 42

Measurement of Losses.—There are two principal methods: (*a*) the motor-current method and (*b*) the retardation method. In the motor-current method the machine is run light at various speeds as a motor, by applying varying P.D.s to the armature and keeping the excitation constant at the desired value throughout the test. Since the machine is on no load, the back E.M.F. is almost equal to the applied P.D. and varies directly as the speed, the field being constant. Since the flux is constant, the torque will vary with the armature current. This torque exerted is that required to overcome friction and windage, hysteresis and eddy-current losses. The hysteresis loss is at any speed $n = a\,n$, where n = revolutions per second, therefore the hysteresis torque is a constant quantity proportional to "a", and independent of the speed. The component of the no-load current required to produce this torque is also constant. The friction torque is also nearly constant, so that the friction and hysteresis current is a constant quantity and = OA in fig. 42. The eddy-current loss = $B\,n^2$, and there-

fore the eddy-current torque is Bn, and the torque varies as the speed. The eddy-current component therefore varies directly as the speed, and when plotted in relation to it gives the straight line O D. If we plot, therefore, the total current input to the armature as ordinates, and revolutions per second as abscissæ, we get the straight line A B.

$$\text{The hysteresis and friction loss} = \text{G F} \times E_a \text{ watts,}$$
$$\text{and the rate of loss by eddies} = \text{E F} \times E_a,$$

where E_a is the back E.M.F. corresponding to the speed O G. The objection to the method lies in the fact that the friction and windage loss is not strictly proportional to the speed. It is essential to run the machine for some considerable time to allow a steady temperature of the bearings to be reached, and also to set the brushes correctly.

If two machines be connected rigidly together, and one whose losses are known at various speeds be used to drive the other, then the friction and windage loss is determined from the excess of power input to the motor above that required to run the motor alone at the given speed, the field of the generator not being excited. On exciting the generator the excess power gives the losses due to friction, windage, hysteresis, and eddies.

The second test is that due to Routin. The armature is first run up to speed and the driving power cut off. It is then allowed to come to rest, and its speed is measured every few seconds. If this speed be plotted in relation to time, then the tangent gives the angular accleration at the given time.

Let k = radius of gyration of the armature in centimetres,
m = mass of armature in grammes,
ω = angular velocity in radians per second.

At any instant of time during the retardation the rate of change of angular momentum is equal to the retarding torque,

$$\text{or torque} = -mk^2\frac{d\omega}{dt} \text{ dyne centimetres.}$$

The rate at which kinetic energy is liberated is

$$= -mk^2\frac{d\omega}{dt}\omega \text{ ergs per second}$$

$$= -mk^2\frac{d\omega}{dt}\omega \times 10^{-7} \text{ watts.}$$

If the retardation curves be plotted with the field unexcited and without the brushes, and then with the field excited to various strengths, and then with the brushes down, the different losses can be separated and tabulated. To determine the bearing friction and windage the retardation curve is obtained by coupling the machine to a small motor and running up the machine above its full speed. The belt is then thrown off. A low-reading voltmeter is connected across a pair of small brushes on the com-

mutator and the voltage read every three seconds or so. The speed of the machine is proportional to the voltage due to residual magnetism. The machine is then excited and run up to full speed as a motor. The armature circuit is broken and the readings of voltage again taken as the speed falls.

To separate the losses due to hysteresis and eddy currents the watts due to the same are divided by the corresponding voltage, and the current so obtained is plotted with voltage as abscissæ. The intersection on the ordinate axis measures the current required to overcome the torque due to hysteresis, and the line drawn parallel to the line of total current and passing through the origin is the current required to overcome the torque due to eddy currents. The moment of inertia of the armature may be obtained by suspending it bifilarly and measuring the time of oscillation.

Let $l' =$ length of the two parallel wires,
$a =$ distance of each wire from the axis of suspension,

$$\text{then the periodic time} = 2\pi\sqrt{\frac{l k^2}{g a^2}},$$

from which k^2 can be obtained.

If the armature be large it may be suspended on knife-edges, and set swinging with a small weight attached at a radius "r" from the axis of the shaft.

$$\text{Then } k^2 = \frac{m_1}{m_2}\left\{g r \frac{t^2}{4\pi^2} - (r - r_1)^2\right\} - r_1^2,$$

where m_1 = mass of the attached weight,
m_2 = mass of armature,
t = time of a complete oscillation,
r = radius of small mass,
r_1 = radius of the shaft.

Efficiency Tests.—The first test to be described is that due to Swinburne. Here the machine is run light as a motor with a P.D. across the brushes equal to the full-load E.M.F., the exciting current being adjusted to give the normal speed. The power W_0 absorbed by the machine represents the hysteresis, eddy current, and shunt losses and friction losses and armature losses. These losses, less the no-load armature copper loss, are assumed to remain constant at all loads. The losses being known,

$$\text{the efficiency} = \frac{VI}{VI + W_0 + W_2},$$

where I = full-load current.
W_0 = no-load losses.
W_2 = variable copper losses.
VI = output as generator.
V = potential difference.

The iron losses do not remain constant on load, and hence the method is not reliable. A very convenient method of determining the efficiency of

a generator is to couple it to a standardized motor. If η_1 is the efficiency of the motor when absorbing a power W_1, the brake H.P. is $\eta_1 W_1$, and the efficiency of the generator is given by $\frac{W_2}{\eta_1 W_1}$, where W_2 = full-load output of the generator.

With large machines the expenditure of energy necessary to carry out such a test is expensive, and very often impossible. When two exactly similar machines are available, the Hopkinson effitiency test is most usual. The two machines are coupled rigidly together, so that one acts as generator and the other as motor. The generator supplies part of the power to drive the motor, and the losses are supplied from some external source. The two machines are driven at their normal speed, and at approximately their normal voltage; the field of the one is slightly weakened by a rheostat on its field circuit, so that its internal E.M.F. is less than the terminal E.M.F. of the other machine. One machine then acts as motor, driving the other as generator, and, by weakening the field of the motor sufficiently, full-load current is caused to flow through each. In the original Hopkinson test the power was supplied mechanically.

If W = power supplied mechanically,
R_d = resistance of dynamo armature,
R_m = resistance of motor armature,

Then (if the fields are separately excited) $W - (I_d^2 R_d + I_m^2 R_m)$ = losses due to friction, hysteresis, and eddy currents on the two armatures. The field of one is slightly stronger than that of the other, but the error involved in assuming the losses equally divided between the two machines is very small if the machines are above 30 kilowatts capacity, and having efficiencies over 90°. The efficiency of the dynamo then equals

$$\frac{V_1 I_D}{V_1 I_d + I_d^2 R_d + I_{fd}^2 R_{fd} + \frac{W - (I_d^2 R_d + I_m^2 R_m)}{2}},$$

where V_1 = dynamo-terminal P.D,
and I_{fd} = dynamo-field current.

If the dynamo supplies the magnetizing current of both machines, then, if they are shunt wound, the efficiency of the dynamo

$$= \frac{V_1 I_1}{V_1 I_1 + I_d^2 R_d + \frac{M}{2} + V_1 I_{fd}},$$

where $M = W - I_d^2 R_d - I_m^2 R_m - V_1 (I_{fd} + I_{fm})$.

I_1 = total current supplied to the motor by the generator.
I_d = dynamo-armature current.
I_m = motor-armature current.
I_{fd} = dynamo-field current.
I_{fm} = motor-field current.

The losses are frequently supplied by a dynamo electrically, or from a battery or other source. This dynamo is coupled either in series or in parallel with the two machines to be tested. This auxiliary dynamo need only be of small size. In the series arrangement it must be able to take the full-load current of both machines, and its voltage

$$= \frac{\text{total losses on both}}{\text{current in the armatures}}.$$

In the parallel arrangement it must have an E.M.F. equal to the full E.M.F. of the machine to be tested.

Fig. 43 represents the Kapp modification of the Hopkinson test, the

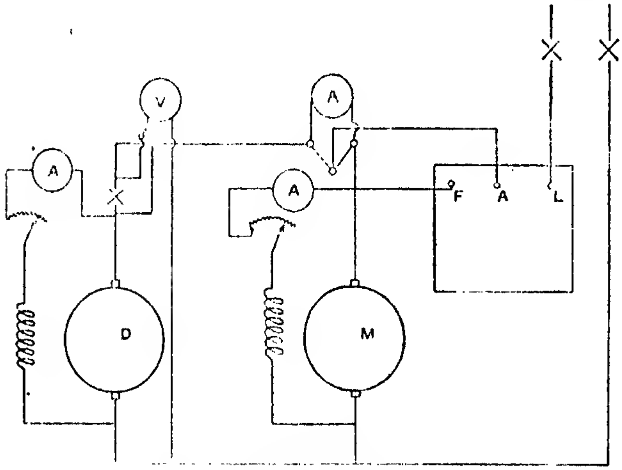


Fig. 43.—Hopkinson Efficiency Test

losses being supplied from the mains. The order of procedure is as follows:—

(*a*) Run the set at normal speed with the motor connected to the mains.

(*b*) Close the shunt field switch of the generator, and regulate the excitation until the generator voltage equals that of the bus-bars, and then close the circuit-breaker which puts the incoming machine in parallel with the bus-bars.

(*c*) Strengthen the field of the generator until the full-load output is given, keeping the speed of the set constant by regulating the motor field.

Let W_G = generator output = circulating amperes × line volts,
W_E = input from external circuit,
W_M = motor input = $W_G + W_E$,

then the combined efficiency $= \dfrac{W_G}{W_G + W_E}$.

If the excitation is approximately the same in each machine, then the efficiency of either

$$\eta = \sqrt{\frac{W_G}{W_G + W_M}}.$$

It will be noticed that a special switch S is used, which throws the ammeter either into the motor circuit for measuring the motor current, or into the generator circuit for measuring the generator current. The efficiency of the set

$$= \frac{\text{generator amperes} \times \text{volts}}{(\text{motor-armature amperes} + \text{motor-field amperes})\ \text{volts}}.$$

Coupling of Dynamos.—Electrical energy is frequently transmitted over considerable distances at high pressures by direct current. The direct-current dynamo is difficult to design for high pressures owing to commutation troubles, and a maximum limit might be set at 4000 volts. By coupling such machines in series total line pressures of 27,000 volts and 57,600 have been used in France by M. Thury for transmission purposes. All that is necessary is to join the positive terminal of one to the negative terminal of the other. Much more frequently, however, machines are connected in parallel. In central electrical stations and in other large plants the energy is supplied by a number of machines feeding into a common pair of bus-bars. The coupling of shunt machines in parallel is easily effected without disturbing those already working. The incoming machine is excited and run up to speed, and its voltage is adjusted to equality with that of the bus-bars. The positive terminal of the machine is connected to the positive bus-bar and the negative terminal to the negative bus-bar. The distribution of the load depends essentially on the armature internal E.M.F.s and the resistances of the armatures.

The terminal P.D.s being equal to that of the bus-bars, it follows that the distribution of current in each is such that the resistance drops in each when subtracted from the internal E.M.F.s give equality of P.D.s. It will at once be seen that the regulation of load is effected by altering the internal E.M.F. of any machine. This can be brought about in two ways: (*a*) By changing the speed or (*b*) by altering the shunt excitation. Such shunt machines when working in parallel have a natural tendency to equalize the load on each. When one machine is slightly overloaded, due to an increase of speed, and its terminal P.D. exceeds the internal E.M.F. of another machine, then a reverse current is driven through the armature of weak E.M.F., and it is driven as a motor in the same direction as before with consequent increase of internal E.M.F. This increase of E.M.F. causes the other machine to take some of the load. The governor of the prime mover acts also in such a manner to equalize the load. Automatic cut-outs are connected between each dynamo and the bus-bars, which cuts a dynamo out of circuit before a reverse current passes through it. Compound dynamos for traction loads are frequently connected in parallel. To prevent a reverse current passing through the series coils, and demagnetizing the fields or reversing the polarity, it is essential to connect the

inner ends of the series coils by a connecting lead of very low resistance. The condition for running compound machines in parallel of different outputs is that the drop of volts over the series windings and leads should be the same when the machines are carrying their correct share of the current. In other words, the external characteristics at the bus-bars must be alike.

CHAPTER VI

EXAMPLES OF MODERN GENERATORS

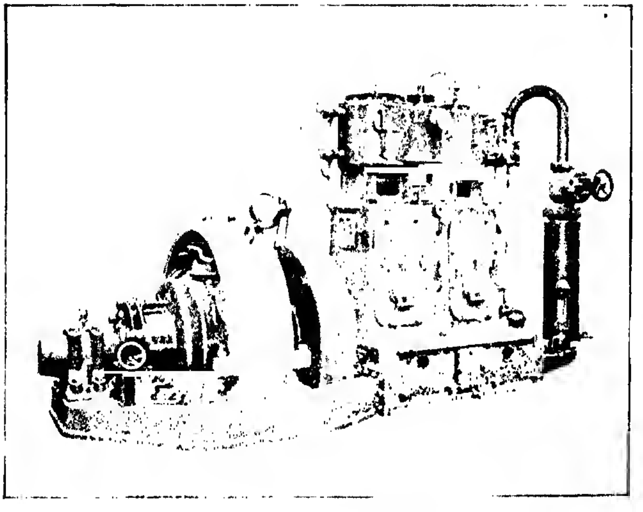
An example of the proportioning of the length and diameter of multipolar machines is illustrated in the photograph in fig. 44. It is shown coupled to a Scott & Mountain enclosed high-speed engine, with forced lubrication, running at 325 revolutions per minute.

A few particulars of this machine are given below, and it will be seen on reference to these that the length of the armature over core discs, gap-area, magnetic flux per pole, radial depth of armature, are exactly the same as those of the crank-shaft generator just described.

Number of poles	6
Kilowatts	200
Revolutions per minute	325
Terminal volts	500 to 600
Amperes	335
Armature—	
Diameter over all	39 inches
Internal diameter	22 ,,
Length over iron	13 ,,
Effective length of magnetic iron	10.8 ,,
Number of ventilating-ducts	2
Width of each duct	0.5 inch
Magnet Core (cast steel)—	
Length of pole-face	13 inches
Length of pole arc	14.5 ,,
Pole arc ÷ pitch	0.7
Radial length	12.25 inches
Bore of field	39.625 ,,
Depth of air-gap	0.3125 inch
Yoke (cast steel)—	
Outside diameter	72 inches
Commutator—	
Diameter	28 inches
Rubbing length	6 ,,
Number of segments	480

Brushes

Number of sets	6
Number in one set ...	3
Contact size of each brush ...	1.05 square inches
Armature no-load voltage	500
Number of inductors	960
Inductors per slot	6
Style of winding	6-circuit single or lap
Mean length of one inductor	1.08 yards
Resistance between brushes ...	0.0025 ohm
Average voltage between commutator segments ...	7.5 volts
Armature ampere turns	8040
Ampere turns to overcome armature reaction	6370
Reactance voltage (full load)	2.8



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Bobbin Windings

Ohms per bobbin	34
Size of wire	0.108 inch
Number of shunt turns per bobbin	3104
Section of series wire	0.33 square inch
Number of series turns per bobbin	11
Ampere turns per circuit (shunt)	18,400
Ampere turns per circuit (series)	7716
Total ampere turns per circuit (full load) ...	26,116

The number of poles is 6; the diameter of the armature, 39 inches; and length, 13 inches. If tests and figures are available relating to the performance of the 39-inch machine, the probable behaviour of a 78-inch machine can be predicted with considerable accuracy. In the same way

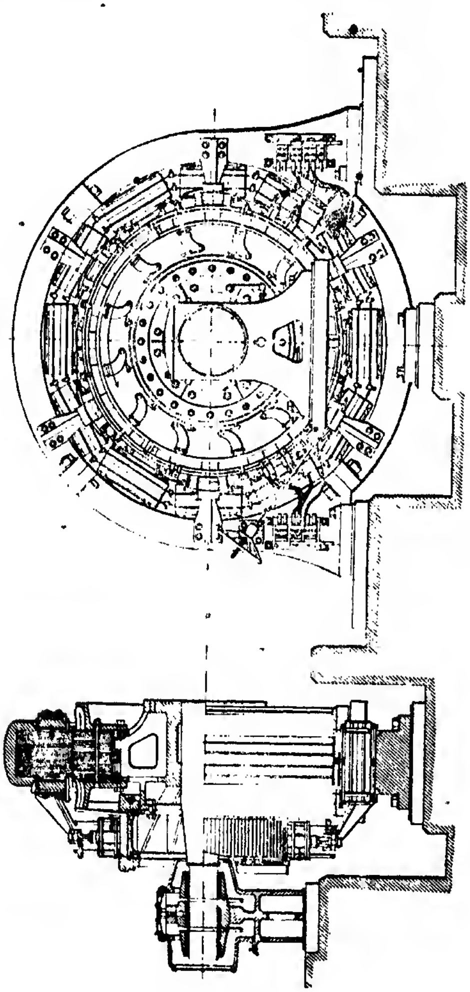


Fig 45 —840-Kilowatt Crank-shaft Generator

the behaviour of a machine having, say, 20 poles and an armature 130 inches diameter and 13 inches long could be predicted from the tests of the 39-inch machine.

Fig. 45 is a drawing of a machine built by Messrs. Crompton & Co., of Chelmsford, for power work. The design is exceedingly massive and

substantial from a mechanical point of view. Messrs. Crompton & Co. have kindly furnished the following particulars:—

Output	840 kilowatts
Volts	600
Amperes	1400
Revolutions per minute	230
Number of poles	12
Overload capacity	1900 amperes for two hours

Armature—

Diameter	90 inches
Length over core discs	18 „
Net effective length of iron	15 „
Number of slots	250
Inductors per slot	6
Length of air-gap (iron to iron)	$\frac{3}{8}$ inch
Total number of inductors	1500
Number of commutator segments	750
Diameter of commutator	58 inches

The armature is parallel-coupled and fitted with equalizing rings. The flux density at the root of the teeth is 100,000 C.G.S. lines per square inch, and in the iron of the armature body about 32,300 lines per square inch. These values are properly kept low in order to reduce the core loss of the machine, as the speed, 230 revolutions per minute, is high for a machine of the above diameter, and gives a frequency of alternation of 23 per second.

The magnets are made up of laminated pole cores cast into the cast-iron yoke, and fitted with cast-steel pole-shoes. The winding of the bobbins is compound, and it will be noticed that the series coils are wound outside, but quite independent of the shunt coils, with a space left for ventilation between.

The densities in the various portions of the magnetic circuit are as follows:—

Flux density in the air-gap	45,000	lines per sq. inch	
„ „ wrought-iron magnet cores	107,000	„	„
„ „ cast-iron yoke	37,000	„	„
„ „ teeth	100,000	„	„
„ „ armature body	32,300	„	„

The electrical efficiency of the above machine is as high as 98.5 per cent, and the guaranteed full-load commercial efficiency is 96.5 per cent. The guaranteed temperature rise above that of the atmosphere is 70° F. after thirty-six hours' run at full load. The above machine is, of course, fitted with a slotted drum-wound armature and carbon brushes.

The British Thomson-Houston Company have designed and erected some very large direct-current generators. Fig. 46 is a photograph of the field frame of a generator designed for an output of 2700 kilowatts at 75 revolutions. The following are some particulars of this machine, kindly supplied by the British Thomson-Houston Company, Limited, of Rugby.

The generator is an M.P. 36-pole machine, giving an output of 2700 kilowatts at 575 volts, full load, at a speed of 75 revolutions per minute. The diameter of the armature is 21 feet 8 inches, and of the commutator 15 feet 7 inches. The outside diameter of the frame is 31 feet, the total

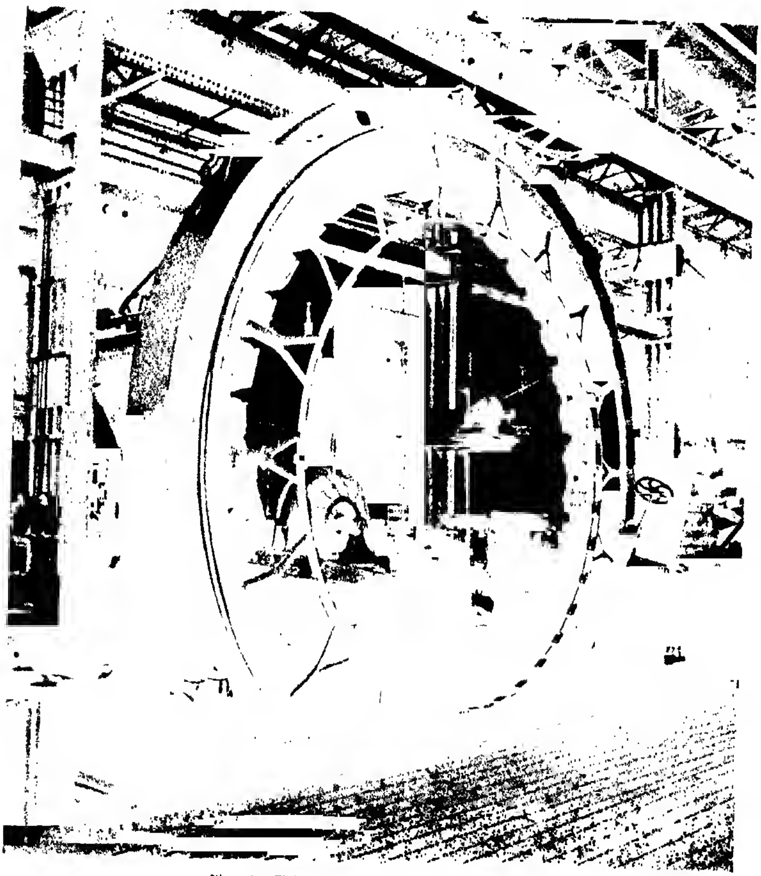


Fig. 46. – Field Frame of 2700-Kilowatt Generator

weight of the armature is 160,000 lbs., and the total weight of the entire machine is 310,000 lbs.

Magnetic Frame and Poles.—The magnet frame is of cast iron, and divided horizontally, so that the upper half can be lifted to permit inspection of or repairs to either armatures or fields. On account of the large size of the frame, the lower and upper halves were each subdivided into three sections to facilite transportation.

The magnet cores are 36 in number, and of cast steel, the pole-pieces

being laminated and securely fastened to the same; they are fastened to the magnet yoke by bolts which can easily be removed, so that inspection or repair of any particular pole-piece with its field winding can be readily made. The pole-faces are composed of laminations of two lengths, arranged alternately, so as to produce the graduated field effect given by a chamfered pole-piece. In the old generator this effect was produced by chamfering the solid pole-face.

The field coils are wound on spools, having a double sheet-iron body and malleable-iron flanges, the shunt winding being next the magnet frame and the series winding below it.

Armature.—The armature spider is of cast iron, and so designed that shrinkage strains are avoided. The armature core is built up of laminations, japanned before being assembled, to prevent eddy currents. These are secured to the spider by dovetail projections, and are held in place by end flanges of cast iron, which also serve to support the end windings. Space bolts are inserted at equal intervals between laminations to provide ventilation ducts. None of the bolts entering into the construction of the armature pass through the laminations. The armature spider is so arranged that it produces a fan effect, forcing the air through the ventilating-space between the laminations and into the windings, thus reducing heating of the generator. The armature is multiple-circuit drum-wound, and provided with equalizer rings, connected to points of equal potential at the back of the winding. The rings are secured with insulation pieces attached to the arms of the spider on the back of the armature, and are readily accessible for inspection. They tend to compensate for differences in the magnetic strength of the poles, due either to difference in material or length of air-gap, which effect tends to improve the commutation of the generator. Armature coils are made in halves, back connection being made by soldering two ends together. This form of winding has the advantage that, in case it is necessary to remove one-half of the coil for repairs or inspection, the other half may be left in place without disturbing the adjacent coils.

Commutator.—The commutator is supported on an extension of the armature spider. The commutator bars are of hard-drawn copper, and made with a dovetail on the lower edge to secure them in place.

The side mica used in the construction of the commutator is of soft quality and as thin as the requirements for insulation will permit, so that even wear of mica and copper is secured

Brush-holders and Brushes.—The brush-holders are so constructed that the tension of any brush can be readily adjusted, or the brush removed, while the machine is in operation. The brush is inclined slightly forward of a radial position, the angle in reference to the commutator being such as to largely eliminate the friction of the brushes in the guides, causing it to better follow any inequalities due to the movements of the commutator.

Insulation.—The insulation between the field coils and magnet frame, and also between armature windings and the armature core, is tested at 4000 volts alternating current for 10 seconds, or 2000 volts for a period of 60 seconds.

The temperature of the generator is guaranteed not to rise more than

63° F. after a run of twenty-four hours at full rated load. The rise will not be more than 99° F. with an overload of 50 per cent for two hours, following a run of twenty-four hours at rated load.

The generator is further guaranteed to carry an overload of 50 per cent at rated voltage for two hours, and 100 per cent momentarily, without movement of the brushes, without injurious sparking.

The generator is compounded for 525 volts no load and 575 volts full load, full load being 4695 amperes.

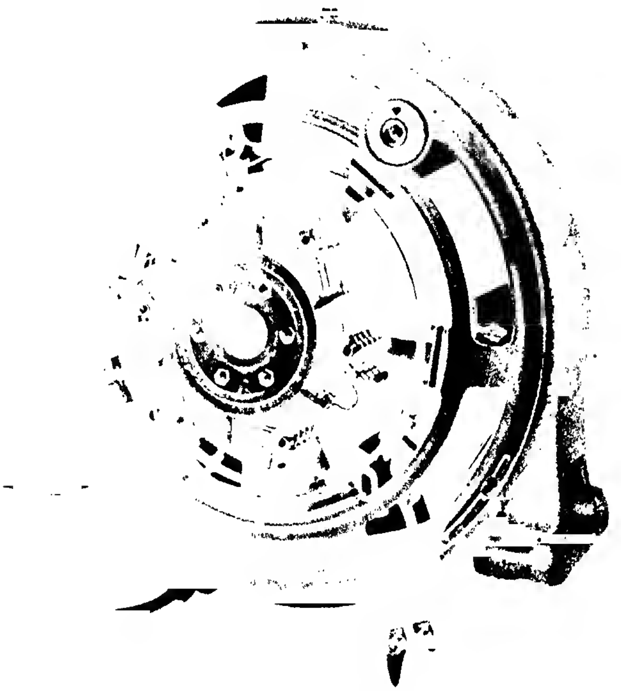


Fig. 47. Generator for Direct Coupling to Engine

Fig. 47 shows another form of generator, manufactured by the above company for direct coupling to engines. These machines have iron-clad armatures of the slotted drum-wound type. The field frame is cast from specially selected soft iron of the highest permeability, while the pole-pieces are of soft steel and are solid. The bobbins are of the round pattern. The armature windings, which are separately insulated, consist of straight copper bars requiring but two joints for each turn, and repairs can easily be made. The conductors are held in the slots by wood wedges, rendering the use of binding-bands on the core unnecessary. The commutator sleeve, which is of cast iron and is pressed on the armature spider, is independent of the shaft.

The following is the standard basis of rating of the above generators. The temperature of no part of the armature or field coils as measured by the thermometer after a run of twenty-four hours at full rated load with normal conditions of ventilation will rise more than 60° F. above that of the surrounding air. The guaranteed rise of the commutator is 72° F.

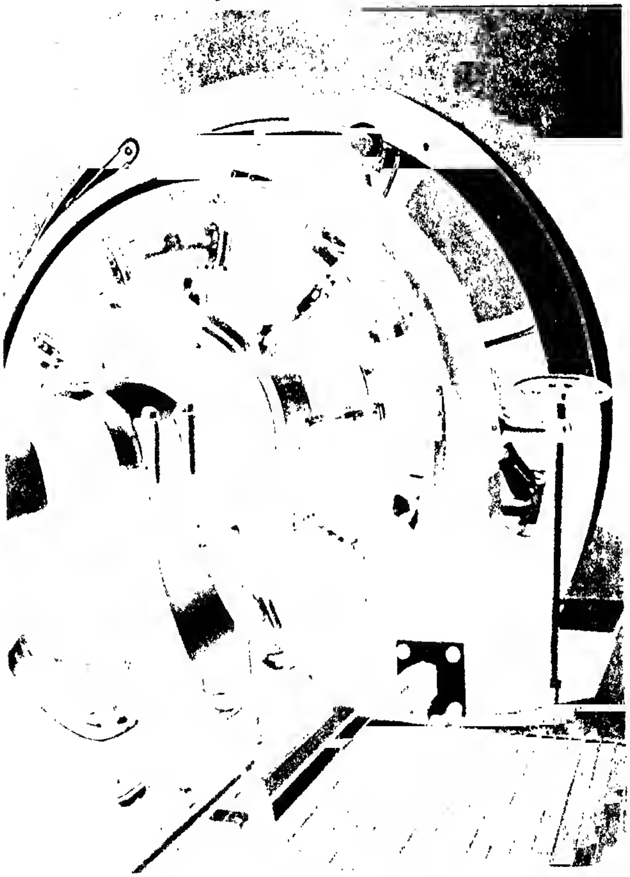


Fig. 46. 400-Kilowatt Crank-shaft Generator

They, however, introduce a proviso that the temperature of the air must not exceed 77° F., and in the event of this temperature exceeding 77° F. the above rise in temperature is to be corrected 0.28 per cent for each degree Fahrenheit that the temperature of the surrounding air differs from 77° F. With the load then increased 50 per cent above the rated amperes and at the rated volts for two hours, the temperature of no part of the generator

will be more than 99° F. above the atmosphere. These generators will carry an overload in current of 100 per cent momentarily without injurious sparking.

Fig. 48 is a photograph of a 400-kilowatt crank-shaft generator, showing the general type of machine built by The British Westinghouse Company. Particulars and data of the machine illustrated are not available, as the company is averse to publishing such data. However, the general design of the generators built by this company consists of a circular yoke of cast iron carrying inwardly-projecting poles of laminated soft steel. The field castings are divided vertically and set upon a guide-plate. This vertical division of the field affords excellent facility for inspection or removal of the armature or field coils without the necessity of removing the armature from its bearings. These machines are generally compound wound, and are over-compounded to raise the pressure at full load in accordance with the best standard practice. The shunt and series coils are removable at will. Another feature of these machines is that the armature core is built up upon a cast-iron spider which also has the commutator sleeve cast solid with it. Larger machines have retaining-wedges of hard fibre driven into the slots as in the case of some of the other machines described in this paper. Carbon brushes are, of course, used with all these machines. The brush-gear is carried in the approved method, i.e. from brackets projecting from a ring concentric with and supported by the field yoke.

4. Alternating-current Generators

CHAPTER I

GENERAL PRINCIPLES

With exception of "homopolar" dynamos all electric generators in practice are primarily generators of alternating current. For the purpose of obtaining direct current a commutator is required to convert the E.M.F. and current from alternating to direct. The presence of a commutator in direct-current machines makes it almost essential that the armature and commutator should be the revolving members, and the field-magnets and brushes remain stationary, so that the latter may receive attention whilst the machine is in motion.

With an alternator this restriction does not exist, and either the magnets or armature may be made to revolve. There are some obvious advantages in making the armature stationary. The insulation of the coils is not subject to mechanical stresses due to centrifugal force, and the current is led away from the armature through stationary conductors. Slip-rings are of course required for conveying the current to the field-magnet coils, but the current is a small one and of low voltage. The arrangement of having the armature windings stationary enables them to be easily insulated for generating at high voltage, and this, together with the easy transformation of voltage, forms the chief reason why alternating currents are used for high-tension distribution in preference to direct.

The E.M.F. of an alternator is produced by the relative motion between the magnets and the armature windings, and depends for its amount on the magnetic flux per pole of the magnets, the number of turns in the armature windings, and the relative speed of the poles to the latter. As a basis for calculation we shall suppose a simple form of alternator to consist of a rectangular coil consisting of one turn, A, fig. 1, rotated at a uniform speed in a uniform magnetic field whose direction is shown by the arrow-heads. As the coil rotates, the flux which passes through it varies from a maximum N when the coil is in the horizontal position to 0 when the coil is in the vertical position. If θ is the angle by which the coil has moved from the horizontal position, then the amount of flux enclosed by the coil is N cos θ. If a curve of flux passing through the coil be plotted with angular position of the coil as abscissæ, this curve will follow a cosine law (fig. 2). The abscissæ may represent either angular position of the

coil or time in seconds (to another scale), since the rotation of the coil is at a uniform rate.

When the flux through the coil is maximum, the E.M.F. acting in the coil is 0, since for the moment the coil sides are moving along the lines without cutting them.

When the coil has moved through the angle θ the flux through it has decreased from N to $N \cos \theta$. The coil sides are now cutting the lines of force, and an E.M.F. is set up in the coil whose direction is into the paper, indicated by the cross on the right side of the coil, the dot on the other side serving to indicate that the E.M.F. is there out of the paper. This E.M.F. becomes a maximum when the coil is in the vertical position, the flux through it being 0, after which it dies away until the coil again

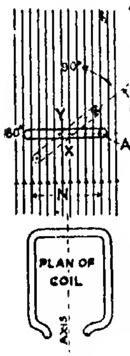


Fig 1

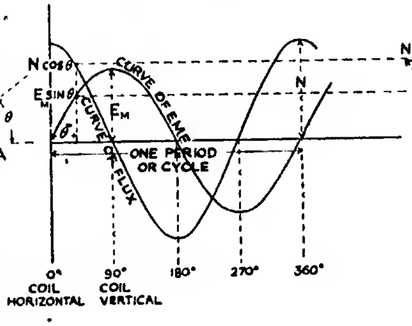


Fig 2

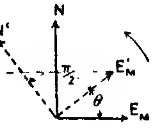


Fig 3

becomes horizontal, when it is again zero. On further rotation of the coil it again increases, but in the opposite sense, and falls to zero. Actually the E.M.F. varies according to a sine law, and the curve of E.M.F. is shown in fig. 2, the maximum value being marked E_M. We have chosen to represent the E.M.F. curve above the axis of abscissæ, or in a positive direction during the period that the flux is changing from a positive to a negative maximum. This is because the direction of E.M.F. is such as to try by the current it would produce to maintain the flux through the coil in a direction from side X to side Y (fig. 1), i.e. in the initial direction, which is positive. In this way the direction of E.M.F. and flux are given some definite relationship to one another.

The mathematical expression for the instantaneous value of the E.M.F. is $E_M \sin \theta$, and of the flux $N \cos \theta$ or $N \sin \left(\theta + \frac{\pi}{2}\right)$.

A more convenient way of representing quantities which vary according to a sine law, than by means of complete sine curves, is to indicate them by the rotating vectors from which the curves are developed.

In fig. 3 such vectors are shown, N for the flux curve and E_M for the E.M.F. These rotate anti-clockwise at the same speed as the coil, and

their horizontal projections represent the instantaneous values of flux and E.M.F. respectively. The position of the vectors after the coil has moved through angle θ is shown by the ones dotted and their projections shown on the curves of flux and E.M.F. The angle between the vectors is $\frac{\pi}{2}$ radians or 90°. Of course all alternators do not produce E.M.F.s which vary in a sine law, nor does it follow that an E.M.F. varying as a sine will always produce a current varying in the same way. In all ordinary cases, however, it is assumed that E.M.F. and current vary as a sine, partly to simplify mathematical problems dealing with the subject

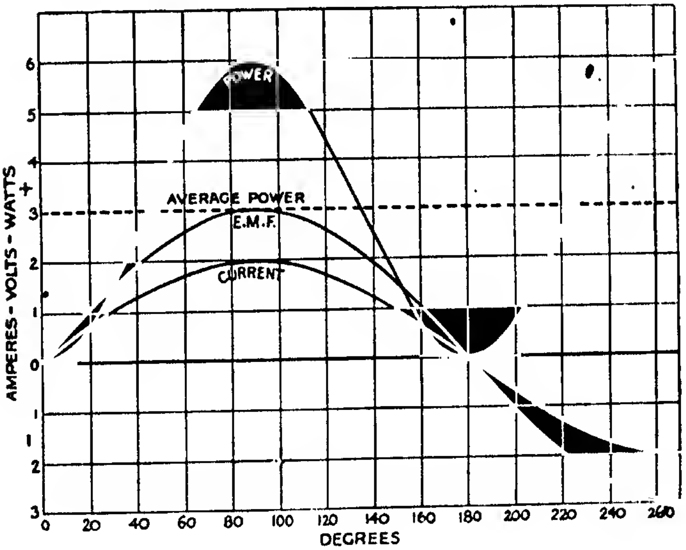


Fig. 4

and partly because such a variation is aimed at in practice, as it produces the best and simplest results. Thus most modern designs of alternators produce approximately sine waves of E.M.F.

The next thing we shall consider is how to express the power of an alternator in terms of the E.M.F. and current. The power of a direct-current dynamo is simply the E.M.F. in volts multiplied by the current in amperes, and is expressed in watts. The power of an alternating current will vary at each instant, and can be expressed by a curve whose ordinates are the product of the instantaneous values of E.M.F. and current at each instant of time. Such a curve is plotted in fig. 4, and we have here assumed for the present the simplest case in which the current and E.M.F. rise and fall together or are in phase. The average power is simply the mean height of the power curve, and is shown by the dotted line. This would be called the output in watts. At first sight it might appear that

the average power could be obtained by multiplying the average E.M.F. by the average current, but this is not the case. The average power is the product of the root mean square (written R.M.S. for short) values of the current and E.M.F. The root mean square of the current is obtained by plotting a curve whose ordinates are equal to the square of the ordinates of the current curve at the corresponding moments. The mean value of this curve is then found, and the square root of this value is the R.M.S. current. The R.M.S. value of the E.M.F. is found in a similar way. If the current and E.M.F. curves are sine curves, or ones to which the mathematical equation can readily be stated, the R.M.S. values can be obtained without recourse to such a long method as that described.

Except for a rectangular wave the R.M.S. value is invariably somewhat greater than the mean value. Thus for a sine wave the mean value is $\frac{2}{\pi}$ or 0.637 time the maximum value, whereas the R.M.S. value is $\frac{1}{\sqrt{2}}$ or 0.707 time the maximum. The ratio of the R.M.S. value to the mean value of a wave therefore depends on the wave form, and is termed the "form factor" of the wave.

The average value of the E.M.F. of an alternator can be readily obtained whatever the wave shape may be, provided the maximum flux which the coil includes during a revolution is known. The average

$$\text{E.M.F. is} = \frac{\text{total change of flux}}{\text{time in seconds for change to take place}} \times 10^{-8} \text{ volts.}$$

We should choose for the time taken that during which the flux through the coil changes from a maximum in the + direction to a maximum in the other, for then the total change of flux is known and is 2 N, and the E.M.F. makes one complete pulsation (fig. 2). The time taken is that of one-half period, so that for f cycles per second the time is $\frac{1}{2f}$ seconds. The average E.M.F. is therefore $\dfrac{2\text{ N}}{\frac{1}{2f} \times 10^8} = 4 f \text{N} \times 10^{-8}$ volts, or for a coil of S turns $= 4 \text{S} f \text{N} \times 10^{-8}$ volts. This is quite irrespective of irregularities in the wave of E.M.F. In order to obtain the R.M.S. value, however, the shape of the E.M.F. wave must be known and the form-factor k found. The E.M.F. would then be

$$\text{E} = 4\,k\,\text{S}f\text{N} \times 10^{-8} \text{ volts (R.M.S.).}$$

For a sine wave the form-factor (from the figures stated above) is $\frac{0.707}{0.637} = 1.11$. Hence the formula for R.M.S. value of the E.M.F. of an alternator producing a sine wave is $\text{E} = 4.44\,\text{S}f\text{N} \times 10^{-8}$ volts.

CHAPTER II

TYPES OF ALTERNATORS

When alternating current was first used it was almost entirely on the single-phase system for lighting. Early examples of single-phase alternators are the Ferranti and the Mordey, both of the coreless-disc type. The principle of the former is illustrated in fig. 5. Disc-shaped coils wound with copper strip were rotated between alternate N. and S. poles, and in this way the magnetic flux caused to alternate in the coils. Then if the total flux issuing from any pole is N and the total number of turns in series S, the E.M.F.

$$E = 4kSfN \times 10^{-8} \text{ volts},$$

where f is the frequency or number of times a coil passes a pair of poles per second. Both the Ferranti and Mordey alternators are obsolete, as they were expensive, mechanically weak as regards the fixing of the armature coils, and unsuited for the high tensions now in use.

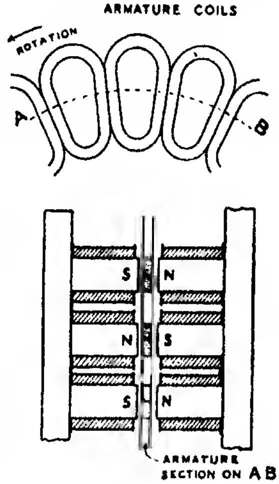


Fig. 5. Ferranti Alternator

The type of synchronous alternator which is now in common use for slow and medium speeds is shown in fig. 6, which shows a three-phase generator of medium speed. The same general construction is used in high-speed turbo-alternators, but the latter are much longer and of smaller diameter than slow-speed ones.

The armature core, which is stationary, is held in a cast-iron frame between two flanges, one of which forms part of the frame, the other being loose. Insulated bolts pass through the stampings of the core and draw the loose flange tightly against them. The outer edges of the laminations bear against ribs in the frame. The laminations are made in segments, and so arranged that the joints of adjacent layers lie on different radii. They are of about 0.5 millimetre thickness, and insulated from one another by tissue-paper and black varnish. A number of ventilation spaces are left between them, and the frame has a large number of openings to allow the cooling air free exit.

The poles are bolted on to a fly-wheel rim, which forms in the larger sizes the sole fly-wheel of the engine. In some designs only the pole tips are laminated, to prevent loss from eddy currents, the pole core being solid. When the poles are laminated throughout they are either dovetailed into the rim and fixed with wedges, or, as shown in the figure, have a rectangular hole in the centre to receive a steel block into which the bolts are screwed. In larger sizes the frames are divided into two parts for

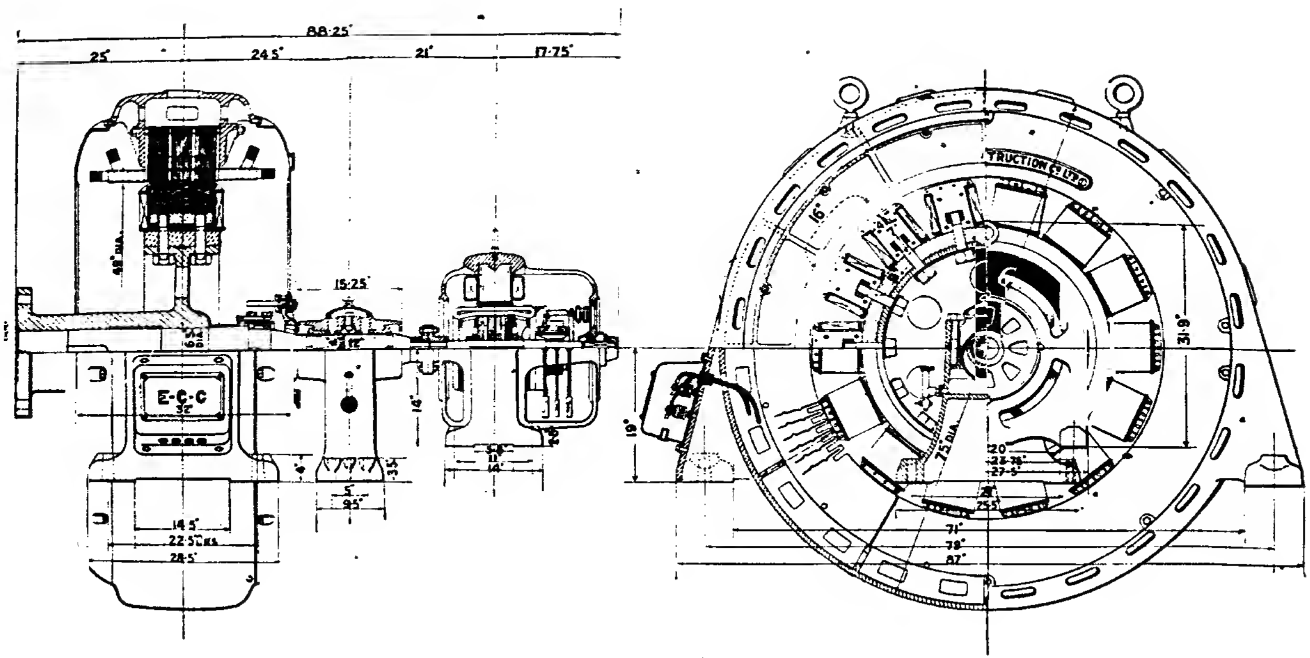


Fig. 6 – Sectional Arrangement E.C.C. Alternator

convenience in handling, as illustrated in fig. 7, the sections being bolted together, as shown at C.

The rotor is also split in two and bolted together. The rim is further held together by steel rings, which fit into grooves, as shown at L, and are shrunk on to the projections of both halves of the rim formed by the grooves.

Larger sizes still, say over 20 feet diameter, are divided into four sections or quadrants and fixed together in a similar manner. Slow-speed alternators of large outputs are not used in conjunction with steam-engines nowadays, turbo-alternators having replaced them, but their use in conjunction with gas or Diesel engines is becoming common, more especially on the Continent. The same design may be used as for steam-engines, but a very much heavier fly-wheel is required for explosion engines, owing to the large variation of crank effort. In some cases, to prevent the rotor from being excessively heavy on this account, the alternator is made of larger diameter than is warranted from the standpoint of electrical design. As the fly-wheel effect increases as the square of the radius of gyration a small increase in the diameter will permit of a considerable reduction in the weight of the fly-wheel rim. Another alternative is to use a separate fly-wheel, but this often involves the use of another bearing. Still another way of obtaining a large fly-wheel effect without making the weight of the rotor excessive is to use an alternator of the externally revolving field type.

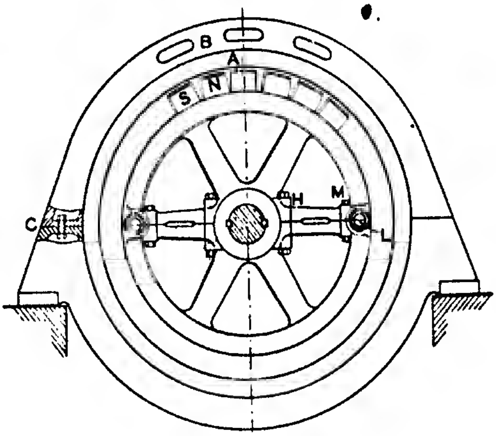


Fig. 7.—Slow-speed Alternator

Horizontal alternators of this type were originally installed in the Niagara Falls power-station, the problem of oiling the vertical shaft of the water turbines being made easier by having the internal armature stationary, whereby the possibility of oil being thrown on to the windings was overcome.

A vertical alternator with externally revolving field, made by the Oerlikon Company, is illustrated in fig. 8. The alternator is three-phase, and is specified to give 1540 kilowatts at 140 revolutions per minute, 42 cycles, 8650 volts. The armature is held by a framework bolted down to the floor, and the shaft carrying the fly-wheel and internal poles passes clear through the frame to a bearing at the outer end of which is carried the exciter.

Another type of alternator, which came into use chiefly on account of its simple construction, is the "Inductor Type", so called because the magnets which rotate are not wound, but have their magnetism *induced* in them by a fixed field coil.

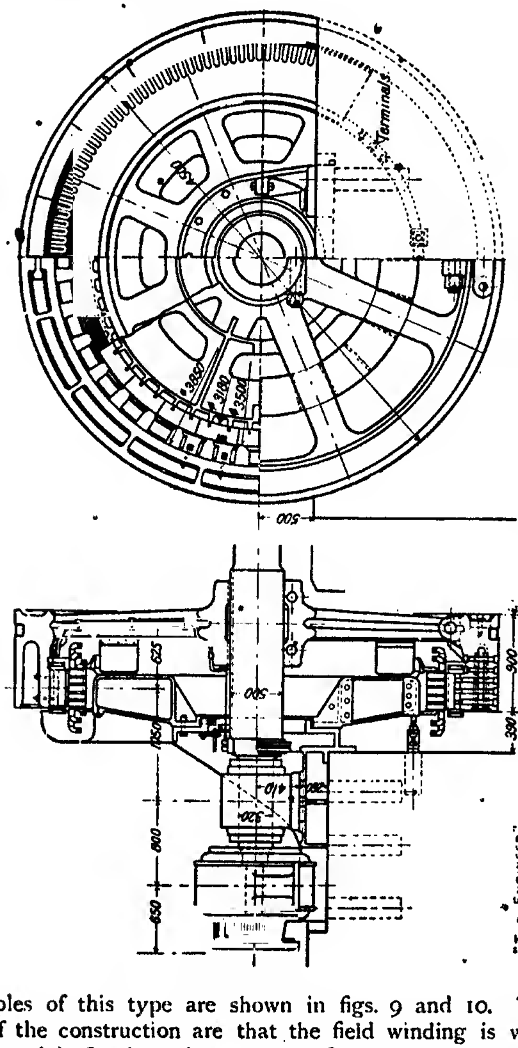


Fig. 8.—Alternator with Externally Revolving Field, made by the Oerlikon Company

"The Engineer"

Examples of this type are shown in figs. 9 and 10. The essential features of the construction are that the field winding is wound in one large coil, and is fixed to the armature frame, and the poles facing the armature are all of similar polarity. The effect is the same as if all the poles of one kind were removed from an ordinary alternator. It would be necessary in this case to provide a return path between the armature and

the magnet system for the lines of force. Fig. 9 shows how this is accomplished in the inductor type, the dotted line indicating the path of the flux. It will be noticed that two air-gaps are required. By treating an ordinary alternator in this way we should still obtain an alternating E.M.F. but of only half the amount, so that the output would be only half what it was

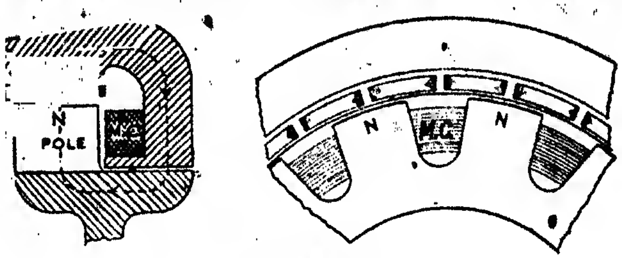


Fig. 9.—Inductor Alternator

M.C., Magnet coil.

before. There would be room, however, to increase the size of the remaining poles, and so obtain a larger flux per pole, whereby the E.M.F. and output might be restored to their original values. This would necessarily increase the weight of iron to carry the flux, and so render the machine very heavy and costly in material. Another way would be to increase the number of turns in the armature winding, but this would lead to very poor

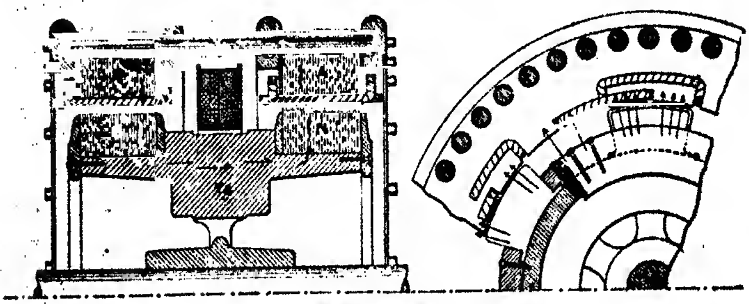


Fig. 10.—Two-Phase Inductor Alternator

Y_1, Armature yoke. AA, Armature core. Y_2, Magnet yoke

regulation (see Chapter IV). The advantage of the inductor type, therefore, lies in the simplicity of the construction of the rotating part and the saving of field copper by winding it in one large coil. This latter has the disadvantage that if the field breaks down repair is difficult. Owing to its many disadvantages the inductor type is seldom used now.

Fig. 10 shows a two-phase generator of this type, in which the return path of the flux is used to excite a second armature.

A form of generator suitable for high speeds, which has the merit also of simple construction of the rotor, is the "Induction" generator. This

consists of an induction motor (q.v.) driven above its synchronous speed, whilst the stator is connected to an alternating-current supply. Under this condition the energy flow of the motor is reversed, and it becomes a generator. The rotor may be of the squirrel-cage type, consisting of a number of copper bars placed in slots in the outer periphery of a laminated core. The ends of the bars are joined together at both ends of the rotor by means of short-circuiting rings into which the bars are bolted, soldered, or sometimes cast. This makes a very simple construction, capable of withstanding high speeds without much to go wrong.

Such a generator, however, has the great disadvantage that it depends entirely upon another generator of the synchronous type to maintain the voltage, as it is incapable of supplying any magnetizing (or wattless) current, and draws magnetizing current from the synchronous generator for its own field excitation. On this account its use is limited to systems in which the power-factor is high, otherwise the synchronous generator becomes overloaded with wattless current. It is sometimes used in conjunction with turbines worked from the exhaust steam of reciprocating engines coupled to synchronous generators which supply the necessary magnetizing current and set the frequency of the circuit.

CHAPTER III

ARMATURE WINDINGS

Alternators are rarely made with more than three phases, and even single or two-phase are not common nowadays. This is because of the general adoption of three-phase current for the transmission of electrical power, owing to its economy in copper over single and two-phase. The windings of alternators, whether of the single-, two-, or three-phase type, generally consist of a number of coils whose sides are embedded in slots, which are usually either open, with wedges of wood to retain the winding (fig. 11), or semi-enclosed (fig. 12). The semi-enclosed type gives a smoother curve of E.M.F. but involves more labour in winding as each conductor has either to be dropped singly into the slot through the opening or pushed through if the opening is not large enough to pass the conductor. Sometimes the slots are completely closed.

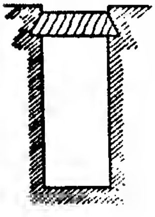


Fig. 11

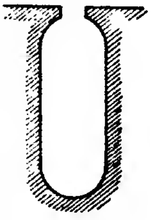


Fig. 12

The arrangement of the coils in single-, two-, or three-phase multipolar alternators is shown in fig. 13. Only two poles developed in a straight line are shown in each case. The coil in (*a*) is shown with its two sides under the centres of adjacent N. and S. poles. By the time a coil side has come under the centre of the next pole moving up to it, the E.M.F. induced

in it has changed from a maximum in one direction to a maximum in the other. In the simple alternator upon which we based our calculations this necessitated a rotation of the coil through 180°. The angular distance from pole-centre to pole-centre is therefore referred to as 180°—electrical degrees as distinguished from the actual number of degrees, which is $\frac{180^\circ}{p}$, where p is the number of pairs of poles. The distance measured in inches or centimetres is called the pole pitch. The E.M.F. induced in the coil at any position of the coil side relative to the pole is shown by the curve in (*d*).

In a two-phase generator there are two windings, the E.M.F. induced in one being 90° out of phase with that induced in the other. To obtain this result the coils of one phase are placed 90 electrical degrees, or one-

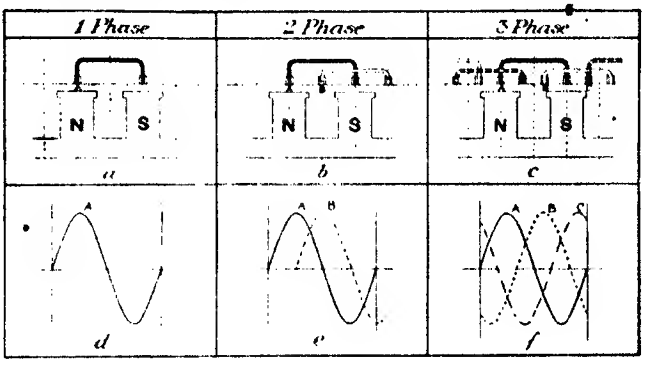


Fig. 13

half of a pole pitch, from the coils of the other, as shown in (*b*). The two E.M.F.s produced are shown in (*e*).

In a three-phase machine three windings produce three E.M.F.s at 120°-phase difference, and the coils are spaced at a distance of two-thirds of a pole-pitch. This is shown in (*c*) and the corresponding E.M.F.s in (*f*).

In figs. 14 and 15 two styles of single-phase 4-pole windings are shown by the coils coloured black, namely, *half-coiled*, in which there is one coil per pair of poles, and *whole-coiled*, in which there is a coil opposite every pole. The coils are shown connected in series, though for low voltages they may be paralleled. The E.M.F. produced by either style is the same for the same number of turns in series. The mean length of a turn in the half-coiled winding is slightly greater than in the whole-coiled, so that the resistance and amount of copper used is correspondingly more. In a single-phase alternator the armature is usually slotted throughout and a fraction, say two-thirds, of the slots are wound. In this case only half are wound. If a similar winding (shown red) is placed in the remaining slots, a *two-phase* winding is obtained. In order to clear the end connections of the first winding the end connections of the second must be bent up, as shown by the black coil in (*b*) fig. 13. For three-phase, the windings are

almost invariably half-coiled, as the clearing of the end connections becomes complicated with whole-coil windings. By the use of half-coiled windings the end connections are simply and easily arranged in two planes. A six-pole three-phase winding is shown in fig. 16. It will be seen that every alternate coil of each phase has its end connections bent up to clear the others. With an odd number of pairs of poles one coil must be specially shaped to clear both straight and bent coils, and a winding with six poles (or three pairs of poles) has been chosen in order to show this; as will be seen, the right-hand coil is of special shape.

The windings of a three-phase alternator may be connected in either of two ways to give three-phase current. Were they not so connected the windings would have six ends and give six-phase current. The two methods of connection are called "star or gamma" and "mesh or delta", the names suggesting themselves from the diagrammatic methods of representing them.

Fig. 17 shows the star connection, in which the three-phase windings have one common junction, called the star point,

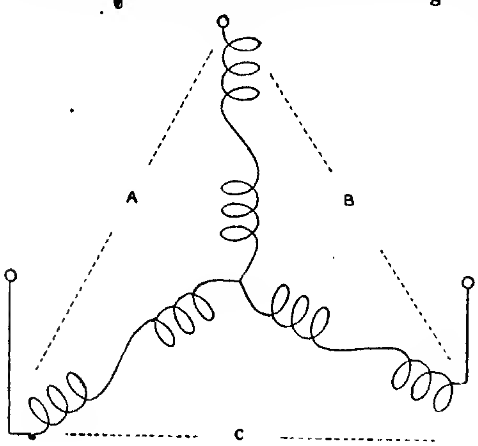


Fig. 17

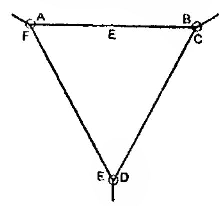


Fig. 18

the free ends being connected to the generator terminals. This is the most common form of connection. The delta or mesh connection is shown diagrammatically in fig. 18. The three junctions of the phases are connected to the generator terminals.

The mesh connection has the disadvantage that if the voltage in each phase is not equal, circulating currents pass round the mesh and cause waste of energy. Each phase, when mesh connected, has to produce the total line voltage, and therefore a larger number of turns is required per phase than with the star connection, which involves more labour in the winding. To connect the winding shown in fig. 16 in star the ends 1′, 2′, 3′, would be joined to form the star or neutral point. For mesh connections the ends 1 2′, 2 3′, 3 1′, would be joined to form the three terminals of the winding.

Barrel windings having the same appearance as direct-current windings are often employed where the slots are open, more especially in turbo-alternators, as the end connections are very compact. The diagram of

a six-pole, three-phase winding of this style is shown in fig. 19. The appearance of the winding is shown in fig. 20, which represents the stator of a turbo-alternator in process of being wound.

The power of an alternating current is the product of the R.M.S. values of current and E.M.F. only if they are in phase. We shall now find an expression for the power when the current and E.M.F. differ in phase by an angle a.

Let the current and E.M.F. at any instant be i and e respectively, so that

$$e = E_m \sin \theta,$$
$$\text{and } i = I_m \sin (\theta - a).$$

That is to say, the current lags behind the E.M.F. by angle a.

To get the activity of the circuit, or the mean power given to the circuit, we have

$$\begin{aligned} e\,i &= E_m I_m \sin \theta \sin (\theta - a) \\ &= \frac{E_m I_m}{2} [2 \sin \theta \sin (\theta - a)] \\ &= \frac{E_m I_m}{2} [\cos a - \cos (2\theta - a)]. \end{aligned}$$

Now the mean value of $\cos (2\theta - a)$ over a complete period is obviously zero, as the values in the second and third quadrants are negative and neutralize the positive values in the first and fourth quadrants. Hence the activity or power in a single-phase generator

$$\begin{aligned} W &= \tfrac{1}{2} E_m I_m \cos a \\ &= E\, I \cos a, \end{aligned}$$

where W = watts output,
E = effective E.M.F. generated per phase,
I = effective current flowing through the circuit,
a = phase angle between current and E.M.F.

In a two-phase generator, where each phase gives an E.M.F. of E volts and a current of I amperes,

$$W = 2\, E\, I \cos a.$$

In a three-phase generator

$$W = 3\, E\, I \cos a.$$

Since, however, the output of a three-phase generator is usually referred to the terminal E.M.F. and the line current, we obtain the following expression:—

$$W = \sqrt{3} \times I_l \times E_l \cos a.$$

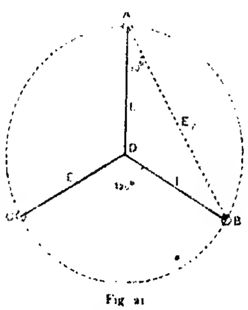


Fig 21

To prove this, consider a generator in which the three windings are joined in star. In such a system the voltage between the extremities of two windings, as BC, fig. 21, is greater than that generated in each phase—that is, the voltage between AB, BC, and CA is greater than between AD, BD, and CD.

Referring to the diagram we find

$$E_l = 2 E \cos 30 = 2 E \frac{\sqrt{3}}{2} = E\sqrt{3}.$$

The current leaving any of the collector rings is the same as flows in the windings, thus $I_l = I$.

The total power of the three phases is obviously three times as great as that of one phase, thus

$$W = 3 \times E I \cos \alpha$$

(α always referring to the phase relation between current and E.M.F. in each phase).

Inserting in this equation the values of line current and E.M.F., that is, substituting for I and E values of I_l and E_l as obtained from above equations, we get

$$W = 3 \times \frac{E_l}{\sqrt{3}} \times I_l \cos \alpha = \sqrt{3}\, E_l I_l \cos \alpha$$

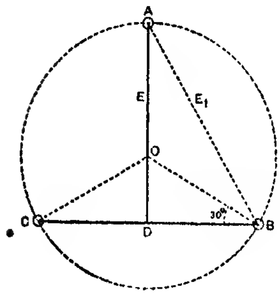


Fig. 22

which is the equation given above.

When the phases are mesh-connected the voltage between the terminals is obviously that of each phase, in other words, $E_l = E$.

Any current drawn from a terminal is, however, made up of the two currents of adjacent phases, and since the currents generated in each phase are displaced from one another, the resultant current is not the sum of the currents in each phase, that is, not $2 \times I$, but, as can be ascertained from the diagram, fig. 22,

$$I_l = \sqrt{3}\, I.$$

Substituting for these values the values of I_l and E_l, as above, we get as before—

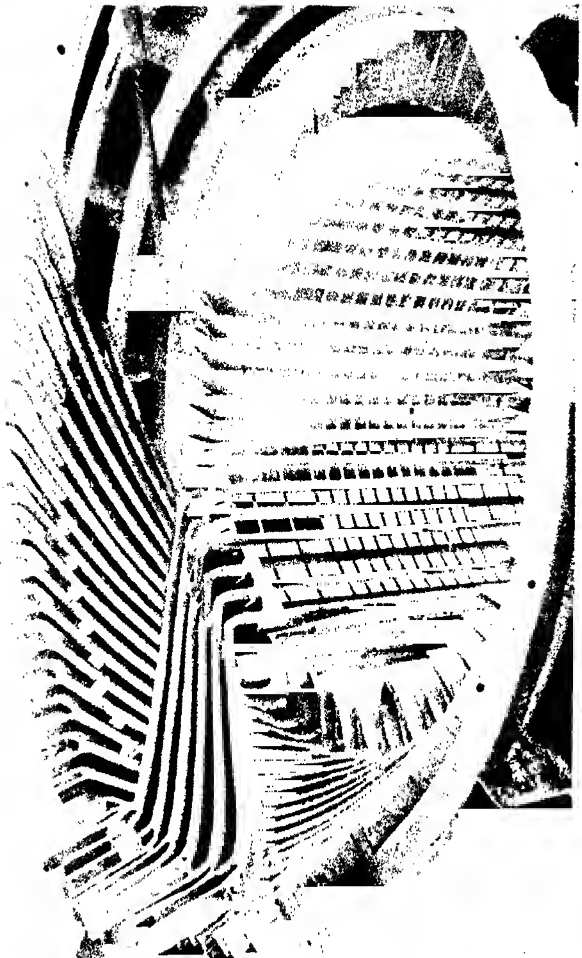
$$W = 3 \times E \times I \cos \alpha$$
$$= 3 \times E_l \times \frac{I_l}{\sqrt{3}} \cos \alpha = \sqrt{3}\, E_l I_l \cos \alpha.$$

CHAPTER IV

SPEED REGULATION—ARMATURE REACTION

Alternators may be classified as low, moderate, or high speed. Low- and moderate-speed alternators are used in conjunction with reciprocating steam- or gas-engines. High-speed ones are used with steam-turbines.

The speeds must be such that with the frequency specified the number of poles are even (i.e. there must be an equal number of N. and S. poles).



If f = frequency, n = revolutions per second, p = number of pairs of poles, then

$$f = pn \text{ or } n = \frac{f}{p}.$$

The highest possible speed for 50 cycles per second is 3000, and for 25 cycles 1500 revolutions per minute, for in both cases the number of poles would be two. Turbo-alternators of 50 cycles are made with two poles occasionally up to outputs of 5000 kilowatts, and 25 cycles up to much larger outputs. Two-pole designs are more expensive than four-pole, in spite of their higher speed, which, however, permits of a cheaper and more efficient steam-turbine. Usually the maximum speeds are 1500 revolutions per minute for 50 cycles and 750 for 25 cycles.

Moderate-speed alternators range in speed from 200 to 400 revolutions per minute, and are used in conjunction with high-speed reciprocating engines. Slow-speed alternators range from 70 to 150 revolutions per minute.

One of the most important factors in the design of alternators is the securing of good regulation of voltage with load, that is to say, the pressure must not fall greatly when the alternator is suddenly called upon to meet a heavy load, nor must it rise much above normal when a heavy load is switched off.

Regulation is affected by three causes, namely:—

1. Volts absorbed by resistance of armature windings.
2. Volts absorbed by reactance „ „
3. Reaction of armature ampere turns on the field ampere turns.

The first of these is of small account compared with the other two, and is of most importance when the armature current is in phase with the E.M.F. The third cause has considerable effect when the armature current is much out of phase with the E.M.F. To explain why this is so, we shall take as an example the simple three-phase winding illustrated in fig. 23, and show how the armature current affects the magnetic circuit. For the sake of simplicity, only one slot per pole per phase is shown and one conductor per slot numbered according to the phase to which it belongs. The current in phase 1 is supposed to be at its maximum passing out from the alternator, and the current in the other two phases will therefore be one-half the maximum passing into the alternator. The exact flow of current under these conditions can best be understood by referring to the winding diagram (fig. 16), and supposing the winding to be star-connected.

The current in fig. 23 is indicated by a cross where it is flowing away from, and by a dot where it is flowing towards the reader, the depths of the markings giving an idea of the intensity of flow. The currents in the three phases are represented by the vectors shown to the left-hand side, the vector numbered 1 indicating the current in phase 1, and so on. We have chosen to represent the pole-centre just over the phase having maximum current. The actual relative position between the pole and the currents flowing in the winding will depend on the nature of the load. Thus with a

heavy lagging current the pole-centre would be forward of the position shown, in the direction of rotation of the poles (indicated by the arrow), and *vice versa* if the current were leading by a large amount. The magnetic effect of the armature current is shown by the lines of force round the armature conductors, and, as will be seen, the result under the conditions shown is to distort the main field and virtually shift it backwards by an amount, a, which depends upon the strength of the armature ampere turns relative to the field ampere turns. This does not affect the strength of the main field to any extent. If, however, the N. pole-centre were placed in the

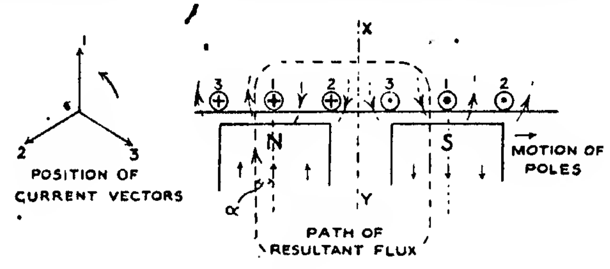


Fig. 23

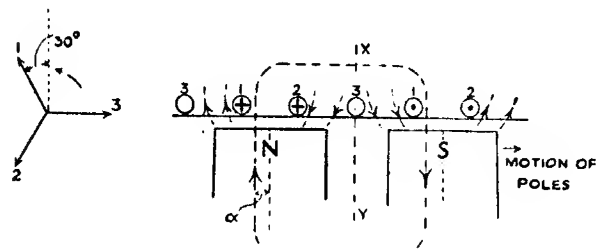


Fig. 24

position X Y, the magnetic field produced by the armature would directly oppose the main field. The main field under these circumstances would be reduced and the voltage would be lowered. This relative position between the pole and currents in the armature would occur if the current in each phase lagged behind the E.M.F. by 90° if the load were purely inductive. Exactly the opposite would take place if the current were leading the E.M.F. by 90°.

We have so far chosen to represent only one state of the currents in the armature, at a certain instant when the current in phase 1 is maximum. It is easy to show that the relative positions of the pole-centre and centre of the band of armature currents is not appreciably altered by movement of the pole. For as the poles move along, the current dies away in some conductors and increases in others, so as to preserve this relative position. For example, in fig. 24, the pole is shown as having moved 30° from the original position. In the same time the vector diagram of currents must have rotated 30°, and it will be seen that the current in phase 3 is now

nothing, in phase 1 it has decreased slightly, and in phase 2 it has increased somewhat. Though the distribution of current in the conductors is now somewhat different, the relative position of the pole and the currents is the same, and the effect is still a distorting one. This is very nearly the condition of affairs when the terminal E.M.F. and current of the alternator are in phase (i.e. load non-inductive, such as lamps, &c.) or the E.M.F. and current are maximum at one and the same instant.

When the current lags behind the E.M.F. by 90° the main field is considerably weakened by the currents in the armature, and the E.M.F. is only maintained by strengthening the main field by the field regulator. A lag of 90° is never attained in practice. This would correspond to a power-factor of 0. Usually with a power and traction load the power-factor is 0.8, which corresponds to a lag of 37 electrical degrees. In this case the effect on the main field will be a mixed one, partly distorting and partly weakening. The weakening effect is still considerable, however, and has to be taken account of in the design of the field-magnet and armature windings.

It is evident that when the current is lagging the change of voltage from no load to full load will depend mainly on the relative strength between the ampere turns of the field and those of the armature tending to demagnetize the field. The greater the ratio of field to armature ampère turns the less will be the change of voltage with change of load, in other words the *stiffer* the field the better will be the regulation, especially when the current is a lagging one. The value of this ratio will vary according to the conditions of the load and the quality of regulation specified. If the load is non-inductive the ratio must still be within a certain minimum to prevent undue distortion of the field, which in itself throws the position of maximum E.M.F. and current in a backward direction relative to the pole and introduces back ampere turns in the armature. Accordingly we may assign a minimum value to this ratio below which it would be inadvisable to go, otherwise bad regulation would be sure to ensue. On the other hand, were the ratio made too large, the machine would become too expensive, owing to the excessive amount of active material required, both of copper and iron. The reason more material is required may be seen as follows: Suppose we have designed an alternator with a certain ratio of field-strength to armature-strength and have got all the active material into as small a space as possible without danger of overheating taking place. We could increase the ratio of field to armature ampere turns either by making the number of turns in the armature fewer or the number of turns in the field greater. If we chose the first way, then the E.M.F. of the machine would be lowered. In order to bring it up to the same value as before, the flux would have to be increased. This would necessitate an increase in the size of the magnets and radial depth of the armature—in fact an increase in the total volume of the machine. If the second method were adopted, more copper space would be required for the field windings, and the machine would have to be made larger. In either case an increase of material and cost is involved.

The value of the ratio is therefore fairly definite for a given alternator, and an experienced designer will know approximately what value to assign for a specified voltage regulation.

CHAPTER V

PRINCIPLES OF DESIGN

Space does not permit of a very full discussion of the design of alternators here, but a few of the fundamental principles and a worked-out design will be given to indicate as far as possible the methods used.

The output of an alternator in kilo-volt-amperes is proportional to E I, where E is the E.M.F. per phase and I the current per phase.

Now $E = 4.44\, S f N \times 10^{-8}$, so that for an alternator of given output and frequency we may write, omitting the constant quantities,

$$\text{output is} \propto E\, I \propto S\, I\, N,$$

where S I is a quantity which multiplied by a constant gives the ampere turns in the armature winding, and N is the flux per pole. The expression S I N is therefore constant for a given alternator, and we have the choice of making a machine with either a large flux and few armature ampere turns, or small flux and many armature ampere turns. This means that the *field* ampere turns may be small when the flux is large, and *vice versa*, since the ratio of field to armature ampere turns is more or less fixed according to the quality of regulation required. Various considerations determine the limits within which the ratio between these two quantities (S I and N) must lie, but a little thought will show that with a heavy flux the volume of iron and consequently weight of the machine must be great, whereas with a large amount of field and armature ampere turns and small flux the weight of iron would be less, but that of the copper actually increased in spite of the fact that the mean length of the windings is smaller. A simple example will make this clear. Suppose a square pole of 3-inch side had four ampere turns, the flux it could carry would be proportional to the area of cross-section of the pole, or 9. If now we reduce the pole side to 2 inches, the flux the pole could carry would be proportional to 4. The ampere turns × flux being constant, the ampere turns would have to be increased in the ratio of $\frac{9}{4}$, and would therefore be equal to 9. To keep the example simple we shall suppose the windings in both cases to carry one ampere, so that the turns in the first case are 4 and in the second 9. The length of turn in the first case will be slightly more than, but approximately, 12 inches and in the second case about 8 inches. The total length of wire in the first case will be 48 inches and in the second case 72 inches, and as the same current has to be carried in both cases these figures will very nearly represent the relative weights of copper. Another objection to having a large number of ampere turns is that the slots

would be very deep and the poles long radially. This would cause a great deal of leakage flux between the poles, and the armature to be highly inductive, increasing the drop of voltage with load and reducing the power of running in parallel with other machines. Also special means would have to be adopted to keep the windings cool.

Suppose that the ratio between S I and N is fixed, we have still to choose between making the diameter of the alternator large and the axial length small, or vice versa. If the diameter is made very large, the peripheral speed would become too high for the material to stand the strain, and the amount of material used for the weight of the framework would become excessive, making an expensive machine. On the other hand, if the diameter were made very small, the length of the machine would necessarily be greater to carry the flux, and this again would waste active material and necessitate a large number of vents to keep the machine cool.

A coil with a given number of ampere turns surrounding a pole carrying a given flux will have its mean length round the pole least when the pole is round, since a circle is the figure which has least periphery for a given area enclosed. A round pole is therefore the most economical in copper, and is easily cooled. Nevertheless few alternators have round poles, as the diameter necessary to permit their use is generally too large for the speed, especially if the frequency is high. An armature coil cannot be made round, but the axial length of armature for minimum length of armature coil enclosing a given area or flux is about the same as that for which round poles could be used.

Great length of armature therefore leads to waste of copper, bad ventilation, high inductance of the armature winding, and abnormal leakage of flux from pole to pole, owing to the proximity of the poles and great length of iron exposed.

Slow-speed alternators having a large number of poles are made of large diameter and small length, whereas high-speed alternators such as turbo-alternators are limited in diameter, owing to the centrifugal forces set up in the rotor, and this to such an extent that there is small accommodation for the poles even though they are few in number. Hence the ratio of diameter to length of an alternator depends largely on the speed. The ratio of axial length to pole-pitch does not vary very greatly, however, and the aim of the designer is to make it as high as possible without exceeding the limit of peripheral speed or adding too largely to the structure, so that the pole section may approximate to the ideal circular section for economy of copper. The ratio ranges from 1 to 2 with slow-speed machines and 2 to 3 with high-speed turbo-alternators, the lower values being obtained in designs of low-frequency alternators which have few poles (except for two-pole generators, the design of which is quite special).

Output Coefficient.—In designing an alternator to a given specification it is necessary to start with a tentative value of the diameter D and length l of the armature parallel to the shaft. If approximate values of D and l have been deduced from some previous design (not necessarily of the same output or speed), the designer would discover whether any

patterns in stock were of suitable dimensions for constructing the framework of the alternator. Even if the patterns would admit of slightly larger dimensions than absolutely necessary, it would probably pay to make a larger alternator rather than incur the expense of new patterns. Exactly how the diameter and length can be deduced from a previous design depends on certain laws connecting the dimensions with the K.V.A. output and speed of machines.

Fig. 25 shows a two-pole alternator developed in a straight line, the circumference at the air-gap being π D centimetres and the length l centimetres in a direction perpendicular to the paper. Obviously if we extended π D to include two other poles, as shown by the dotted lines, the output would be doubled, assuming the length l and peripheral speed v have remained the same. Thus the output is proportional to π D, and this is irrespective of whether the poles are increased or not; for if we now reduce the alternator to the original size, but retain four poles instead of two, the original output is obtained (fig. 26).

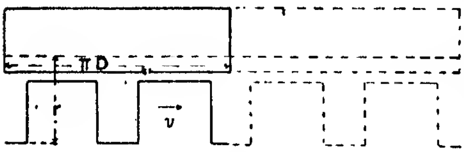


Fig. 25

This is seen by taking account of the various quantities which influence the output. Let the output of the four poles in fig. 25 be E I $\propto$ S I fN, where S I = armature ampere turns per phase, f = frequency, and N = flux per pole. In the four poles in fig. 26 we find only half the space for the armature ampere turns (the slot depth being the same), half the flux per pole, since the poles are half the size, and double the frequency, since the peripheral speed is the same. The output is therefore $\propto \frac{1}{2}$S I $\times 2f \times \frac{1}{2}$N $= \frac{1}{2}$S I fN, or reduced to half.

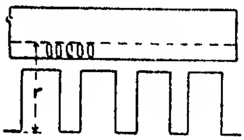


Fig. 26

Again, the output is $\propto$ to l, for any increase of l permits a proportionate increase of N, and consequent increase of the output. Lastly, if the peripheral speed is increased the frequency f is increased in proportion, and therefore the output. So that the output is proportional to all three quantities, and is therefore proportional to their product, i.e.

$$\text{Output in K.V.A.} \propto D\,l\,v,$$

$$\text{but } v = \pi D \times \text{r.p.m.}$$

$$\therefore \frac{\text{Output in K.V.A.}}{\text{r.p.m.}} \propto D^2 l,$$

$$= \xi D^2 l,$$

where ξ is a constant called the output coefficient. The greater ξ is, the larger the output which can be obtained from given dimensions.

From this it would appear a simple matter to deduce the main dimensions of any alternator, of which the output and revolutions are given, from

one already designed. But this is not the case, since certain considerations which have not been taken into account cause ξ to be anything but constant.

One of the most important considerations, apart from the question of heating, which we shall take up later, is that we have assumed the alternator developed in a straight line instead of being circular. If the two-pole alternator shown in fig. 25 is made circular, as in fig. 27, with the same periphery as before, we find that the space for the poles and field winding is far too small, and the diameter will have to be considerably increased to get them in at all, that is to say, larger dimensions are required for the same output and revolutions, so that the value of ξ is necessarily lower. This is characteristic of all alternators having few poles, such as turbo-alternators. Two-pole turbo-alternators are sometimes used, but their design is very uneconomical on account of the low output coefficient and great weight of iron in the armature core. The same section of core is required as for the straight armature core, but the total length round the poles is greater. Four-pole alternators suffer from the same defects, but not quite so much, as they more nearly approach the straight type. Multipolar alternators of large diameter approximate fairly closely to the straight type, and therefore have a large and more constant output coefficient. In fact, with a constant pole-pitch the greater D is the better the value of ξ, for the conditions are always more nearly approaching those of the straightened-out alternator. But even if the number of poles is constant, and the pole-pitch therefore variable, the value of ξ is still greater with greater diameter, for the distance r (fig. 25) from the root of the poles to the top of the slots increases with the diameter under these circumstances, and the greater r is the more the ampere turns which can be placed in the slots and on the poles and the greater the output in proportion. It would perhaps be better to say that r must necessarily decrease with decrease of diameter, otherwise the depth of the slots and radial length of the poles would become too great in comparison with the pole-pitch for economical design. The effect will be seen by comparing the four poles of fig. 26 with those of fig. 25, r being the same in both cases. In fig. 26 the poles are close together and long, and the slots deep.

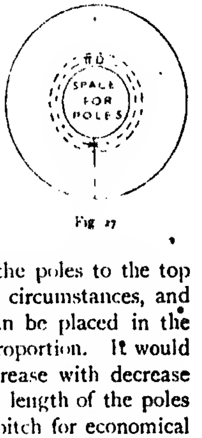


Fig. 27

An alternator of low periodicity and of medium or slow speed has an advantage over one of high periodicity in this respect. For example, a 25-cycle alternator has half the number of poles of a 50-cycle alternator of the same speed. If the diameter and length are the same for both, the possible output of the 25-cycle alternator will be greater than that of the 50-cycle one (provided it can be kept cool), for the pole-pitch is greater and r may therefore be greater in proportion. Alternators of low periodicity have therefore a greater output coefficient than high-period ones of the same output and speed, except where the number of poles is abnormally low. It does not follow that the low-period machine is the cheaper of the two, for the flux per pole is greater, there being fewer and larger poles, and the

section of armature core and magnet yoke must be correspondingly heavier to carry the larger flux. In fact, 25-cycle generators are, as a rule, more expensive than 50-cycle ones of the same output and speed.

The assumption has been made that the number of ampere turns on the field and armature can be made proportional to the space available for them. This is true if the current density in the windings is constant, which, however, is not the case, since a large coil must work at a lower current density than a small one unless extra precautions are used to keep it cool. The space occupied by insulation has also not been taken account of, and as this is considerable in high-tension machines the value of ξ must be lower on this account.

It will be evident that the value of the output coefficient depends on a number of factors all of which vary, and though it would be possible

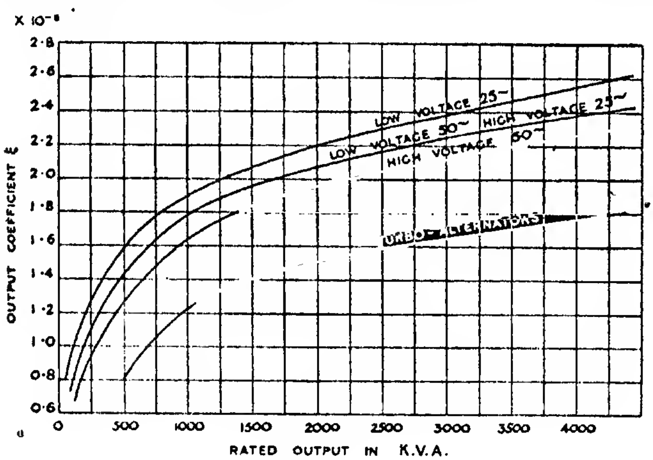


Fig. 28

to deduce the effect of these in each case, it is customary to plot a curve of ξ as a function of the output, the values being obtained from a number of previous designs of alternators. Such curves are shown in fig. 28.

These curves can only be taken as approximate for a preliminary rough calculation of the dimensions, and many cases arise in which the output coefficient will be quite different from the average value given by the curves, owing to some factor in the specification. The usual method of fixing the main dimensions is as follows. The value of $D^2 l$ is obtained from the formula

$$D^2 l = \frac{\text{K.V.A.}}{\xi \times \text{r.p.m.}},$$

where D = diameter of armature in centimetres,

l = length of armature in centimetres,

The value of D is then obtained from the formula

$$\frac{\pi D n}{100} = \text{peripheral speed in metres per second.}$$

$$n = \text{revolutions per second.}$$

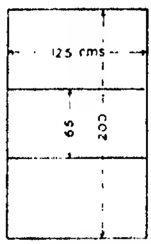
The usual values of the peripheral speed are as follows:—

Output in K.V.A.	Peripheral speed in metres per second. Slow and medium speeds.	Turbo speeds.
0–100	20–24	—
100–500	24–28	—
500–1000	28–32	60–70
Above 1000	32–40	70–90

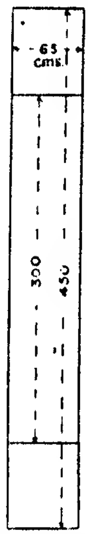
Once D and $D^2 l$ are fixed, the value of l may be found.

The output of an alternator is generally limited by either regulation of voltage or rise of temperature. If too much output is demanded of it, either the drop of voltage will be greater than can be compensated for by field regulation or the temperature greater than the insulation of the windings can withstand for a prolonged period.

The temperature should never be allowed to exceed 80° C., otherwise the insulation will gradually char and deteriorate. Usually the *rise* of temperature is specified not to exceed 40° C. above the surrounding air after continuous running. In hot climates a lower rise of temperature may be specified if there is danger of the ultimate temperature being excessive. At high altitudes it is found that machines operate with a higher temperature-rise than at sea-level. This is due to the rarity of the atmosphere whereby less heat is carried away by the cooling air. Roughly the temperature-rise will be about 2½ per cent greater for every 1000 feet above sea-level.



3000-Kilowatt Turbo-alternator, Cooling Surface, 1600 Square Decimetres



3000-Kilowatt Slow-speed Alternator, Cooling Surface, 3500 Square Decimetres

Fig. 29

Alternators rated to rise 40° C. after continuous running can generally be run for two hours at 25 per cent overload without danger of overheating. With slow-speed and medium-speed alternators the surrounding air has easy access to the windings and ventilating-ducts, and there is little difficulty in keeping the temperature-rise within the limit before trouble occurs due to bad regulation. On the other hand, with high-speed turbo-alternators extra precautions must be taken to thoroughly cool the rotor and stator, otherwise the output would be limited by overheating long before bad regulation occurred. This is because the turbo-alternator is so much smaller in size

for its output than a slow-speed machine. The steady temperature a body will attain above atmosphere is proportional to

$$\frac{\text{heat produced in body}}{\text{cooling surface of body}}.$$

The heat is produced in this case by the copper and iron losses in the active material. These losses, though differently distributed, are approximately the same taken as a whole for a turbo-alternator and slow-speed alternator of the same output. The turbo-alternator, however, being much smaller, has far less cooling surface. An idea of the relative sizes of a slow-

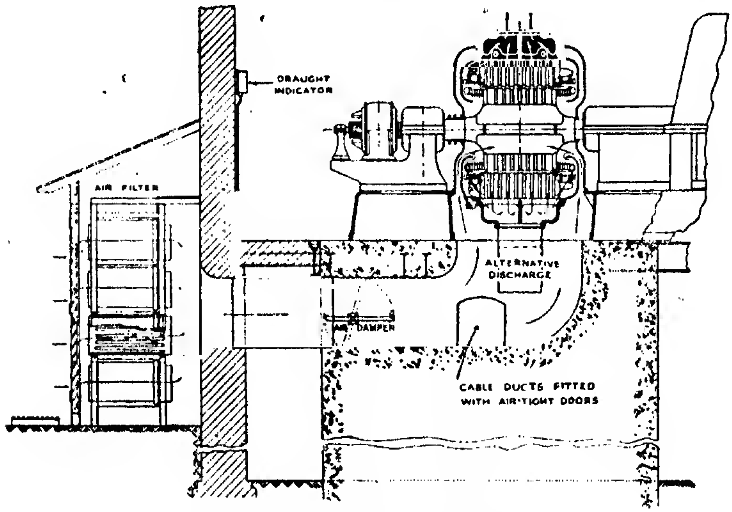


Fig. 30. - Ventilation System of a Turbo-alternator

speed and turbo-alternator of the same output may be obtained from fig. 29, which shows the outlines of two alternators of 3000 kilowatt output.

In order to keep down the temperature-rise in turbo-alternators the ventilation system has to be made highly efficient by guiding a strong air-blast through a large number of ventilating-ducts in the rotor and stator. The cool air passes first to the rotor, as owing to the cramped space in which it works it requires more cooling than the stator. In fig. 30 the cooling arrangements adopted by the British Thomson-Houston Company for turbo-alternators are shown. Air is drawn in from below the alternator by the suction produced by the rotor revolving at high speed. After passing through the rotor and stator the air enters an annular space round the stator core, from which there is an outlet at the top into the engine-room or an alternative one at the bottom to the outside, which is used when the heated air is not desired in the engine-room, the top outlet being closed.

The air is not allowed to pass the bearings, so that it cannot become charged with oil vapour, which has a very detrimental effect upon insulation.

It is always advisable to have the air filtered in some way, so that it is freed from dust particles, which would otherwise collect on the windings and produce choking of the air-ducts, surface leakage, and probable break-down. The air-filter shown in fig. 30 consists of a number of boxes each containing a broad strip of cloth wound in a zigzag fashion so as to form deep corrugations and afford plenty of surface for the air to pass through. The boxes are removable for cleaning purposes.

In case of fire occurring in the filter, and reaching the alternator, a damper is provided between the filter and the alternator for cutting off the air-supply. The damper is weighted so as to remain open, but on emergency it can be closed by pulling up a handle attached to a chain which passes through the floor near the alternator.

Another way of filtering the air, which is coming into use, is to pass it through fine sprays of water which catch all the dust particles. Danger of fire in this case does not exist.

CHAPTER VI

ANALYSIS OF A MEDIUM-SPEED ALTERNATOR DESIGN

Design of 250 K.V.A. medium-speed alternator made by the Electric Construction Company.

Specification.—

Output = 250 K.V.A. three-phase (star-connected).
Speed = 428 revolutions per minute.
Frequency = 50 cycles per second.
Voltage = 6600 between terminals (= 3810 to star).
Exciter: direct-coupled, voltage = 75.

The current per phase will be

$$\frac{\text{K.V.A.} \times 1000}{\sqrt{3}\,E} = \frac{250000}{\sqrt{3} \times 6600} = 22 \text{ amperes.}$$

$$\text{Number of poles} = \frac{120 \times f}{R} = \frac{120 \times 50}{428} = 14.$$

Main Dimensions.—

Armature: Internal diameter (D) = 49 inches or 125 cms.
External diameter = 65 inches or 165 cms.
Length parallel to shaft (l) = 10 inches or 25.4 cms.
Number of vents = 2 – ½ inch wide or 1.27 cms.

This gives an output coefficient of

$$\xi = \frac{\text{K.V.A.}}{D^2 l n \times 60} = \frac{250}{125 \times 125 \times 25.4 \times 428} = 1.47 \times 10^{-6}.$$

$$\text{Peripheral speed} = \frac{\pi \times 125}{100} \times \frac{428}{60} = 28 \text{ metres per second}$$

$$\text{Pole pitch } \tau \text{ at air-gap} = 28 \text{ centimetres.}$$

$$\text{Ratio } \frac{l}{\tau} = \frac{25.4}{28} = 0.91.$$

Net length of iron in core

$$= (10 \text{ inches} - 2 \times \tfrac{1}{2} \text{ inch}) \times 0.9 = 8.1 \text{ inches} = 20.6 \text{ cms.}$$

The factor 0.9 is introduced to allow for insulation between stampings.

Armature Winding and Slots.—A rapid approximation to the number of conductors in the armature is obtained from the usual number of "ampere wires" per centimetre of armature periphery. This varies according to the current density in the wires and the depth of the slots, and is of course greater in large alternators than small. The usual values are

For alternators from 100 to 500 kilowatts, 120 to 150.
" " 500 to 1000 " 150 to 200.
" " 2000 kilowatts upwards, about 220.

The current per phase is = 22 amperes.
The number of conductors = 3024.

$$\text{Ampere wires per centimetre of periphery} = \frac{22 \times 3024 \times 7}{125 \times 22} = 170.$$

At least three slots per pole per phase should be used if possible, though sometimes two are used where there is a large number of poles and space is cramped. In this case there are 126 slots, and so the slots per pole per phase are $= \frac{126}{3 \times 14} = 3$. Each slot is 1.9 inches $\times$ 0.52 inch, and contains $\frac{3024}{126} = 24$ conductors of 0.104 inch d.c.c.

The slots are insulated with micanite tube $\frac{1}{8}$ inch thick, and 20 millimetres of paper are placed between the two layers.

An approximation to the size of conductors and slot to contain them can be roughed out from the usual current densities used, which average the following values:—

Current in Amperes.	Current Density in Amperes per square inch.
Up to 25	2800–2000
25 to 150	2000–1800
150 to 300	1800–1600.

$$\text{The size of conductor is then} = \frac{\text{current per conductor}}{\text{current density}} \text{ square inches.}$$

The size and number of conductors being known, a slot can be provisionally

dimensioned to contain them with the necessary insulation. In this case the current density works out at 2600 amperes per square inch.

A detailed drawing of the slot is shown in fig. 31. The micanite tube is required for the high voltage at which the alternator works.

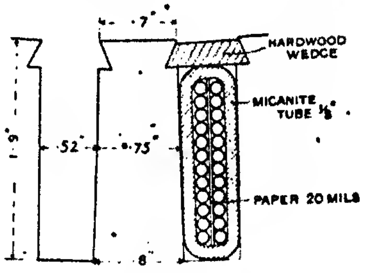


Fig. 31

The total armature reaction per pole is expressed in terms of the R.M.S. armature current per phase and the turns per pole per phase. The magnetomotive force of the armature is not the same throughout a pole pitch, but is greatest at the point, which is enclosed by the greatest number of armature ampere turns. The distribution of magnetomotive force for the instant the currents in each phase are as shown in fig. 23, is represented in fig. 32 by the full-line curve. In an actual alternator the conductors of each phase coil are spread over a number of slots, so that the curve is not so abrupt, but approaches more to the sine curve shown dotted.

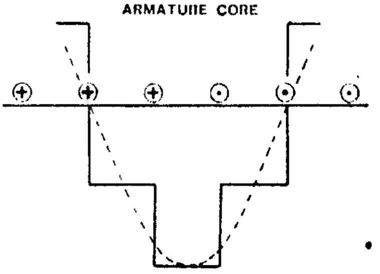


Fig. 32

The maximum magnetomotive force at the instant shown is due to the maximum current in one phase and half this current in other two. If S_p is the turns per *pole* per phase and I the R.M.S. current, then the magnetomotive force is

$$\left(\sqrt{2}\,I + \frac{\sqrt{2}\,I}{2} + \frac{\sqrt{2}\,I}{2}\right) S_p = 2\sqrt{2}\,I\,S_p.$$

With the distribution of current in the phases shown in fig. 24 it would be $= \frac{\sqrt{3}}{2}\sqrt{2}\,I\,S_p = 1.73\sqrt{2}\,I\,S_p$. Thus it varies slightly as it moves round synchronously with the poles, and on the average is about $1.86\sqrt{2}\,I\,S_p$. The effective armature reaction must be considerably less than the maximum. If we suppose it to act in direct opposition to the field pole, which would be the case when the current is lagging by 90°, the effect on the field would be less the greater the spread of the pole over the pole pitch, due to the falling off of the armature magnetomotive force to either side of the centre. As the ratio of pole arc to pole pitch does not vary much in different designs, we may in general take the effective armature reaction to be $1.5\sqrt{2}\,I\,S_p$. The total armature reaction will therefore be $1.5\sqrt{2} \times 22 \times 36 = 1680$ ampere turns per pole.

Calculation of Flux per Pole.—

$$\text{Number of turns per phase} = S = \frac{3024}{2 \times 3} = 504.$$

$$\text{Phase voltage} = \frac{6600}{\sqrt{3}} = 3810.$$

$$\text{Therefore } E = 3810 = 4.44\, S f N \times 10^{-8}.$$

Due to the winding being spread over several slots, the E.M.F. induced by a given flux is slightly less than if the windings were concentrated in one slot. A factor called the *breadth factor* has to be introduced to allow for this, and has values as follows:—

Three-phase	Slots per pole per phase	=	1	2	3	4
	Breadth factor	=	1	0.966	0.96	0.958
Two-phase or single-phase, with 50 per cent of slots unwound	Breadth factor	=	1	0.924	0.91	0.906

The factor we must take is obviously 0.96.

$$\therefore\ E = 4.44 \times 0.96\, S f N \times 10^{-8},$$

$$\text{and } N = \frac{3810 \times 10^8}{4.44 \times 0.96 \times 504 \times 50} = 3.54 \times 10^6.$$

The flux passing out of each pole is greater than that entering the armature, due to leakage of lines from pole to pole. The ratio of field flux to armature flux is called the *leakage factor*, and may be calculated approximately from the reluctance of air-paths between the poles. We shall assume a value of 1.2 as being a fair average for this type of alternator. The field flux would then be

$$3.54 \times 1.2 \times 10^6 = 4.24 \text{ megalines.}$$

We should now fix trial values of the dimensions of the poles and depth of armature core, so as not to oversaturate the poles or produce too high a flux density in the armature core. The poles are $9\frac{1}{2}$ inches $\times$ $4\frac{3}{4}$ inches in cross-section (see fig. 33), and built up of laminated iron sheet riveted together. The area for the flux is therefore

$$9\tfrac{1}{2} \times 4\tfrac{3}{4} \times 6.45 \times 0.95 = 277 \text{ square centimetres.}$$

The factor 0.95 is introduced to allow for insulation between the laminations. The flux density in the pole is therefore

$$= \frac{4.24 \times 10^6}{277} = 15300 \text{ lines per cm.}^2$$

which is about the limit compatible with economy of field copper.

The armature core has an external diameter of 65 inches and length

parallel to the shaft of 10 inches with two ½-inch vents. The cross-section of the core above the slots is therefore

$$\frac{65-(49+2\times 1.9)}{2}\times 6.45\times(10-2\times \tfrac{1}{2})\times 0.9 = 317 \text{ sq. cms.}$$

The factor 0.9 is used here instead of 0.95, to allow for insulation between stampings, as they are thinner (about 0.5 millimetre thick) than the pole laminations.

Since the flux divides into two paths on entering the armature core, the flux density is

$$\frac{3.54\times 10^6}{2\times 317} = 5600 \text{ lines per square centimetre.}$$

The limit of core density for 50 cycles is about 8000, and for 25 cycles 11000 lines per square centimetre.

We must now predetermine the saturation curve of the alternator in

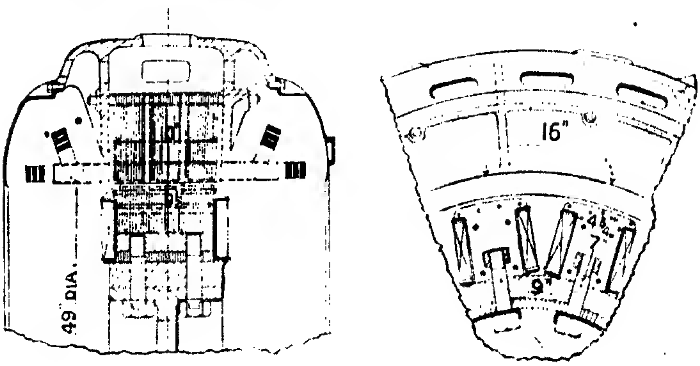


Fig. 33

order to find the ampere turns required on the field, and also the air-gap necessary to produce satisfactory regulation. We shall begin with a point at which the E.M.F. is normal, or 3810 volts per phase, and the armature flux therefore 3.54×10^6 lines. The dimensions of the magnetic circuit are given in fig. 33.

1. *Core.*—The flux density of the core is 5600 lines per cm.² The mean length of magnetic path per pole is 8 inches = 20.4 centimetres. The ampere turns per centimetre required are found from the magnetization curve for stampings in fig. 34 and = 1.6. The total ampere turns are therefore

$$= 1.6 \times 20.4 = 33.$$

2. *Teeth.*—The dimensions of the teeth are given in fig. 31. The polar arc is 7 inches = 17.8 centimetres. To allow for fringing of the lines at the horns of the pole we add on an air-gap length at each horn to the polar arc. In designing for the first time, an air-gap length would have to be

assumed, and corrected afterwards if necessary. The air-gap in this case is 0.22 inch = 0.56 centimetre. The virtual polar arc is therefore

$$17.8 + 2 \times 0.56 = 19 \text{ centimetres.}$$

The number of teeth under the pole is therefore $= \dfrac{19}{3.11} = 6.1.$

The net length of armature core is 20.6 centimetres. The width of tooth at the root, middle, and top is 2.03, 1.91, and 1.79 inches respectively.

$$\text{Maximum B at tip of tooth} = \frac{3.54 \times 10^6}{1.79 \times 20.6 \times 6.1} = 15700.$$

$$\text{Mean B at middle of tooth} = \frac{1.79}{1.91} \times 15700 = 14700.$$

$$\text{Minimum B at root of tooth} = \frac{1.79}{2.03} \times 15700 = 14000.$$

The ampere turns per centimetre in each case, as obtained from fig. 34, are 18, 11, and 8.5 respectively, the mean of which is 12.5 ampere turns per centimetre. The total ampere turns for the teeth are therefore

$$12.5 \times 4.8 = 60.$$

Were the teeth highly saturated a correction would have to be made on account of the fact that a proportion of the flux passes into the armature

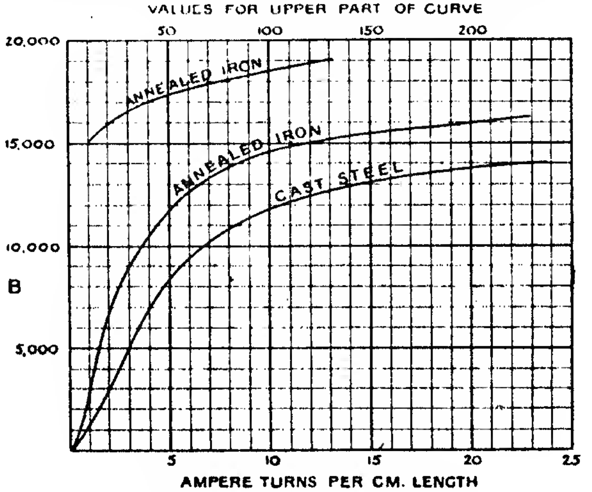


Fig. 34.—Magnetization Curves of Cast Steel and Annealed Iron Sheet

by way of the slots. This proportion becomes noticeable at about 18,000 lines per square centimetre (maximum), and is about 4 per cent at 22,000, being slightly more for wide slots and narrow teeth than for the reverse. Though the actual difference between the apparent and real flux densities

may be small the effect upon the ampere turns is considerable, and too high a value would be obtained without this correction. Such densities can only be used on low-periodicity machines, otherwise there would be considerable loss and heating in the region of the slots.

3. *Pole.*—The flux density in the pole is 15,500.

$$\text{Ampere turns per centimetre} = 16,$$
$$\text{Magnetic length of pole and pole-shoe} = 7 \text{ inches} = 17.8 \text{ centimetres},$$
$$\text{Ampere turns for pole} = 16 \times 17.8 = 285.$$

4. *Yoke.*—The section of the yoke is

$$11 \text{ inches} \times 3 \text{ inches} = 33 \text{ square inches} = 213 \text{ square centimetres.}$$

$$\text{B in the yoke is } \frac{4.26 \times 10^6}{2 \times 213} = 10{,}000 \text{ lines per square centimetre.}$$

The yoke is of cast steel, so the ampere turns per centimetre from fig. 34 are $= 7$.

$$\text{Length of yoke per pole} = 4\tfrac{1}{2} \text{ inches} = 11.5 \text{ centimetres.}$$
$$\text{Ampere turns for yoke} = 11.5 \times 7 = 80.$$

5. *Air-gap.*—The distribution of flux in the air-gap with a slotted armature is indicated in fig. 35, where t is the tooth width, w the slot width, and g the gap. The tooth resembles a pole facing a smooth armature core, and in order to obtain the true flux density under the tooth-head an addition will have to be made to t to allow for fringing of the lines at the edges of the tooth. The addition will be greater the larger the air-gap g, but obviously cannot exceed w altogether or $\frac{1}{2}w$ on either side. The virtual tooth width will therefore be $t + kg$, where k is a factor which will be the same for all similar ratios of $\frac{w}{g}$ but will vary with $\frac{w}{g}$ in some way which can be found experimentally or by calculation. A curve showing this variation is plotted in fig. 36. Where w is large compared with g there is little interference between neighbouring teeth, and the addition kg becomes nearly a constant times the gap $= 3g$. On the other hand, where $\frac{w}{g}$ is small, k and $\frac{w}{g}$ become more nearly equal to one another, and the case approaches that of a smooth core armature. The addition becomes $kg = \frac{w}{g}.g = w$ or $\frac{1}{2}w$ at either side.

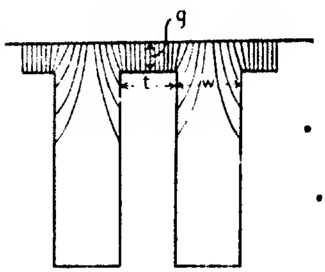


Fig. 35

In this case $\frac{w}{g} = \frac{0.52 \text{ inch}}{0.22 \text{ inch}} = 2.36$ and k from the curve in fig. 36 is $= 1.54$. Hence the virtual tooth width is $t + 1.54g$.

The virtual breadth of gap parallel to the shaft is found in a similar way. The pole is $9\frac{1}{2}$ inches long, and there are two $\frac{1}{2}$-inch vents in the

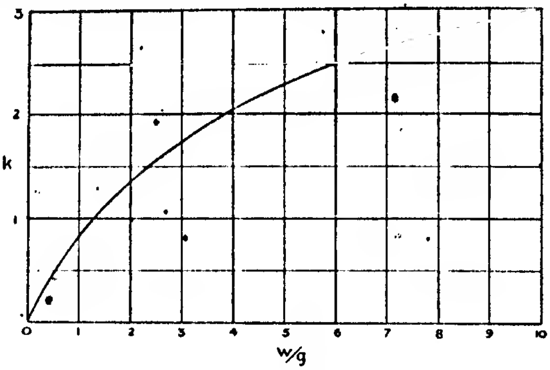


Fig. 36

armature, so that the net length of iron exposed is $8\frac{1}{2}$ inches. The ratio of width of vent to gap is about 2.3, so that $k = 1.5$. For each fringe,

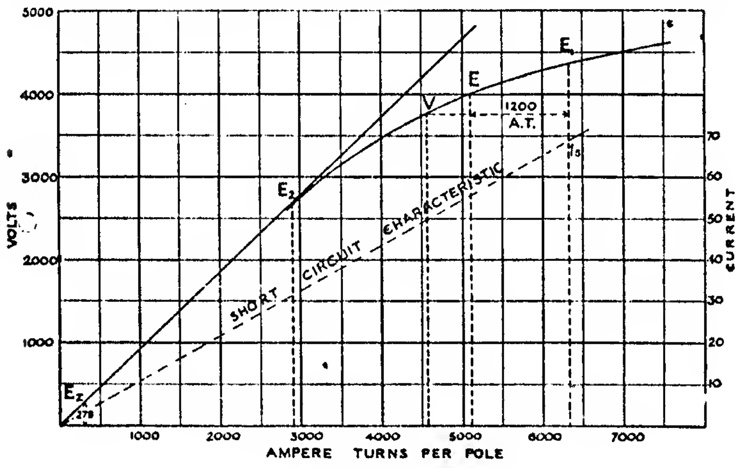


Fig 37

therefore, we add on $\frac{1}{2}kg = 0.75g$; and since there are six fringes the virtual breadth of the gap is

$$8.5 + 6 \times 0.75 \times 0.22 = 9.49 \text{ inches} = 24.1 \text{ centimetres.}$$

The ampere turns required for the gap are then

$$0.8 \cdot \frac{Ng}{(l + kg) \times 6.1 \times 24.1} = 0.8 \cdot \frac{3.54 \times 10^6 \times 0.56}{(1.79 + 1.54 \times 0.56) \times 6.1 \times 24.1}$$
$$= 4100.$$

The ampere turns for a number of voltages are tabulated in Table I, and from these the no-load saturation curve is plotted in fig. 37.

We may now proceed to determine the regulation of the alternator. In fig. 38, V is the terminal voltage per phase and I the armature current lagging, so that the power-factor $\cos\phi = 0.8$. The induced E.M.F. E is the vector sum of the terminal voltage V, and the E.M.F. Iz consumed by the armature impedance per phase $= z$. The vector Iz is composed of Ix at right angles to I, and Ir in phase with I, x and r being the reactance and resistance of the winding per phase. An estimate of the value of x may be made by calculation (see Hawkins and Wallis, *The Dynamo*), or it may be deduced from experimental determinations on other machines. For a machine with open slots, like this one, Ix may be assumed to be about 7 per cent, and Ir 2 per cent of the terminal voltage per phase V. This makes the induced E.M.F. E about 5 per cent greater than V.

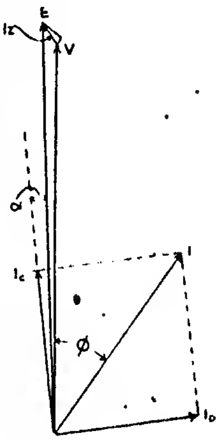


FIG. 38

TABLE I.—AMPERE TURNS PER POLE

E.M.F. per Phase.	Flux per Pole in Megalines.		Ampere Turns per Pole.					Total.
			Armature.		Air-gap.	Magnets.		
	Armature.	Pole.	Core.	Teeth.		Pole.	Yoke.	
3810	3.54	4.25	33	60	4100	285	80	4558
4200	3.9	4.7	37	140	4500	780	90	5647
4380	4.07	4.9	41	235	4700	1280	112	6368
4570	4.25	5.1	44	350	4925	1860	126	7305

The value of E may be found by direct measurement or from the approximate formula

$$E = V + Ir\cos\phi + Ix\sin\phi.$$

Regulation.—The current I may be split into two components, one I_c, the cross-magnetizing component which attains its maximum value in the conductor under the centre of the pole, and the other I_D, the demagnetizing component lagging behind the former by 90 electrical degrees. The latter acts in direct opposition to the main field, so as to weaken it, whilst the former shifts the main field bodily backwards, so that the centre of the field and position of E is behind the pole-centre by the angle α. For a power factor of 0.8 we may assume the angle α to be about

4 degrees. The demagnetizing component I_D is now found by measurement to be 0.71 I, so that, since the total armature reaction is 1680, the demagnetizing component is $0.71 \times 1680 = 1200$ ampere turns (fig. 38).

Referring to the saturation curve at no load the voltage of the alternator is $V = 3810$ volts per phase. At full load the E.M.F. required to be induced is $E = 5$ per cent greater than V or 4000 volts. The total ampere turns per pole required are those necessary to produce E plus the extra amount required to balance the demagnetizing ampere turns of the armature. We therefore have to add on 1200 ampere turns, giving altogether 6350 ampere turns required to maintain the voltage at 3810 volts at full load. If full load is suddenly switched off, the voltage will rise to $E_1 = 4400$ volts, a rise of 16 per cent above normal.

Allowance has not been made here for the fact that the pole carries a slightly greater flux than corresponds to 4000 volts, for the leakage lines are produced by the total ampere turns, and will therefore be greater than allowed for. Hence slightly more than 6350 ampere turns will be required at full load to allow for the greater flux density in the poles and yoke, and the rise of voltage slightly more than 16 per cent.

On test, the alternator required 4750 ampere turns at no load and 6750 at full load, and the voltage rise was found to be 16½ per cent. The fall of voltage on switching on full-load current when the alternator is working at no load with a voltage $= V$ is found by exactly the reverse process to be $E_2 = 2700$ volts or a fall of 29 per cent.

Calculation of the Field Winding.—For this purpose an approximate value of the mean length l_m (centimetres) of the winding per pole would have to be assumed in the first place, and corrected afterwards if necessary. We shall take the correct length, however, which is given us as 83 centimetres.

The voltage across each pole is $\frac{75}{14} = 5.35$ volts.

Let I_F = field current at full load,
s_F = turns per field coil,
a = section of wire used in square centimetres,
$\rho = 2 \times 10^{-6}$ (specific resistance of hot copper).

Resistance per coil is then

$$r = \rho \frac{l_m s_F}{a}.$$

$$\text{Voltage per coil} = I_F r = I_F s_F \rho \frac{l_m}{a} = 5.35.$$

$$\therefore a = \frac{I_F s_F \rho l_m}{5.35}$$
$$= \frac{6350 \times 83 \times 2 \times 10^{-6}}{5.35}$$
$$= 0.2 \text{ square centimetre}$$
$$= 0.031 \text{ square inch.}$$

Wire 0.18 inch square has a section of 0.032 square inch, but to allow

for overload and greater field current required 0.19 inch square d.c.c. wire has been used, the current being regulated by a regulating resistance in the field circuit.

The number of turns s_F is settled by the question of temperature rise permissible (or the efficiency if a high one is specified). The ampere turns for a given voltage per coil and section of wire vary inversely as l_m, and, as this does not vary much with s, they may be taken as almost constant for a wide range of s_F. In this case 144 turns per coil have been wound and arranged 6×24.

The resistance r is therefore

$$= 2 \times 10^{-6} \times \frac{144 \times 83}{a},$$

$$a = 0.036 \text{ square inch} = 0.23 \text{ square centimetre},$$

$$\therefore r = 2 \times 10^{-6} \times \frac{144 \times 83}{0.23} = 0.104 \text{ ohm}.$$

$$I_F = \frac{6350}{144} = 44 \text{ amperes}.$$

The watts lost per coil

$$= I_F^2 r = 44^2 \times 0.104 = 200 \text{ watts}.$$

The cooling-surface of the coil is about 1800 square centimetres. The temperature-rise is not only a function of the watts lost per unit area, but also of the peripheral speed, since a greater speed produces a greater windage and cooling effect. From tests the following empirical formula has been devised:—

Temperature-rise $C^\circ = K \dfrac{W}{A(1 + 0.1 v)}$ where K is a coefficient varying from 7 to 10,

W = watts lost per circuit,

A = cooling surface per pole in square decimetres,

v = peripheral speed of rotor in metres per second.

We shall take K to be 9, since the cooling is not so favourable as for strip copper wound on edge, but not unfavourable otherwise since the poles are not crowded.

$$\therefore \text{ Rise of temperature } C^\circ = 9 \times \frac{200}{\frac{1800}{100}(1 + 0.1 \times 28)}$$

$$= 30^\circ \text{ C.} = 54^\circ \text{ F.}$$

Temperature Rise of Armature Core. The rise of temperature is proportional to the watts lost per square decimetre of cooling surface, or

$$= K \frac{\text{watts lost}}{\text{area of cooling surface in square decimetres}},$$

where K varies from 2 to 2.3. In this case we may take the lower value, as the conditions for cooling are favourable.

The watts lost include the iron losses in the core and teeth and the

copper losses in the slots. The cooling surface is the whole outside surface of the core, including the surface exposed in the air-gap, and, in addition, *half* the exposed surface in the vents. Only half the surface is taken, because it is confined and not so efficient in cooling effect as the remainder.

The loss per kilogram of iron in the core and teeth is given for various flux densities and frequencies in the curves in fig. 39. At full load the flux

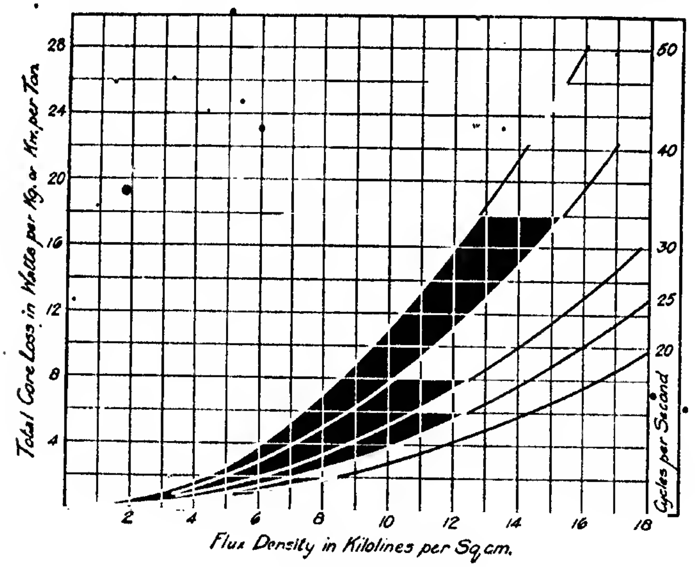


Fig. 39

densities are 5 per cent greater than at no load, since the induced E.M.F. is 5 per cent greater, so that the flux density in the core is $5600 \times \frac{105}{100}$ $= 5900$ lines per cm.² The loss per kilogram from the curves is 3.8 watts. The weight of the core is

$$\frac{\text{Area of core in sq. cms.} \times \text{mean circumference} \times 7.8}{1000} \text{ kilograms.}$$

$$= 317 \times \pi\left(\frac{65 + 52.8}{2}\right) \times 2.54 \times 0.0078 = 1180 \text{ kilograms.}$$

$$\text{Total core loss} = 1180 \times 3.8 = 4500 \text{ watts.}$$

In the same way for the teeth—

$$\text{Mean flux density} = 14700 \times \frac{105}{100} = 15500 \text{ lines per cm.}^2$$

$$\text{Loss per kilogram is} = 26 \text{ watts.}$$

Weight of teeth

$$= 126 \times 8.1 \times 7.5 \times 1.9 \times (2.54)^3 \times 0.0078 = 190 \text{ kilograms.}$$

$$\text{Total loss in teeth} = 190 \times 26 = 5000 \text{ watts.}$$

The watts lost in the armature windings

$$= (\text{armature current})^2 \times \text{resistance per phase} \times 3.$$

About 20 to 50 per cent more than this value is taken for polyphase alternators, due to extra losses incurred by eddy currents in the windings. In this case the conductors are small and all in series, so we shall take the lower figure.

The length of wire in the slot is 10 inches. The pole-pitch $\tau = 11$ inches. The mean length of an armature turn is

$$2(10 + 2\tau) = 2(10 + 2 \times 11) = 64 \text{ inches} = 162 \text{ centimetres.}$$

Resistance per phase (hot)

$$= 2 \times 10^{-6} \times \frac{162 \times 504}{\frac{\pi}{4} \times (10.4)^2 \times 6.45} = 3 \text{ ohms.}$$

$$\text{Total loss} = (22)^2 \times 3 \times 3 \times \frac{120}{100} = 5200 \text{ watts.}$$

The loss in the slots only will be

$$\frac{20}{64} \times 5200 = 1630 \text{ watts.}$$

The total core losses are

$$= 4500 + 5000 + 1630 = 11130 \text{ watts.}$$

The cooling-surface of the core is

$$= \pi(49 + 65) \times 9 + \frac{\pi}{4}(65^2 - 49^2) \times 4$$

$$= 3220 + 5740 = 9000 \text{ square inches}$$

$$= 580 \text{ square decimetres.}$$

The temperature rise is therefore

$$= 2 \times \frac{11130}{580} = 38^\circ \text{C.} = 68^\circ \text{F.}$$

On test the maximum rise at any part of the alternator was found to be 60° F. by thermometer.

Efficiency.—The friction and windage losses may be assumed to be about 2 per cent of the output in kilowatts. The K.V.A. output is 250, so that at 0.8 power factor the output is 200 kilowatts. The friction and windage losses are therefore

$$\frac{2}{100} \times 200 \times 10^3 = 4000 \text{ watts.}$$

The exciter volts = 75, and amperes = 44.

The power required for excitation of the field is therefore

$$75 \times 44 = 3300 \text{ watts.}$$

Assuming an exciter efficiency of 80 per cent, the total power required for field excitation, including exciter losses,

$$= \frac{3300 \times 100}{80} = 4125, \text{ say } 4200, \text{ watts.}$$

The total losses are now

Core loss	=	4500
Tooth loss	=	5000
Armature copper loss	=	5200
Field loss	=	4200
Friction and windage	=	4000
Total	=	22900 watts
	=	22.9 kilowatts.

$$\text{Efficiency} = \frac{\text{output}}{\text{output} + \text{losses}} = \frac{200}{200 + 22.9} = 90 \text{ per cent.}$$

Single-phase Design.—The design of a single-phase alternator differs but slightly from that of a polyphase. It is usual to have the armature completely slotted, so that the same slotting machine will do for either single or polyphase. The coil sides only occupy from a half to three-quarters of the slots per pole, as no appreciable advantage is gained by filling up the remainder, the spread of the coil being too great for the additional conductors to be effective. In consequence of this the output of a single-phase alternator is some 20 to 30 per cent less than that of a polyphase alternator of the same dimensions.

The armature reacts on the poles when the machine is loaded, just as in a polyphase alternator, but the reaction pulsates between zero and a maximum value in unison with the current as the poles rotate, and is thus much more variable than that of a polyphase armature in which it is comparatively steady. In order to damp out the variations which would be produced in the flux, "amortisseur", or damping-coils, consisting of bars of copper inserted axially through the pole-tips and short-circuited on each other at the ends, are fitted much in the same way as the bars in a squirrel-cage rotor of an induction motor. Any change of flux in the poles sets up a current in these coils which tends to oppose the change, so that whilst the average effect of the armature reaction acts on the field, violent pulsations are prevented. For single phase the formula for armature reaction on p. 243 becomes $0.5 \sqrt{2}\, S_1 I_a$.

CHAPTER VII

TURBO-ALTERNATORS

Turbo-alternators have displaced entirely the large slow-speed alternator where steam power is used, owing to the much greater economy in space, both of the steam-turbine and turbo-alternator. In fact, the large slow-speed alternator is now only used in conjunction with large gas-engines, the gas-turbine so far not being a commercial success.

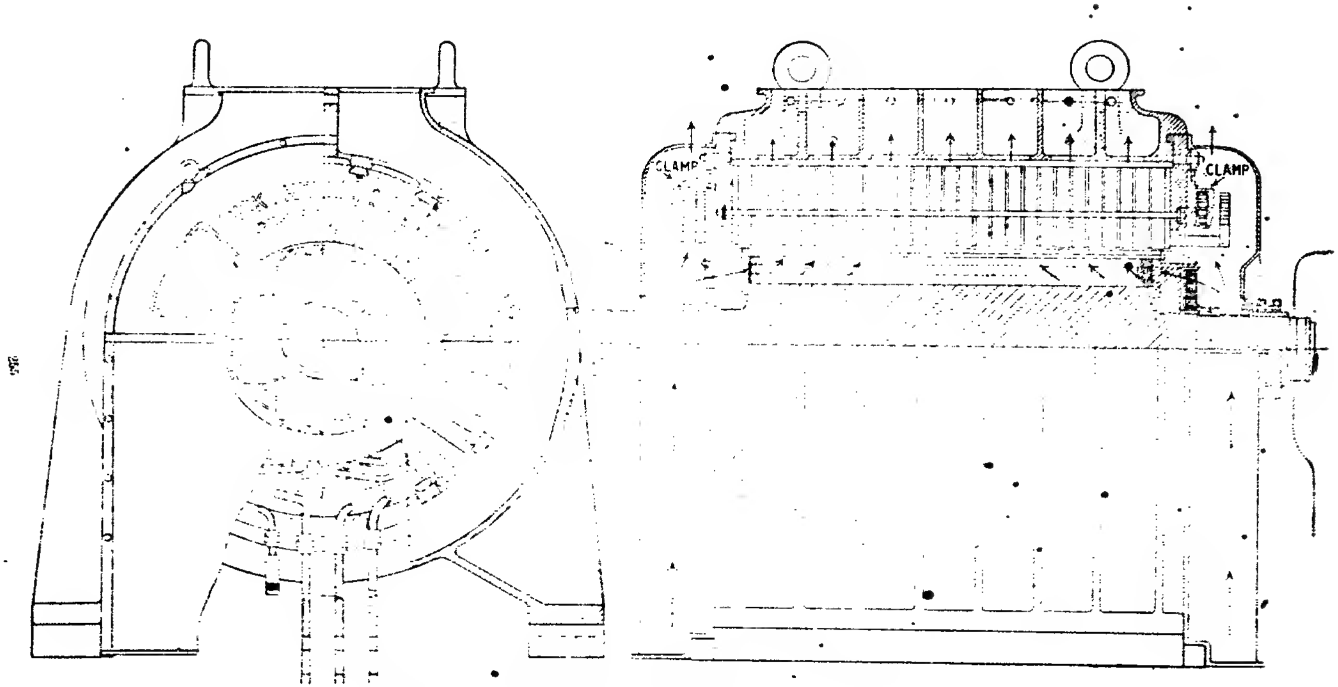


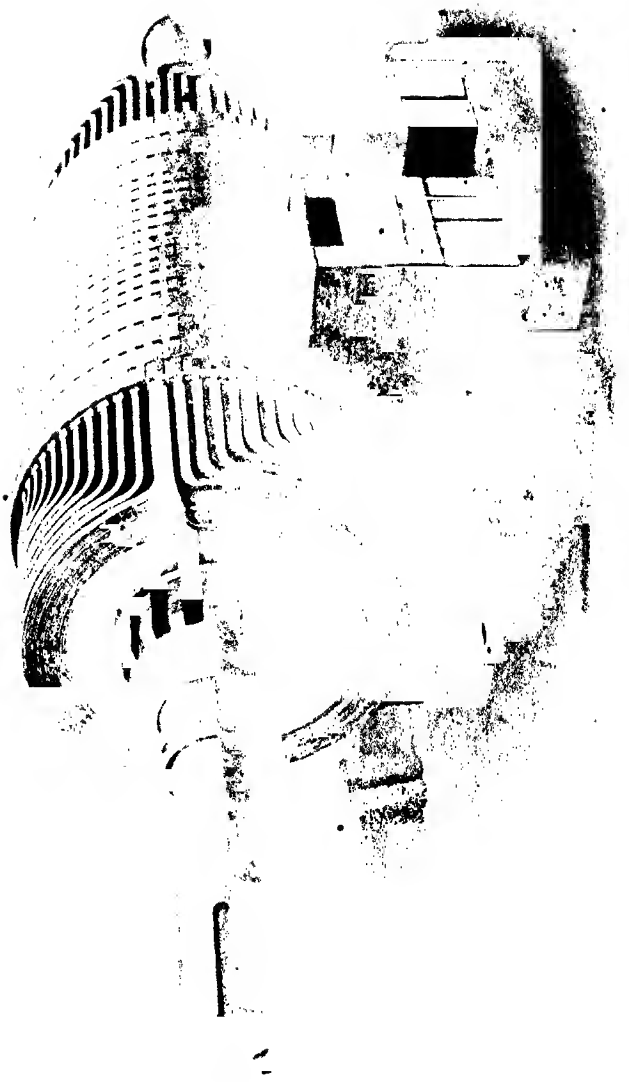
Fig. 40.—6250-K.V.A. at .8 Power-factor, 6000–6600 Volts, 50 Cycles, 1500 Revolutions per Minute

The usual speeds of turbo-alternators have already been given on p. 231. Not only is the speed in revolutions much higher than that of the reciprocating engine, but the peripheral speed of the rotor is greater also than that of the slower-speed alternator coupled to a reciprocating engine. This is possible owing to the small diameter of the rotor, which enables it to be constructed in a much more rigid manner, and also to the use of the highest-grade materials. Peripheral speeds up to 18,000 feet per minute are considered safe, but at such speeds special precautions have to be taken to hold in the field windings, for the centrifugal forces set up are very great and may be over a ton for every pound of material at the periphery.

The output coefficient of a turbo-alternator is lower than that of a slow-speed one, largely due to the cramped space in which the rotor must work. It has been explained on p. 237 that where the poles are few in number the large angle by which they are inclined to one another causes a considerable reduction in the field-winding space, and consequently in the output for a given value of $D^2 l$. To obtain the required output either the diameter D may be increased, giving more room for the field, or the axial length l, which increases the area of pole-face and permissible flux N, and therefore admits of fewer ampere turns on the field and armature, since S I N is constant for a given alternator (see p. 234). In a turbo-alternator the diameter is limited by the peripheral speed, and the output if large can only be obtained by increasing the axial length all out of proportion to the diameter. This makes the design wasteful in copper, since the poles are far from having the ideal circular section, ventilation is more difficult, and more material is required to preserve the rigidity of the rotor and stator. In many cases a lower speed with more poles would actually give a more economical design of turbo-alternator, but the efficiency and cost of the steam-turbine would suffer so much that as a whole the higher speed is better. For this reason two-pole 50-cycle alternators, running at 3000 revolutions per minute, are often used up to outputs of 5000 K.V.A. in spite of the cost of the alternator, since the next possible speed is 1500 revolutions per minute or just one-half.

Rotors may have definite poles, like those of slow-speed alternators, in which case they are called "salient" pole type, or they may consist of a cylinder with the field winding placed in slots grouped at certain parts of the surface, so as to form poles between these parts. Such rotors are termed "non-salient" or smooth type. In the earlier turbo-alternators definite pole rotors were exclusively used, following the general practice with slow-speed alternators. For large outputs, however, the non-salient type has a distinct advantage in the matter of space economy, and it has almost entirely displaced the salient type. An example of a modern salient-pole type of rotor is illustrated in fig. 40, which shows a 6250-K.V.A. four-pole turbo-alternator, constructed by Dick, Ker, & Co. The poles are forged solid with the hub, making a very strong construction, and the field winding is held in against centrifugal force by V-shaped clamps between the poles.

In the non-salient pole rotor the field winding is better situated than in



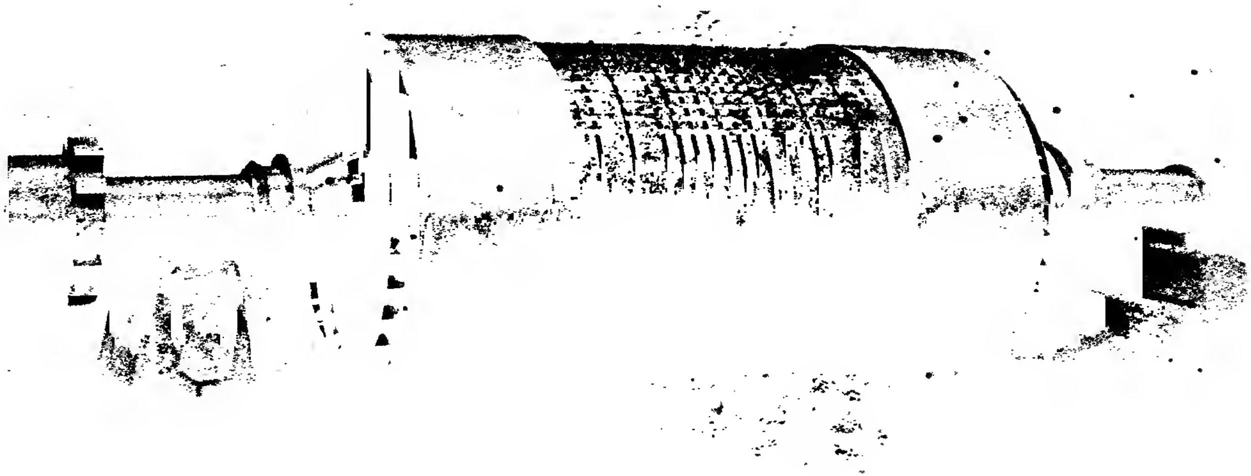


Fig. 42.—Two-pole Rotor, Complete

the salient type, being at the surface, and so reducing the leakage flux and giving the maximum room for the iron of the pole-core and also for the copper. In the salient type the field-winding space encroaches too much upon the space for the iron, and a large amount of space is wasted by the holding-in device. The field winding being distributed in the smooth core rotor, more surface is exposed for cooling, and the smoothness of the core produces silent running. The magnetomotive force is greatest at the pole-centre, and tapers off at either side, thus giving a flux-distribution approaching a sine law. Due to the distribution of the winding and the smooth symmetrical surface all round the rotor the displacement of the flux from the pole-centre on load does not produce undue distortion of the flux wave with consequent peak in the E.M.F. wave.

The slots in the rotor may be totally enclosed, or open with steel wedges bridging the opening to hold in the winding. In some cases bronze wedges are used to reduce the leakage flux, but this destroys the continuity of the flux-distribution curve and introduces harmonics into the E.M.F. wave. This may be overcome by skewing the slots so that they are not parallel to the axis of the shaft, and no rotor slot can at any time be completely opposite a stator slot. A non-salient-pole rotor half complete is shown in fig. 41. The rotor has only two poles, one at the top and the other at the bottom, in the position shown, and is part of a 7500 K.V.A. 25-cycle alternator running at 1500 revolutions per minute, manufactured by the British Thomson-Houston Company. The hub and shaft are in one piece, and the core consists of groups of steel laminations separated by distance-pieces so as to form a large number of vents. The field winding is placed in slots and held in against centrifugal force by long metal wedges. The first two or three slots on either side of the pole-centre are wedged with steel, the remainder being wedged with bronze to keep down leakage flux. The same rotor completed is shown in fig. 42. The end connections of the windings are held against centrifugal force by steel end-cups screwed on to the hub. The part of the cups next the core and for about 4 inches away from it, is made of bronze to prevent leakage. The cups completely cover the end connections, and fans are attached at both ends for ventilating the end connections of the stator.

The end view of a 4-pole cylindrical rotor for a 25,000-kilowatt 25-cycle alternator running at 750 revolutions per minute, and constructed by Messrs. Parsons & Co., is shown in fig. 43. The axial length over the pole-face is 7 feet 6¼ inches, and the diameter 6 feet 7 inches. The core is made of 32 boiler-plate steel discs separated by ventilating-ducts mounted on a hub or spider, which is forged in one piece with the shaft, and machined out so as to form the axial air-passages. Bronze keys hold the windings against centrifugal force, and the slots are skewed to eliminate harmonics in the E.M.F. wave.

A rotor in process of being wound is shown in fig. 44, in which a section of the winding-department of Messrs. the British Westinghouse Company is illustrated.

Balance.—A turbo-alternator may give trouble by the whipping of the rotor if it is not perfectly balanced. Any rotor will start to whip at a

certain speed—called the "critical" speed—at which the natural frequency of transverse vibration of the rotor considered as a beam is equal to the number of revolutions in the same time. The slightest want of balance is sufficient to produce whipping at this speed, and the longer and more flexible the rotor the lower the speed at which it will occur. Rotors for alternators of large output are therefore often made of solid steel throughout, so that their stiffness may ensure the critical speed being above the normal. In this case the slots must be milled out of the solid. But even with this

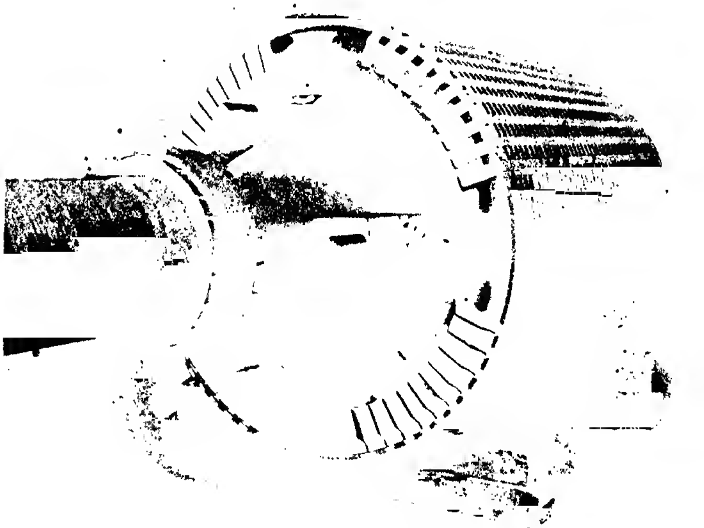


Fig. 43.—Rotor of 25,000 Kilowatt (Parsons & Co.)

construction, according to Behn-Eschenburg, it is not possible to make an alternator of greater output than 5700 K.V.A. at 3000 revolutions per minute, or 19,000 K.V.A. at 1500 revolutions per minute, without the rotor being so long that the critical speed is attained or exceeded at normal speed. It is however possible to work either above or below the critical speed without fear of vibration, and some makers design the rotor so that the critical speed is below the normal, and provide special means for preventing any whipping which might occur in running up to speed or stopping down. This is usually done by allowing the bearing at one end some lateral play all round, so that the rotor can revolve round its true centre of mass.

It is essential that a rotor be properly balanced before being put on

Fig. 44 Winding a Turbo-alternator Rotor, British Westinghouse Company

duty. Static balance is obtained by supporting the two ends of the shaft on horizontal runners or knife-edges, and either adding weights or drilling out metal at suitable places until the rotor shows no tendency to roll one way or the other whatever its position. Static balance does not ensure

"dynamic" balance, or balance under running conditions, unless the material is evenly distributed along the rotor. For example, the cranks of a bicycle are in static balance, but when rotated fast set up a considerable wobble. Further, the centrifugal forces set up on rotation often cause a slight movement of the end-connections of the windings which disturbs the balance. To prevent this as far as possible, the rotor is temporarily completed and run at a moderate speed in order to let the winding pack itself together. The end bells holding in the windings are then taken off and the final covering of tape passed round the end connections.

Dynamic out-of-balance is observed by mounting the rotor in a couple of bearings each of which is mounted on a ball race at right angles to the shaft, and is free to oscillate horizontally between two stop-blocks. The movement of the bearing is controlled by springs or rubber cushions between the bearing and the stop-blocks to prevent shock. The drive is either by a vertical belt or flexible shaft. If the rotor is unbalanced it will oscillate horizontally, and a piece of chalk or pencil brought up to the shaft will mark the side of the rotor to which weight must be added. The proper point is just where the pencil leaves off marking the shaft. If necessary, one end at a time may be balanced by fixing one of the bearings so as to prevent it from oscillating.

The stator of a turbo-alternator is designed on much the same lines as that of a slow-speed alternator, the chief difference being its extreme length in comparison with its diameter, and the large number of ventilating-spaces required for cooling. The frame is so constructed as to guide the cooling air systematically through the rotor and stator. In the stator shown in fig. 40 the direction of the cooling air is shown by arrows.

The cross-section of a 25,000-kilowatt alternator, built by Parsons & Co., is shown in fig. 45. The cast-iron casing is made in four sections. Two sections bolted rigidly together form the bottom and two the top half, and the core is divided along the horizontal centre line so that the top half may be lifted off. The winding is of the type shown in fig. 16, and two of the phases cross the magnetic joint between the top and bottom halves. In the case of the coils of those phases which bridge the joint, each conductor is provided with a separate bolted joint so that the top half of the winding can be easily separated from the bottom. The core is built of stampings arranged in forty-four groups with a ventilating-space adjacent to each, and is 92 inches in length.

It is very essential that the end connections of stator windings in turbo-alternators be strongly clamped, to prevent their being displaced by the stresses set up by the momentary current due to a sudden short circuit or bad synchronizing. The short-circuit current obtained by gradually shorting an alternator is about three to four times full-load current, and is much less than that obtained on suddenly shorting it. In the latter case an initial rush of current up to twenty or more times full-load current may flow. Both these currents may be approximately predetermined from the design of the alternator.

On a short circuit the armature current is equal to the induced E.M.F. ÷ armature impedance. As the armature reactance is much greater than

its resistance, the current will lag nearly 90° behind the induced E.M.F., with the result that the armature ampere turns will almost directly oppose the field ampere turns (see p. 232), but will be slightly less than the latter by a sufficient amount to permit the flux which induces the E.M.F. to pass into the armature.

The gradual short-circuit current of the 250 K.V.A. slow-speed alternator whose saturation curve is plotted on p. 248 can readily be found as follows: The field ampere turns necessary to produce full-load current under short circuit are first found. With full field-strength the short-

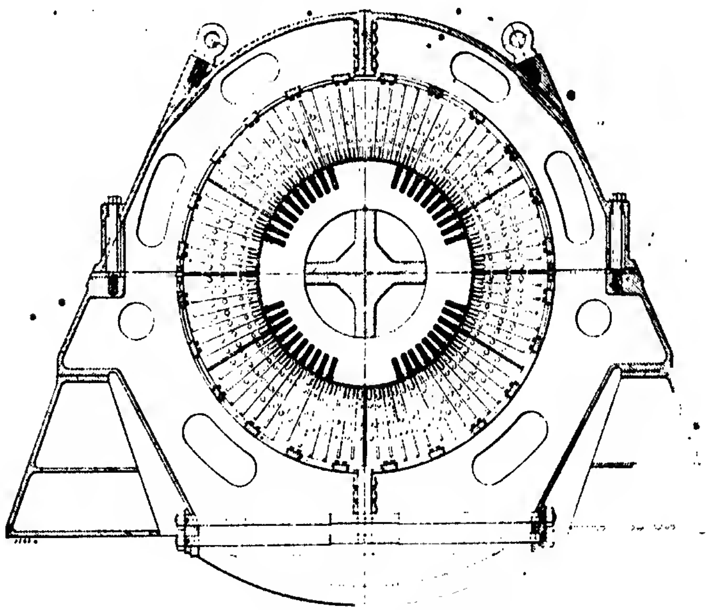


Fig. 45.—Cross-section of 25,000 Kilowatt Alternator (Parsons & Co.)

circuit current is then greater in proportion to the field. A curve showing the relation between the short-circuit current per phase and the field ampere turns is called the short-circuit characteristic, and is approximately a straight line.

In this alternator the impedance voltage is $\sqrt{(2)^2 + (7)^2}$ per cent $= 7.3$ per cent of the main volts, or $\frac{7.3}{100} \times 3810 = 278$ volts per phase. The ampere turns necessary to produce this are found from the saturation curve at the point marked E_z ($= 278$ volts) to be $= 300$ ampere turns per pole. The total armature reaction per pole is 1680 ampere turns. The field has therefore to produce $300 + 1680 = 1980$ ampere turns per pole in order to cause the full-load current per phase I ($= 22$ amperes) to flow in the armature. A line joining the origin to I erected at 1980 ampere turns gives

the short-circuit characteristic, and this produced to meet the vertical from the full-load ampere turns gives the short-circuit current $I_s = 68$ or three times full-load current. The initial current on a sudden short circuit is driven by the total induced E.M.F. E, as some interval of time, however small, is required for the armature reaction to alter the main flux and weaken it down to the normal value under short circuit. The initial short-circuit current per phase would therefore be

$$= \frac{E}{z}, \text{ where } z = \text{impedance per phase.}$$

The current, however, does not rise instantly to this value, on account of armature inductance, and, as demagnetization of the main field is meanwhile taking place, it is somewhat less.

The initial short-circuit current of the alternator can be easily worked out as follows:—

$$\begin{aligned}
\text{Let } I &= \text{full-load current per phase} = 22 \text{ amperes,}\\
I_{sc} &= \text{initial short-circuit current per phase.}\\
\text{Then } Iz &= \text{impedance voltage} = 7.3 \text{ per cent of } 3810\\
&= \frac{7.3}{100} \times 3810.\\
\therefore\ z &= \frac{7.3}{100} \times \frac{3810}{I},\\
\text{and } I_{sc} &= \frac{3810}{z} = \frac{100}{7.3} \times I, \text{ or about fourteen times full-load current.}
\end{aligned}$$

In a turbo-alternator the impedance voltage is even a smaller percentage of the main voltage than with a slow-speed machine, so that the initial current is anything from twenty to thirty times the normal. Now a conductor in which a current is suddenly started tends to repel any solid metal near it due to a secondary current which reacts upon the primary one being induced in the metal in the opposite direction. The end connections of the armature windings will therefore repel the ironwork near them, and the phases will also exert considerable mechanical forces on each other during a short circuit, and considerable displacement and damage might be done were they not thoroughly clamped. Various devices are used for this purpose, according to the style of winding used. For coil windings either glands or clamps, as shown in fig. 49, are used, whilst for barrel windings insulated steel rings are used, to which the end connections are bound by cord both inside and outside, as shown in fig. 20 and fig. 46, which shows the same winding as fig. 20 completed.

The modern tendency is to keep the initial current low by designing turbo-alternator windings with a fairly high inductance at the expense of good regulation, which is unnecessary since the perfection of automatic regulators. With very large generators it is not possible to make the inductance high enough, so that external inductance between the generator and the bus-bars is necessary. This may consist of an ordinary choke

coil with a laminated iron core, but in order that it may choke efficiently on a short, it is essential that the flux density be very low, say 500 lines per cm^2 under normal working, otherwise the large rush of current will

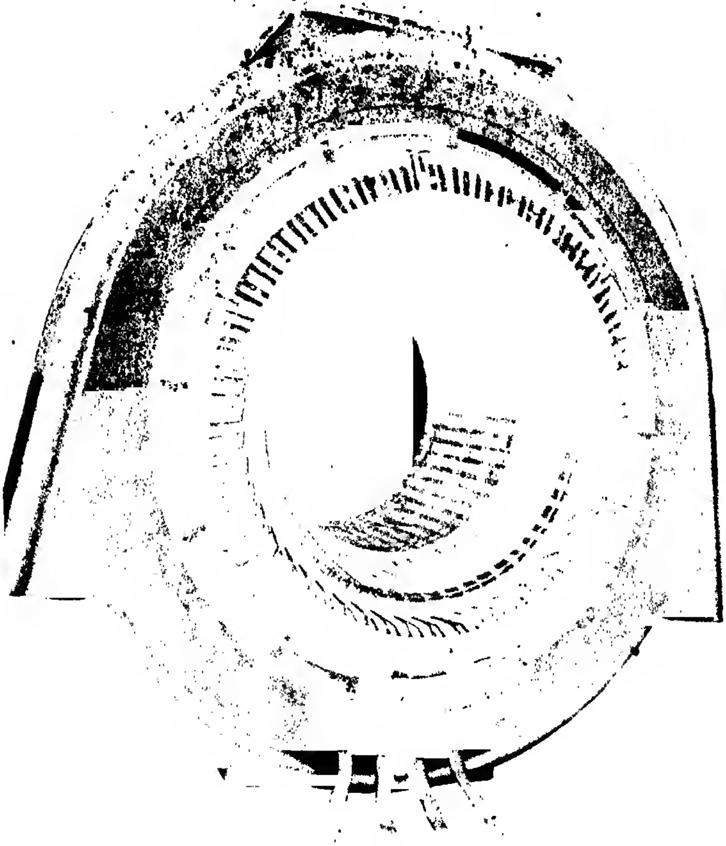


Fig. 4[illegible].—Three-phase Barrel Winding Complete

quickly saturate the iron and greatly reduce the choking effect. In some cases iron is dispensed with altogether, and the coil is wound on a large core of concrete or other non-magnetic substance. The inductance is then constant whatever current passes.

CHAPTER VIII

AUTOMATIC REGULATION

Many ingenious schemes for automatically compensating alternators for voltage drop on load have been devised, amongst which may be mentioned the following:—

1. Compounding the field by means of a current rectified from current and potential transformers in the main circuit, so that the greater the load and lower the power-factor the greater the compensating effect (Heyland and others).

2. Operating on the exciter field by the introduction of alternating current through slip-rings into the armature. The armature reaction so set up is caused to increase the field on load. The exciter must be of the same frequency as the alternator, the field-magnet position adjustable, and the alternating current obtained from the main circuit through current transformers if necessary (E. W. Rice).

3. Utilizing armature reaction to strengthen the field on load. This method is only applicable where the load is practically non-inductive (Miles Walker).

4. Increasing the exciter voltage by decreasing the exciter leakage flux on load. The two limbs of a choke coil bridge the gap between the pole-tips of adjacent N. and S. poles, thus permitting considerable leakage. A current proportional to the main flows round the coil and saturates the limbs of the coil, so increasing the reluctance of the leakage path (Parsons).

Many of these methods have been successfully employed, and are useful in certain circumstances. They are, however, subject to the same defect as direct-current compound generators for constant voltage supply, in that they compensate for load only and take no account of speed variation. Automatic field regulators, on the other hand, tend to maintain a constant voltage whatever the cause of variation. Of these the Tirrell regulator is the most universally employed, due to its rapidity of action and close regulation.

In the simple Tirrell regulator the field rheostat is alternately shorted and thrown into circuit so rapidly that practically a steady average field current is produced. The field current is varied by the rheostat being shorted for a longer or shorter period. When the rheostat is thrown into circuit with the field the generator voltage commences to fall, but before it can fall to any appreciable extent the rheostat is again short-circuited. With the rheostat cut out the volts would rise considerably above normal, but immediately they are above normal the rheostat is again thrown into circuit. In this way the voltage is kept hovering about normal with a variation too small to be noticeable.

With load on, the voltage falls more rapidly when the rheostat is cut in, so that the period during which it is in circuit is shorter. The voltage also takes longer to rise when the rheostat is shorted, so that the time during which it is shorted is greater. The average field current is therefore greater on load. In the case of an alternator the regulator operates on the

field rheostat of the exciter, not the generator, thus reducing the field losses to a minimum.

A diagram of the connections is shown in fig. 47. The exciter control magnet, which is connected across the exciter terminals, consists of a floating solenoid attached to a lever operating the main contacts, the pull of the solenoid being opposed by a spring which tends to keep the main contacts closed. The closing of the contacts operates a relay magnet which short-circuits the field rheostat. The relay consists of a horseshoe electro-magnet, one limb of which is permanently connected across the exciter and the other through the main contacts also across the exciter. The two limbs are wound differentially, so that when the main contacts are closed the relay magnet is demagnetized and the relay contacts are closed by a spring which pulls a lever away from the magnet. This shorts the exciter rheostat and raises the exciter voltage sufficiently to open the main contacts. Magnetism is then re-established in the relay magnet and the relay contacts also open, throwing the rheostat once more in circuit. The relay contacts are shunted with a condenser of suitable capacity to prevent sparking. The main control magnet is connected across the alternator leads through a potential transformer. It consists of a floating solenoid in which the pull on the plunger is independent of its position in the coil, and, as the alternator voltage must remain constant at all loads, the force which balances the pull must be constant. A weight is therefore used on the main control lever instead of a spring. The action of the main control magnet is to shorten or lengthen the time of contact of the main contacts, and thence the relay contacts, according as the voltage is high or low respectively. With a high voltage the plunger is pulled up, the main control lever rotates anticlockwise, so that the upper contact has farther to travel in order to close the relay circuit, and the rheostat remains open-circuited for a longer and longer period. This proceeds until the voltage is again normal. Just the reverse takes place if the main voltage falls below normal, the lower contact rising and the rheostat being shorted for a longer and longer period till the main voltage is normal. The main control plunger is so delicately poised that the slightest alteration of main voltage will start the lever rotating in one direction or the other, and the whole arrangement is capable of maintaining the main voltage within ½ per cent of normal. The main volts may be made to rise when load comes on, so as to compensate for line drop by arranging a current coil on the main control magnet, the current being obtained through a current transformer. Any required amount of compounding may be obtained by moving a switch lever so as to include more or less turns of the current coil.

CHAPTER IX

SYNCHRONIZING AND PARALLEL RUNNING

For the purpose of connecting an alternator in parallel with others it is necessary for the voltage, frequency, and phase of the incoming generator to be the same as that of the bus-bars to which the others are connected. Equality of voltage is readily obtained by adjusting the field regulator when the speed is correct and a slight difference of voltage is not of much consequence. The incoming generator must, however, be in the proper phase relation to the bus-bars, and a synchronizing device or synchroscope is required to indicate the moment at which it is safe to close the main switch between the alternator and the bars. A rotary form of synchroscope is

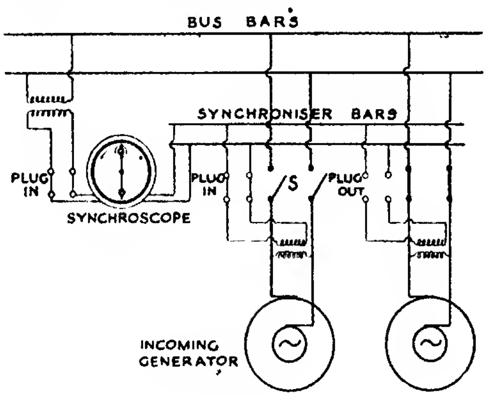


Fig. 48

generally used. As made by Messrs. Everett Edgcumbe this instrument consists of a small two-phase stator, inside which is a rotor also wound two-phase. Both the stator and rotor have a phase-splitting device, consisting of a resistance in series with one phase and an inductance in series with the other, so that when they are connected to a single-phase circuit the current in the inductive phase of the winding lags nearly 90° behind that in the phase connected to the resistance. The device is similar to that used for starting single-phase induction motors, only in this case both the stator and rotor produce rotating fields which move round in the same direction. The stator is connected across a single phase of the incoming generator and the rotor through slip-rings to a pair of bus-bars. It is only necessary to synchronize a single phase of the generator with the bus-bars, as the connections are so arranged during installation that all the phases are in synchronism when one pair is. The general connections are shown in fig. 48. So long as the stator and rotor are supplied with currents at the same frequencies the rotor will remain stationary, but the slightest difference will cause the rotor to move one way or the other, and

a pointer attached to the rotor spindle indicates by its direction of rotation whether the incoming generator is running too fast or slow. A lamp inside the instrument also indicates for distant reading by showing red when the generator is too fast and green when too slow. When the pointer comes into the vertical position the alternator and the bus-bars are in phase and the main switch S may be safely closed. A synchroscope of this description is almost essential with large turbo-generators in order to indicate the exact moment when to close the main switch. Closing the switch at the wrong moment causes a heavy current to flow between the alternator and the bus-bars, which will no doubt pull the alternator into synchronism but will also disturb the whole system and generally act like a momentary short circuit on the bus-bars. Lamps are often used but cannot be relied on for exact synchronizing, whether they are used to indicate synchronism when they are bright or dark. A dead-beat voltmeter may also be used, though it is not so good as the rotary synchroscope, and does not distinguish whether the generator is running fast or slow. Very often both lamps and a voltmeter are installed on the synchronizing panel as a standby in case the rotary synchroscope fails.

Once an alternator is paralleled with the bus-bars it will in ordinary circumstances tend to remain in parallel. We may regard the forces maintaining the rotating-field system in synchronism in the following manner: Suppose the armature were connected to the bus-bars and a laminated iron core substituted for the rotating field-magnets. A magnetizing current would be drawn from the bars, and a rotating armature field produced in the same way as in the stator of an induction motor. If now the field-magnets are substituted for the iron core, and revolved at the same rate as the rotating field, a north pole of the magnets facing a south pole of the armature field, it becomes no longer necessary for the bus-bars to supply magnetizing current, for the field-magnets will themselves *induce* the rotating armature field, a north pole of the field inducing a south pole in the part of the armature facing it, and so on. This corresponds to the alternator synchronized with the bars but unloaded. In order to make the alternator share the load on the bars, it is necessary to adjust the engine governor to supply more steam at the same speed, since the latter is fixed by the bus-bar frequency. Increasing the field-strength will not load up an alternator, as in the case of a direct-current generator, which obtains more driving power by dropping in speed and causing the governor to open the throttle valve. When the alternator field is increased a current is produced by the difference between the alternator induced E.M.F. and the bus-bar volts. This current lags nearly 90° behind the alternator induced E.M.F., due to the inductance of the armature, and is therefore practically wattless. Its effect, therefore, is not to load the alternator, but to react on the field and reduce it to nearly normal. When the field strength is reduced a current opposite to the former flows, or one leading the alternator induced E.M.F. by nearly 90 and so tending to strengthen up the field again. If, however, the driving force supplied to the alternator shaft is increased, so that it tends to run faster than the synchronous speed or speed of the rotating armature field, a load or in-phase current

issues from the alternator which retards it. The load current produces an armature field 90° behind the armature field induced by the poles. The resultant of this field and the one induced by the magnets is a field somewhat behind that of the rotating field-magnets, tending to pull them back. In reality the field-magnets have moved ahead of the synchronous position which is represented by the resultant armature field. In fig. 49 both the armature and rotor fields are depicted as if they were rotating magnets, the position of the rotor field being shown slightly in advance of the synchronous position. The attractive force between the two fields tends to restore the rotor field to the synchronous position. This restoring force at first becomes greater with greater displacement between the magnets, but later reaches a maximum and diminishes as the distance between the magnets weakens their effect upon each other. At this stage the alternator would fall out of step, and cause a bad short circuit on the bus-bars.

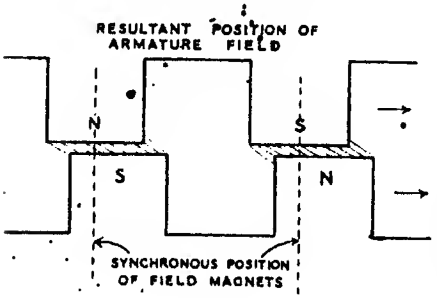


Fig. 49

Evidently it is possible for the rotor to oscillate about the synchronous if jerked in any way, and there will be a natural frequency of oscillation. Should any periodic variation of the engine torque approach this natural frequency, violent oscillations of the rotor may be set up which may jerk the alternator out of step. An unstable governor may produce the same result. To prevent these oscillations, "amortisseur", or damping, coils are fitted to the rotor poles, as described on p. 254, in connection with single-phase alternators. These coils act like the squirrel-cage rotor of an induction motor, and retard by currents induced in them any motion of the rotor poles relative to the armature poles.

Trouble of this nature chiefly occurs in connection with reciprocating engines, and more especially large gas-engines. The driving torque of turbo-alternators is so uniform that it is not generally thought necessary to provide them with damping coils.

INDEX TO VOLUME I

A

B

C

D

E